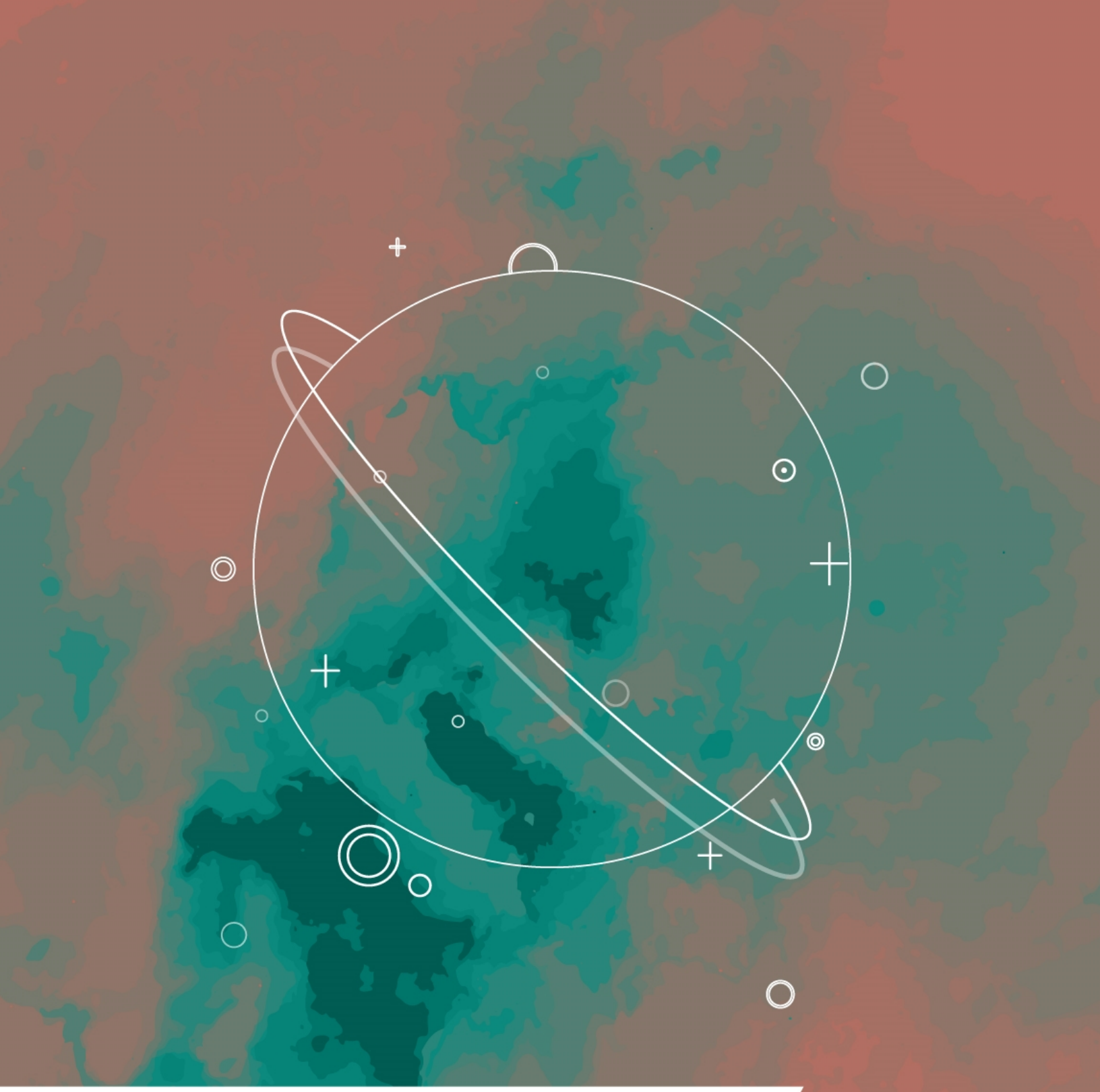

CHAPTER 1

Numbers And Calculation With Numbers

MathU Teaching® Emporium

CONTENTS

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1 NUMBERS AND CALCULATION WITH NUMBERS

Example : Number Formats and Conventions

Study the table below. There are 2 scales in use in the world for indicating large numbers: the "long" scale and the "short" scale. In South Africa we work with both and therefore you have million and billion (two names) on the representation below. South Africa, Canada and Puerto Rico use both scales, because of the fact that English is one of the languages in these countries.

Compare the 2 scales

Long Scale		Short Scale	
Million	6 zeroes	Million	6 zeroes
Milliard	9 zeroes	Billion	9 zeroes
Billion	12 zeroes	Trillion	12 zeroes
Billiard	15 zeroes		
Trillion	18 zeroes		

For Mathematical Literacy we use billion as $1\ 000\ 000\ 000$.

Trillion			**Billion**			**Million**			**Thousands**			**One**		
HT	TT	T	HB	TB	B	HM	TM	M	HT	TT	T	H	T	O

Furthermore:

Study the following number with a decimal fraction:

T	H	T	O	,	t	h	t
3	2	5	4	,	6	7	9

$3 \times 1000 = 3000$
$2 \times 100 = 200$
$5 \times 10 = 50$
$4 \times 1 = 4$
$6 \times \frac{1}{10} = 0{,}6$
$7 \times \frac{1}{100} = 0{,}07$
$9 \times \frac{1}{1000} = 0{,}009$

So we say: 3 thousand 2 hundred 54 fifty four Comma six seven nine

Example : Number Formats and Conventions (continued)

Sometimes one can get confused between the use of a comma and the point. In South Africa the decimal comma separates the whole number from the fraction. In Mathematics the point is used for **multiplication**, e.g. $3\,000\,000{,}453$

Note however that some calculators use a comma to separate the thousands and the point to separate the fractions, e.g. $3{,}000{,}000.453$ while others use spaces e.g. $3\,000\,000.453$; it can also be represented as $3'000'000{,}453$

To indicate an amount of money, separate the Rands from the cents with a comma and use spaces to indicate thousands e.g. R $123\,345{,}45$

1.1 Exercise 1

1. Expand the following number. (Use the example above): $7\ 321{,}146$
2. Write the following numbers in words:
   - 2.1 $289{,}5$
   - 2.2 $4\ 693{,}592$
   - 2.3 $123\ 618\ 698\ 967{,}250$

1.2 Exercise 2

1. Do the following **calculations without** your calculator
   - 1.1 $123{,}456 \times 100 =$
   - 1.2 $20 \div 1\ 000 =$
   - 1.3 $123 \times 10\ 000 =$
   - 1.4 $3\ 023{,}20 \times 1000 =$
   - 1.5 $23{,}33 \div 100 =$
   - 1.6 $10\ 000 \div 100 =$
   - 1.7 $234{,}908 \div 10 =$
   - 1.8 $1\ 200 \div 100 \times 10 =$
   - 1.9 $12\ 009 \div 1000 =$
   - 1.10 $120 \div 40 =$
   - 1.11 $140 \div 10 \times 100 =$

1.12 $0{,}02 \times 0{,}003 =$

1.13 $20 \div 20 =$

1.14 $234\,098 \div 10 =$

1.15 $10^2 =$

1.16 $10^3 =$

1.17 $3\,001{,}1 \div 100 =$

1.18 $10 \times 10 \div 10 =$

1.19 $0{,}012 \div 100 =$

1.20 $17 \div 170 =$

2. One packet of Jelly Beans has 88 Jelly Beans. It weighs 125 g.

   2.1 How many packets will be in a ton?

   2.2 How many Jelly Beans will be in a ton?

   2.3 There are nine different colours in a packet. What is the average number per colour in a packet if the colours are evenly distributed?

   2.4 What is your favourite colour?

   2.5 One packet costs R$12{,}99$. What will you pay for 1 kg Jelly Beans?

   2.6 What will the price be for 1 ton?

   2.7 The profit the factory makes on one packet is R$5{,}49$. What will be the profit in one ton?

3. You buy 6 carry bags every time you go shopping. You shop 3 times a week. You pay 46 cent per bag. What will you spend on carry bags per year?

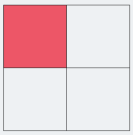
2 FRACTIONS

Definition : Fractions

Fractions are parts of whole numbers. The top number reflects the number of parts you have in comparison with the bottom number. The bottom number reflects how many ***equal*** *parts there are in total.*

Example : Fractions

$\frac{1}{4}$ means one part out of four parts.



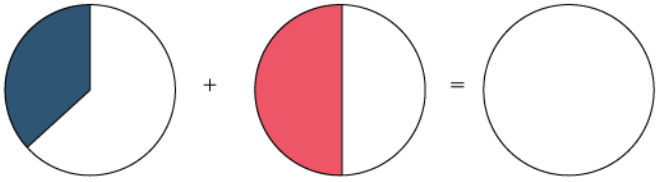
2.1 Exercise 3: Fractions and Integers

1. Bring a Bar One to school. Divide the Bar One into fifths and also share with you friends. Show on the figure below what you have done.

   Write down the fraction of the Bar One that you have eaten.

2. A farmer has a square sized farm and wants to use $\frac{3}{4}$ of the land for grazing. He wants to use $\frac{2}{8}$ of the land to cultivate maize. Calculate the size of each section in m^2 assuming that the size of the farm is 32 hectares.

3. Adding the following fractions with unequal denominators:
   Assume that the figure is a pizza. The coloured part is the part that has been eaten. Colour the last pizza to represent the total part that has been eaten.



   Divide the circles in equal pieces, so that both fractions can be shown. How many equal pieces are there now? Show your calculations.

4. Mixed numbers: (Are made up of a whole number and a fraction)
   For example: $2\frac{1}{5}$ means $2+\frac{1}{5}$ which is the same as $\frac{11}{5}$ (improper fraction) Complete this table:

Mixed Number	Improper Fraction
$3\frac{1}{3}$ →	
$7\frac{1}{2}$ →	
$1\frac{1}{5}$ →	
$3\frac{1}{4}$ →	
←	$\frac{12}{5}$
←	$\frac{185}{19}$
←	$\frac{100}{33}$

5. Do the following without your calculator:

5.1 $\frac{1}{3}+\frac{5}{9}$

5.2 $\frac{3}{4}\div\frac{3}{8}$

5.3 $\frac{3}{9}+\frac{27}{81}$

5.4 $\frac{3}{9}\times\frac{27}{81}$

5.5 $\frac{1}{2}+\frac{2}{3}$

5.6 $\frac{3}{9}-\frac{27}{81}$

5.7 $\frac{1}{2}-\frac{2}{8}$

5.8 $\frac{1}{2}\div\frac{2}{8}\times\frac{3}{6}$

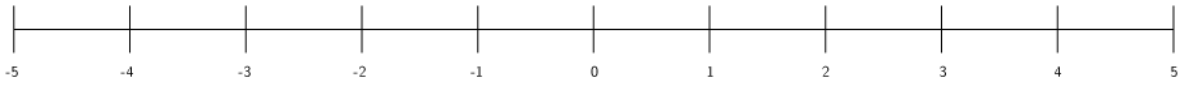
5.9 $\frac{2}{9}+\frac{1}{24}\times\frac{12}{3}$

5.10 $\frac{1}{2}\div\frac{1}{6}\times 2$

3 POSITIVE AND NEGATIVE NUMBERS

3.1 Exercise 4

1. Make use of a number line to do the following without your calculator:



1.1 $-5+3=$

1.2 $-5+5=$

1.3 $-4-1=$

1.4 $0-3=$

1.5 $5-8=$

1.6 $-4+8=$

2. This is a building with a parking lot. Study this picture and answer the following questions.

2.1 Your car is parked at level 7, you are at level 3. How many levels are you from your car?

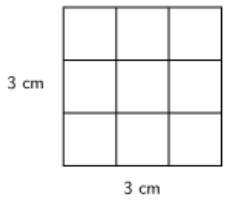
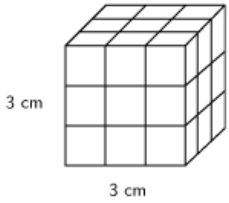
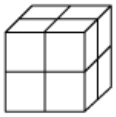
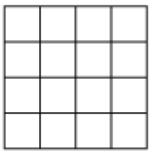
2.2 Your car is parked at level -1 , you are at level 3 . How many levels are you from your car?

2.3 Your car is parked at level 7, you are at level -3. How many levels are you from your car?

4 SQUARE NUMBERS & ROOTS, CUBE NUMBERS & ROOTS

4.1 Exercise 5

1. Complete the following table:

2D Area	3D Volume
 Square: $(3\text{ cm})^2 = 9\text{ cm}^2$ Square root: $\sqrt{9\text{ cm}^2}$	 Cube: $(3\text{ cm})^3 = 27\text{ cm}^3$ Cube root: $\sqrt[3]{27\text{ cm}^3}$
 Square: _____ Square root: _____	Cube: _____ Cube root: _____
Square: _____ Square root: _____	 Cube: _____ Cube root: _____
 Square: _____ Square root: _____	Cube: _____ Cube root: _____

2. Make use of your calculator to do the following: [Always try to estimate the answer first.] (Round off to two decimal places where necessary.)

2.1 $\sqrt{144} =$

2.2 $\sqrt{22} =$

2.3 $\sqrt[3]{8} =$

2.4 $\sqrt{9} =$

2.5 $\sqrt{\frac{16}{4}} =$

2.6 $\sqrt[3]{\frac{27}{8}} =$

2.7 $\sqrt{15} =$

2.8 $\sqrt{32-4} =$

2.9 $\sqrt{9+16} =$

2.10 $\sqrt{9}+\sqrt{16} =$

2.11 $5^2+10^3 =$

2.12 $\sqrt[3]{8}+\sqrt{169} =$

2.13 $10^1+10^3 =$

2.14 $(\sqrt{2})^2 =$

2.15 $\sqrt{2^2} =$

2.16 $\sqrt{2}\times\sqrt{2} =$

2.17 $10\times10^2 =$

2.18 $2^3+3^2 =$

2.19 $0{,}1^3 =$

2.20 $1^3 =$

5 MATHEMATICAL LANGUAGE

5.1 Exercise 6: Mathematical Language

1. Complete the following table:

Word	Sign	Word	Sign	Word	Sign
product		minus		times	
difference		divide		total	

2. Give solutions to the following problems:

   2.1 What is the difference in time between US and South Africa in our summer?

   2.2 You spend R$2{,}34$ and R$3{,}78$ in a tuck shop. What is the total amount that you have spent?

   2.3 What is the difference in time between UK and South Africa in our summer?

   2.4 Subtract 2 from 1.

3. Insert the correct unit:

   3.1 Usain Bolt ran the 100 __ in $9{,}58$ __. Justin Gatlin broke Usain's 100 __ record with $0{,}13$ __ His average speed over the 100 __ is __.

   3.2 The road sign indicates that the distance to Pretoria is 50 __.

   3.3 The weight of the cat is 20 __.

   3.4 The length of my bedroom is 450 __.

   3.5 The area of my bedroom is 12 __.

   3.6 The swimming pool takes 27 __ of water.

   3.7 Every 7 __ an aeroplane departs from Cape Town International Airport.

   3.8 The age of the baby is 9 __.

   3.9 I bought 10 __ lace to sew around my table cloth.

   3.10 My dad bought 21 __ of skirting for our new room in our house.

   3.11 The temperature in Michigan in the USA is $-45°$ __ in the winter.

   3.12 I baked the cake at $180°$ __ in the oven. (South Africa)

   3.13 The restaurant owner calculates the meat portion on 150 __ per person.

   3.14 The volume of the cylinder is 10 __.

   3.15 The capacity of the same cylinder is __.

   3.16 The petrol consumption of the car is 6 __ per 100 __.

   3.17 The BMI of the man is 25 __.

   3.18 The average speed of the car was 120 __.

   3.19 The area of the house is 250 __.

   3.20 There is still 500 __ data remaining on my cell phone.

6 OPERATIONS USING NUMBERS AND CALCULATOR SKILLS

Order of Operations using BODMAS rule:

BRACKETS
SQUARE NUMBERS AND ROOTS
OF
DIVISION AND **M**ULTIPLICATION
ADD AND **S**UBTRACT
Example:

$2 \times 3 + 4 \div 2 + (9 - 1) - \frac{1}{2}$ of 8	First the brackets
$= 2 \times 3 + 4 \div 2 + 8 - \frac{1}{2}$ of 8	of (this is multiplication)
$= 2 \times 3 + 4 \div 2 + 8 - 4$	then multiply and divide from left to right
$= 2 \times 3 + 2 + 8 - 4$	then add and subtract from left to right
$= \underline{12}$	

6.1 Exercise 7

1. $3 \times 7 - 11 \div 2 \times 6 + 1 =$
2. $58 \div 2 + 2 \times 4 - \frac{2}{3}$ of $30 =$
3. $2(2-3)^2 - 6 \div 2 =$
4. $\frac{\sqrt{160-16}}{12} - 32 \div 8 =$
5. $6 \times 8 \div 2 + 3 =$
6. $983,5 - 100 - 10 =$
7. $250 - 25 \times 4 + 100 =$
8. $\frac{3}{5}$ of $205 =$
9. $280 + 24,8 \times 20 \div 2 =$
10. $\frac{2}{3}$ of 120 km $+ 7$ km $=$
11. $\frac{2}{5}\left(1\frac{4}{9}\right) =$

12. $\frac{1}{2} + \frac{2}{3} \times \frac{9}{6} - \frac{1}{4} =$
13. $1 \div 1 \times 1 + 1 =$
14. $17 + 3 \times 2 - 1 =$
15. $325 - 36 \div 3 + 100 =$
16. $(5-4)^2 - \sqrt[3]{27} \times 4 =$
17. $\text{R}450 - \text{R}32,50 \times 10 =$
18. $2\frac{2}{3} + 6\frac{5}{6} =$
19. $\frac{3}{4}(7-2) + 6 =$
20. $\frac{1}{2} \times \frac{1}{3}(36 \div 6) + 3 =$

Take note: When one determines the median of an even number of data, be careful that you consider the order of operations. E.g. If the data is: $1; 3; 4; 5$, the middle value between 3 and 4 is not, $3+4 \div 2$ but rather $(3+4) \div 2$!

6.2 Exercise 8: Estimation

Fill in the following table.

Situation	Value
What is the diameter of a R5 coin in mm?	
What is the perimeter of a R10 in mm?	
How thick is a R5 coin in mm?	
What is the average circumference of a male fist in inches?	
What is the longest whistle time in hours?	
How many letters in the alphabet are there?	
What was the male to female ratio for South Africa, in 2020?	
What is the highest recorded temperature in South Africa in $^\circ$C?	
What is the largest human foot size in cm?	
What is the average female hand length in inches?	
In South Africa, what is the recommended amount of students in a class per teacher?	

7 ROUNDING

7.1 Exercise 9: Rounding off/Rounding up

1. Round off to two decimal places:
   E.g $354{,}7899 = 354{,}79$ **but** $354{,}7824 = 354{,}78$

(i)	$31{,}2536 \approx$	(xi)	$321{,}456317 \approx$
(ii)	$46{,}9859 \approx$	(xii)	$10{,}00023 \approx$
(iii)	$52{,}9092 \approx$	(xiii)	$59{,}995 \approx$
(iv)	$20{,}1073 \approx$	(xiv)	$99{,}99142 \approx$
(v)	$32{,}9991 \approx$	(xv)	$942{,}74321 \approx$
(vi)	$0{,}003 \approx$	(xvi)	$0{,}005 \approx$
(vii)	$2{,}3546 \approx$	(xvii)	$34{,}908 \approx$
(viii)	$9{,}0899 \approx$	(xvii)	$56{,}078 \approx$
(ix)	$76{,}0782 \approx$	(xix)	$7{,}0989 \approx$
(x)	$4{,}0125 \approx$	(xx)	$2{,}321 \approx$

2. Round off to the nearest 10:
   E.g $34{,}56$ = 30 **but** $35{,}56$ = 40

(i)	$3{,}2 \approx$	(xi)	$879 \approx$
(ii)	$45{,}12 \approx$	(xii)	$914 \approx$
(iii)	$123{,}4 \approx$	(xiii)	$915 \approx$
(iv)	$412 \approx$	(xiv)	$999 \approx$
(v)	$5{,}9 \approx$	(xv)	$145{,}89 \approx$
(vi)	$6{,}8 \approx$	(xvi)	$45{,}881 \approx$
(vii)	$9149 \approx$	(xvii)	$657 \approx$
(viii)	$1002 \approx$	(xvii)	$732 \approx$
(ix)	$435 \approx$	(xix)	$106 \approx$
(x)	$299 \approx$	(xx)	$123 \approx$

3. Round off the the nearest 5:

   Use this line to see if the digit is closer to $0{,}5$ or 10 (Only consider the one digit!)

(i)	$1 \approx$	(xi)	$124 \approx$
(ii)	$2 \approx$	(xii)	$65 \approx$
(iii)	$3 \approx$	(xiii)	$71 \approx$
(iv)	$4 \approx$	(xiv)	$10 \approx$
(v)	$5 \approx$	(xv)	$91 \approx$
(vi)	$6 \approx$	(xvi)	$93 \approx$
(vii)	$7 \approx$	(xvii)	$99 \approx$
(viii)	$8 \approx$	(xvii)	$98 \approx$
(ix)	$9 \approx$	(xix)	$33 \approx$
(x)	$11 \approx$	(xx)	$79 \approx$

4. Round off to the nearest cent:

   This is the same as rounding to two decimals.

   E.g $12{,}234 = 12{,}23$ but $132{,}2355 = 132{,}24$

(i)	R$12{,}234 \approx$	(xi)	R$10{,}016 \approx$
(ii)	R$190{,}1254 \approx$	(xii)	R$1{,}712 \approx$
(iii)	R$3{,}534 \approx$	(xiii)	R$80{,}089 \approx$
(iv)	R$2{,}989 \approx$	(xiv)	R$14{,}012 \approx$
(v)	R$5{,}999 \approx$	(xv)	R$4{,}129 \approx$
(vi)	R$1{,}095 \approx$	(xvi)	R$90{,}994 \approx$
(vii)	R$12{,}081 \approx$	(xvii)	R$7{,}0139 \approx$
(viii)	R$41{,}890 \approx$	(xvii)	R$5{,}982 \approx$
(ix)	R$4{,}089 \approx$	(xix)	R$99{,}998 \approx$
(x)	R$9{,}0129 \approx$	(xx)	R$19{,}995 \approx$

5. Round off to the nearest rand:

   E.g $142{,}50 = 143$ **but** $43{,}49 = 43$

(i)	R$143{,}76 \approx$	(xi)	R$3{,}76 \approx$
(ii)	R$2{,}45 \approx$	(xii)	R$1{,}23 \approx$
(iii)	R$4{,}87 \approx$	(xiii)	R$3{,}45 \approx$
(iv)	R$3{,}67 \approx$	(xiv)	R$12{,}50 \approx$
(v)	R$13{,}44 \approx$	(xv)	R$9{,}39 \approx$
(vi)	R$54{,}9805 \approx$	(xvi)	R$12{,}98 \approx$
(vii)	R$12{,}987 \approx$	(xvii)	R$3{,}19 \approx$
(viii)	R$90{,}765 \approx$	(xvii)	R$56{,}54 \approx$
(ix)	R$13{,}655 \approx$	(xix)	R$4{,}45 \approx$
(x)	R$1{,}91 \approx$	(xx)	R$6{,}54 \approx$

6. Round off to the nearest integer:

   E.g $12{,}54 = 13$ **but** $45{,}467 = 45$

(i)	$43{,}5 \approx$	(vi)	$8{,}09 \approx$
(ii)	$89{,}35 \approx$	(vii)	$88{,}45 \approx$
(iii)	$45{,}67 \approx$	(viii)	$14{,}54 \approx$
(iv)	$35{,}12 \approx$	(ix)	$90{,}15 \approx$
(v)	$87{,}94 \approx$	(x)	$89{,}51 \approx$

7. Round up/down:

   Complete the table:

You must organize accommodation for $112{,}4$ people. For how many people would you prepare the accommodation?	
You want $116{,}23$ m^2 to be tiled. For how many square meters would you order tiles?	
You need to buy paint and according to your calculations, you need $4{,}4$ tins of paint. How many tins would you buy?	
You need $1{,}3$ kg rice for a function. How many kilograms would you buy?	
The temperature is $6°$C, round it to nearest 5 degrees.	

8 RATIOS

8.1 Exercise 10: Ratios

1. You bought Oros Squash. It must be diluted in a ratio of $1:4$

   1.1 You have 500 mℓ squash; write down the ratio in ml to mix it.

   1.2 Which fraction of the cold drink is water?

   1.3 You want to mix 1ℓ $(1000$ m$\ell)$ of Oros cold drink. How much squash do you need to pour into your jar to make it drinkable?

2. Write the following ratios in their simplest form in the space provided:

70c to R1	
300 mℓ to 2 litres	
3 kg to 750 g	
90 min to 2 hours	
6 m to 28 m	
4 cm to 40 mm	
18 hours to 1 day	
2 months to 2 years	
$0{,}54$ to $1{,}2$	
$2\frac{1}{2}$ to $1\frac{2}{3}$	
R$4{,}20$ to 30c	
30 min : 2 hours	
4c : R$3{,}24$	

3. Divide in the given ratio.

   E.g. Divide 36 in the ratio $1:5$.

   $1+5=6$

   $\therefore \frac{1}{6}$ of $\frac{36}{1}=\frac{1}{6}\times\frac{36}{1}=\frac{1}{6}\times\frac{36}{1}=1\times 6=$ **6**

   and $\frac{5}{6}$ of $\frac{36}{1}=\frac{5}{6}\times\frac{36}{1}=\frac{5}{6}\times\frac{36}{1}=5\times 6=$ **30**

Divide the following in the given ratio:

72 in $1:3$	
450 in $2:3$	
$2{,}800$ in $2:5$	
896 in $2:5:1$	
$14{,}4$ in $3:4:5$	
240 in $2:3:3$	
48 in $1:2:3$	
$15{,}5$ in $2:3$	
150 in $3:7$	

4. What will the length of each piece of a rope be, if 63 m rope is divided in the ratio $1:2:4$?
5. The ratio of the girls to the boys in a certain school is $3:5$. If the school has a total of 528 learners, calculate the number of boys and the number of girls.
6. Divide R1200 between Ross and Frank in such a way that Frank will have three times as much as Ross.
7. I need $25\,\ell$ of paint to paint a roof. White and yellow paint must be mixed in the ratio $2:3$. How many litres of white paint will I need?
8. Sue plans to jog 35 km in two sessions. The distances in the sessions have to be in the ratio $3:2$. How far will she jog in the first session?
9. Divide R342 between A, B and C in the ratio $4:3:5$. What amount will be allocated to B?
10. Divide R$5{,}50$ in the ratio $\frac{1}{2}:\frac{1}{3}$
11. Divide 8 in the ratio $0{,}4:0{,}1$
12. Concrete for a certain job requires sand and cement in the ratio $7:2$. If 6 m^3 of cement are used, calculate how many m^3 of sand is required.
13. Two women bought a tray of apples. The tray contained 105 apples and A paid R15 and B paid R20. How many apples did each receive?
14. If the cost of 300 mℓ oil is R$1{,}23$, calculate how much you will pay for 200 mℓ and 700 mℓ oil respectively.
15. A father and his son went to a rugby game. The father's ticket cost four times as much as the son's ticket. Together they paid R125. What did the son pay for his ticket?
16. A certain dog food manufacturer recommends that a 14 kg dog must be given two of the included cups per day. How many cups per day will be given to a dog of 21 kg ?

17. $\frac{2}{7}$ of the learners of a certain school are in grade 8. The ratio between boys and girls in grade 8 is $3:2$. The age of the girls in grade 8 is either 13 or 14 or 15 years. The ratio of the number of girls in each age group is $1:10:1$ in the same order as above. If the school has 630 learners in the total, determine how many 14 year old grade 8 girls are in the school.

9 PROPORTION

9.1 Exercise 11: Proportion & Applications

1. Complete the following table:

Number of pens	1	3	5		10		25	100		120
Total cost		R12		R32		R84	R100		R448	

2. Complete the following tables:

   2.1 Two colors of paint are mixed at a specific ratio, as seen in the following table:

Green paint (litre):	2	4	6	8	10			26	30	
Yellow paint (litre):	3			12		24	33			300

   2.2 A car drives at a constant speed and the distance covered is presented in the table:

Number of hours driven:	1	2	3	6	7		10			
Km covered		160	240			680		880	1 640	2 400

   2.3 A recipe requires the following ratio:

Flour (g) per mix:	50			450	500	1 000	2 500	5 000	6 500	
Milk (ml) per mix:		60	120		300			3 000		4 200

3. At the recent Common Wealth Games South Africa won 42 medals. The medals were won in the following ratio: gold : silver : bronze $= 1:2:4$.
   Calculate how many silver medals they won.

4. In an exhibition hall there are 16 new cars. There are 8 white ones, 2 red ones and the rest are metallic. Calculate the ratio between:

   4.1 the white cars to the total number of cars.

   4.2 the red cars to the metallic cars.

5. A rope that is 12 cm long, must be divided in the ratio $5:7$. How many mm will the shortest part of the divided rope be?

6. A chocolate bar contains 21 blocks. I eat 14 blocks. Express the left over chocolate in ratio to the total.

7. On a bottle of cool drink concentrate it indicates that the cool drink should be diluted in the ratio $1:4$. If I need 12 liters of cool drink, how many ml of the concentrate do I need?

8. To pass a test (total 30 marks), you are required to get at least 12. The test's marks are adjusted and now the total is only 25 marks. How much do you need to pass now?

9. In a packet of 350 smarties there are the following colours: 75 black, 125 red, 70 yellow, 18 blue, 12 green and the rest are orange. Express the following as a ratio in the simplest form:

   9.1 black smarties : all smarties

   9.2 orange smarties : all smarties

   9.3 blue smarties : green smarties

   9.4 red smarties : rest of the smarties

10 RATES

Example : Rates

SPEED, DISTANCE AND TIME

$$\textbf{Time} = \frac{\textbf{Distance}}{\textbf{speed}}$$

$$\textbf{Speed} = \frac{\textbf{Distance}}{\textbf{Time}}$$

$$\textbf{Distance} = \textbf{Time} \times \textbf{Speed}$$

10.1 Exercise 12

1. Complete the following table:

Time	Speed	Distance
45 minutes		80 km
	135 km/h	455 km
20 minutes	90 km/h	
$1\frac{1}{2}$ hours	15 m/s	
1 hour 15 minutes		2 m
	20 m/s	40 m

2. Complete the following table: (This is a representation of different types of questions)

Exchange foreign currency to Rand e.g. 12 BRR to Rand	
Given: Rand as unit rate R : BRR **1 : 2**	
Here one must ÷	12 BRR =
Gibe : BRR as foreign currency BRR : R **1 : 0,5**	
Here one must x	12 BRR =
Exchange Rand to foreign currency E.g. 24 Rand to BRR	
Gibe : Rand as unit rate R : BRR **1 : 2**	
Here one must x	R24 =
Gibe : BRR as unit rate BRR : R **1 : 0,5**	
Here one must ÷	R24 =

3. The exchange rate between the South African Rand and the American Dollar is R$9{,}64/\$1$

   3.1 How many $\$$ can be exchanged for R$136{,}12$?

   3.2 You have $\$220$. How many Rand do you have?

4. One British Pound (£) currently costs R$14{,}60$. Calculate the following:

   (a) How many Rand you need to order a book that costs £30?

   (b) You want to travel in Britain. The travel agent's quote is £$1554{,}25$. How many rand is this?

   (c) You saved R$17\ 000$ for spending money. How many Pound is it?

11 PERCENTAGES

Example : Percentages

The term percentage is derived from the Latin words "per centum" meaning per hundred. Therefore percentage is a fraction of which the denominator is always 100.
So $\frac{87}{100}$ will mean: 87 hundredths or 87% or therefore 7% is also written as $\frac{7}{100}$

11.1 Excersice 13: Percentages

1. Express each of the following as a percentage

Common Fraction	Percentage
$\frac{18}{25}$	
$\frac{3}{20}$	
$\frac{27}{50}$	
$\frac{2}{5}$	
$\frac{9}{10}$	
$1\frac{12}{25}$	
$\frac{16}{20}$	
$\frac{1}{10}$	

2. Write the following as a common fraction in the simplest form and then express your answer as a decimal fraction:

Percentage	Common Fraction	Decimal Fraction
19%		
150%		
66%		
25%		
120%		
4%		
18%		
45%		
380%		

3. Study the following examples:

EXAMPLE

30% of 6,6 litre. How many mℓ is it?

$\therefore$ 30% of 6,6 litre

$= 6{,}6 \times 30\%$

$= \underline{1{,}98 \text{ litre}}$

$= \underline{1\,980 \text{ m}\ell}$

$27\frac{1}{2}$% of 32,5 kg. How many gram is it?

$\therefore 27\frac{1}{2}$ of 32,5 kg

$=32{,}5 \text{ kg} \times 27\frac{1}{2}\%$

$=\underline{8{,}9375 \text{ kg}}$

$=\underline{8937{,}5 \text{ g}}$

Complete the following table:

25% of 120	
75% of R100	Express answer in cent
15% of 20 litre	Express your answer in mℓ
0,5% of 8 cm	Express your answer in mm
10% of 55 kg	Express your answer in g
120% of 5 ton	Express answer in kg
33% of R33	Express answer in cent
12% of 6 hours	Express answer in minutes
11,25% of R3 500	Express answer in Rand
250% of 600 ml	Express your answer in ml
90% of 40 eggs	How many dozen is it?
99% of 188 kℓ	How many litres is it?

4. Study the following examples:

EXAMPLE

Use a calculator and express 840 mℓ as a percentage of 3 litres.

First write both terms in the same unit: 3 litres $= 3\,000$ ml

Express the two amounts as a fraction: $\frac{840}{3\,000}$

For percentage, multiply by 100 : $\frac{840}{3\,000} \times 100$

Use a calculator and calculate: $= \underline{28\%}$

(Keys: $840 \div 3\,000 = \times 100 =$)

Use a calculator and express the first amount as a percentage of the second amount

400 g of 2 kg	
63 minutes of $3\frac{1}{2}$ hours	
$3{,}5$ litre of 28 litre	
145 mm of 1 metre	
$42c$ of R36	
$2{,}5$ litre of 1250 mℓ	

5. Study the following examples

EXAMPLE

Increase or **decrease** by a certain percentage

Increase R$6\,000$ by 30%

With calculator: 30% of R$6\,000 =$ R$1\,800$

$\therefore$ Increased amount $=$ R$6\,000 +$ R$1\,800 =$ R$7\,800$

Or Increased amount: original amount $+30\% = 130\%$ of R$6\,000$

$$= \frac{130}{100} \times \frac{6\,000}{1}$$
$$= \frac{130}{100} \times \frac{6\,000}{1}$$
$$= 130 \times 60 = \underline{\text{R}7\,800}$$

Decrease 75 kg by 15%

With calculator: 15% of 75 kg $= 11,25$ kg

$\therefore$ Decreased weight $= 75$ kg $- 11,25$ kg $= 63,75$ kg

Or Decreased weight is: original weight $-15\% = (100\% - 15\%)$ of 75 kg

$$= 85\% \text{ of } 75 \text{ kg}$$
$$= \frac{85}{100} \times \frac{75}{l}$$
$$= \underline{63,75 \text{ kg}}$$

Complete the table:

350 cm is increased by 20%	By how many cm is it increased?
R36 is decreased by 10%	By how many Rand is it decreased?
Decrease 120 minutes by 25%	By how many seconds is it decreased?
Increase R150 by $0{,}2\%$	By how many Rand is it increased?

6. Study the following example:

EXAMPLE

If the price of a certain item increases from R$4{,}50$ to R$5{,}00$, calculate the percentage increase of this item.

Increase $=$ R$5{,}00 -$ R$4{,}50 =$ R$0{,}50$ or $500\text{c} - 450\text{c} = 50\text{c}$

$$\frac{50}{450} \times \frac{100}{1} = \underline{\textbf{11,1 \%}} \quad [\text{Keys: } 50 \div 450 = \times 100 =]$$

OR: $\frac{\text{Selling price-cost price}}{\text{Cost price}} \times 100 = \frac{500-450}{450} \times 100 = 11{,}1\%$

Complete the following table: Calculate the percentage increase if:

25 kg increases with 5 kg	
$12{,}5$ litres increases with $2{,}5$ litres	
300 metres decreases by 30 cm	
R$2\,800$ decreases to R$2\,100$	

Applications - general:

7. A group of 240 people were questioned about the use of the cell phone. 40% of them are Vodacom users, 35% are MTN users and the rest are Cell C clients. Calculate the number of users for each cell phone company for this group of people.

8. During an extensive drought a farmer decides to decrease his herd of cattle with 30%, as his grazing has decreased. If he originally had 640 cattle, calculate how many cattle were left on the farm after the decrease.

9. In ABBA Primary's athletics team there are 54 girls and 90 boys. Calculate the percentage of girls and the percentage of boys in the school's athletics team.

10. During the musical, presented by HS Musica, 90% of the school's learners take part. Of the learners taking part, 60% are girls. Of the girls who are taking part, $12,5\%$ are in grade 8 . There are 800 learners in the school.

    10.1 How many of the school's learners are taking part in the musical?

    10.2 How many of the learners taking part in the musical are girls?

    10.3 How many of the girls taking part in the musical are in grade 8?

11. The human body consists of 206 bones. 25 of these bones are ribs. What percentage of your skeleton are ribs? (Round off to 2 decimal places.)

12. A lady lost weight. She weighed 75 kg, but now she weighs 63 kg. What percentage of weight did she lose?

13. An elastic is stretched from $4{,}5$ cm to $5{,}75$ cm. By what percentage is the length increased?

14. Mr Piet Botha bought 30 sheep for R900 each. He sold 20 sheep for R$1\ 000$ each and the other ones for R$1\ 100$ each. What is the percentage of his total profit?

12 ANSWERS FOR EXERCISES

12.1 Exercise 1

1. $7 \times 1000 + 3 \times 100 + 2 \times 10 + 1 \times 1 + 1 \times 0{,}1 + 4 \times 0{,}01 + 6 \times 0{,}001$

2.1 Two hundred and eighty-nine comma five

2.2 Four thousand six hundred and ninety-three comma five nine two.

2.3 One hundred and twenty-three billion, six hundred and eighteen million, six hundred and ninety- eight thousand, nine hundred and sixty-seven comma two five zero

12.2 Exercise 2

1.1 $12345{,}6$

1.2 $0{,}02$

1.3 $1\ 230\ 000$

1.4 $3\ 023\ 200$

1.5 $0{,}2333$

1.6 100

1.7 $23{,}4908$

1.8 120

1.9 $12{,}009$

1.10 3

1.11 $1\ 400$

1.12 $0{,}00006$

1.13 1

1.14 $23\ 409{,}8$

1.15 100

1.16 $1\ 000$

1.17 $30{,}011$

1.18 10

1.19 $0{,}00012$

1.20 $0{,}1$

2.1 $8\ 000$ packets

2.2 $704\ 000$ jelly beans

2.3 10 per colour

2.4 Any colour

2.5 R$103{,}92$

2.6 R$103\ 920{,}00$

2.7 R$43\ 920{,}00$ profit per ton

3 R$430{,}56$ per year

12.3 Exercise 3

1. $\frac{1}{5}$
2. $80\ 000\text{m}^2$ for maize cultivation and $240\ 000\text{m}^2$ for grazing
3. $\frac{5}{6}$
4. Completed table:

Mixed Number	Improper Fraction
$3\frac{1}{3}$	$\frac{10}{3}$
$7\frac{1}{2}$	$\frac{15}{2}$
$1\frac{1}{5}$	$\frac{6}{5}$
$3\frac{1}{4}$	$\frac{13}{4}$
$2\frac{2}{5}$	$\frac{12}{5}$
$9\frac{14}{19}$	$\frac{185}{19}$
$3\frac{1}{33}$	$\frac{100}{33}$

5.1 $\frac{8}{9}$

5.2 2

5.3 $\frac{2}{3}$

5.4 $\frac{1}{9}$

5.5 $1\frac{1}{6}$

5.6 0

5.7 $\frac{1}{4}$

5.8 1

5.9 $\frac{7}{18}$

5.10 6

12.4 Exercise 4

1.1 -2

1.2 0

1.3 -5

1.4 -3

1.5 -3

1.6 4

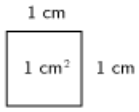
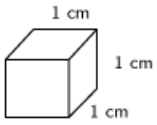
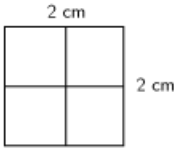
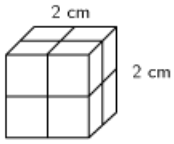
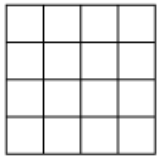
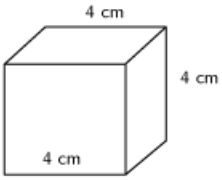
2.1 4 levels

2.2 4 levels

2.3 10 levels

12.5 Exercise 5

1.

 Square: $(1\text{ cm})^2 = 1\text{ cm}^2$ Square root: $\sqrt{1\text{ cm}^2} = 1\text{ cm}$	 Cube: $(1\text{ cm})^3 = 1\text{ cm}^3$ Cube root: $\sqrt[3]{1\text{ cm}^3} = 1\text{ cm}$
 Square: $(2\text{ cm})^2 = 4\text{ cm}^2$ Square root: $\sqrt{4\text{ cm}^2} = 2\text{ cm}$	 Cube: $(2\text{ cm})^3 = 8\text{ cm}^3$ Cube root: $\sqrt[3]{8\text{ cm}^3} = 2\text{ cm}$
 Square: $(4\text{ cm})^2 = 16\text{ cm}^2$ Square root: $\sqrt{16\text{ cm}^2} = 4\text{ cm}$	 Cube: $(4\text{ cm})^3 = 64\text{ cm}^3$ Cube root: $\sqrt[3]{64\text{ cm}^3} = 4\text{ cm}$

2.1 12

2.2 4,69

2.3 2

2.4 3

2.5 2

2.6 1,5

2.7 3,87

2.8 5,29

2.9 5

2.10 7

2.11 $1\,025$

2.12 15

2.13 $1\,010$

2.14 2

2.15 2

2.16 2

2.17 $1\,000$

2.18 17

2.19 $0{,}001$

2.20 1

12.6 Exercise 6

1. Completed table:

Word	Sign	Word	Sign	Word	Sign
product	$\times$	minus	$-$	times	$\times$
difference	$-$	divide	$\div$	total	$+$

2.1 7 hours

2.2 R$6{,}12$

2.3 2 hours

2.4 -1

3.1 Usain Bolt ran the 100 m in $9{,}58$ s. Justin Gatlin broke Usain's 100 m record with $0{,}13$ s. His average speed over the 100 m is $10{,}582$ m/s.

3.2 The road sign indicates that the distance to Pretoria is 50 km.

3.3 The weight of the cat is 20 kg.

3.4 The length of my bedroom is 450 cm.

3.5 The area of my bedroom is 12 m^2

3.6 The swimming pool takes 27 kl of water.

3.7 Every 7 min an aeroplane departs from Cape Town International Airport.

3.8 The age of the baby is 9 months.

3.9 I bought 10 m lace to ew around my table cloth.

3.10 My dad bought 21 m of skirting for our new room in our house.

3.11 The temperature in Michigan in the USA is $-45 \circ$ F in the winter.

3.12 I baked the cake at $180 \circ$ C in the oven. (South Africa)

3.13 The restaurant owner calculates the meat portion on 150 g per person.

3.14 The volume of the cylinder is 10 cm^3.

3.15 The capacity of the same cylinder is 10 ml.

3.16 The petrol consumption of the car is 6 l per 100km.

3.17 The BMI of the man is 25 kg/m^2.

3.18 The average speed of the car was 120 km/h.

3.19 The area of the house is 250 m^2.

3.20 There is still 500 Megabyte data remaining on my cell phone.

12.7 Exercise 7

1. -11
2. 17
3. -1
4. -3
5. 27
6. $873{,}5$
7. 250
8. 123
9. 528
10. 87 km

11. $\frac{26}{45}$
12. $1\frac{1}{4}$
13. 2
14. 22
15. 413
16. -11
17. R125
18. $9\frac{1}{2}$
19. $9\frac{3}{4}$
20. 4

12.8 Exercise 8

Filled table is given below.

Situation	Value
What is the diameter of a R5 coin in mm?	26
What is the perimeter of a R10 in mm?	204
How thick is a R5 coin in mm?	3
What is the average circumference of a male fist in inches?	$8,6$
What is the longest whistle time in hours?	$25,5$
How many letters in the alphabet are there?	26
What was the male to female ratio for South Africa, in 2020?	$0,9709$
What is the highest recorded temperature in South Africa in °C?	50
What is the largest human foot size in cm?	47
What is the average female hand length in inches?	$6,8$
In South Africa, what is the recommended amount of students in a class per teacher?	40

12.9 Exercise 9

1.1 $31{,}25$

1.2 $46{,}99$

1.3 $52{,}91$

1.4 20,11

1.5 33

1.6 0,00

1.7 2,35

1.8 9,09

1.9 76,08

1.10 4,01

1.11 321,46

1.12 10

1.13 60

1.14 99,99

1.15 942,74

1.16 0,01

1.17 34,91

1.18 56,08

1.19 7,10

1.20 2,32

2.1 0

2.2 50

2.3 120

2.4 410

2.5 10

2.6 10

2.7 9150

2.8 1000

2.9 440

2.10 300

2.11 880

2.12 910

2.13 920

2.14 1000

2.15 150

2.16 50

2.17 660

2.18 730

2.19 110

2.20 120

3.1 0

3.2 0

3.3 5

3.4 5

3.5 5

3.6 5

3.7 5

3.8 10

3.9 10

3.10 10

3.11 125

3.12 65

3.13 70

3.14 10

3.15 90

3.16 95

3.17 100

3.18 100

3.19 35

3.20 80

4.1 R12,23

4.2 R190,13

4.3 R3,53

4.4 R2,99

4.5 R6

4.6 R1,10

4.7 R12,08

4.8 R41,89

4.9 R4,09

4.10 R9,01

4.11 R10,02

4.12 R1,71

4.13 R80,09

4.14 R14,01

4.15 R4,13

4.16 R90,99

4.17 R7,01

4.18 R5,98

4.19 R100

4.20 R20

5.1 R144

5.2 R2

5.3 R5

5.4 R4

5.5 R13

5.6 R55

5.7 R13

5.8 R91

5.9 R14

5.10 R2

5.11 R4

5.12 R1

5.13 R3

5.14 R13

5.15 R9

5.16 R13

5.17 R3

5.18 R57

5.19 R4

5.20 R7

6.1 44

6.2 89

6.3 46

6.4 35

6.5 88

6.6 8

6.7 88

6.8 15

6.9 90

6.10 90

7 Completed table:

You must organize accommodation for $112{,}4$ people. For how many people would you prepare the accommodation?	113
You want $116{,}23$ m^2 to be tiled. For how many square meters would you order tiles?	117 m^3
You need to buy paint and according to your calculations, you need $4{,}4$ tins of paint. How many tins would you buy?	5 cans
You need $1{,}3$ kg rice for a function. How many kilograms would you buy?	2 kg
The temperature is $6°$C, round it to nearest 5 degrees.	$5°$C

12.10 Exercise 10

1.1 500ml : $2\,000$ml

1.2 $\frac{4}{5}$

1.3 200 ml

2.1 $7 : 10$

2.2 $3 : 20$

2.3 $4 : 1$

2.4 $3 : 4$

2.5 $3 : 14$

2.6 $1 : 1$

2.7 $3 : 4$

2.8 $1 : 12$

2.9 $9 : 20$

2.10 $3 : 2$

2.11 $14 : 1$

2.12 $1 : 4$

2.13 $1 : 81$

3.1 $18 : 54$

3.2 $180 : 270$

3.3 $0{,}8 : 2$

3.4 $224 : 560 : 112$

3.5 $3{,}6 : 4{,}8 : 6$

3.6 $60 : 90 : 90$

3.7 $8 : 16 : 24$

3.8 $6{,}2 : 9{,}3$

3.9 $45 : 105$

4. 9 m : 18 m : 36 m

5. 198 Girls : 330 Boys

6. Frank R900 : Ross R300

7. 10 l

8. 21 km

9. R$85{,}50$

10. R$3{,}30$: R$2{,}20$

11. $6{,}4 : 1{,}6$

12. 21 m^3

13. 45 forA : 60 forB

14. 200 ml : R$0{,}82$
    700 ml : R$2{,}87$

15. R25

16. 3 cups

17. 90 girls

12.11 Exercise 11

1. Completed table:

Number of pens	1	3	5	8	10	21	25	100	112	120
Total cost	R4	R12	R20	R32	R40	R84	R100	R400	R448	R480

2.1 Completed table:

Green paint (liter):	2	4	6	8	10	16	22	26	30	200
Yellow paint (liter):	3	6	9	12	15	24	33	39	45	300

2.2 Completed table:

Number of hours driven:	1	2	3	6	7	8,5	10	11	20,5	30
Km covered	80	160	240	480	560	680	800	880	1640	2400

2.3 Completed table:

Flour (g) per mix:	50	100	200	450	500	1000	2500	5000	6500	7000
Milk (ml) per mix:	30	60	120	270	300	600	1500	3000	3900	4200

3 12

4.1 $1 : 2$

4.2 $1 : 3$

5. 50 mm

6. $1 : 3$

7. $2\,400$ ml

8. 10 marks

9.1 $3 : 14$

9.2 $1 : 7$

9.3 $3 : 2$

9.4 $5 : 9$

12.12 Exercise 12

1. Completed table:

Time	Speed	Distance
45 minutes	$106{,}7$ km/h	80 km
3 hours 22 minutes	135 km/h	455 km
20 minutes	90 km/h	30 km
$1\frac{1}{2}$ hours	15 m/s	$81\,000$ m
1 hour 15 minutes	$1{,}6$ m/h	2 m
2 s	20 m/s	40 m

2. 2.1 R$6{,}00$

   2.2 R$6{,}00$

   2.3 48 BRR

   2.4 48 BRR

3. 3.1 $\$14{,}12$

   3.2 R$2\,120{,}80$

4. 4.1 R$438{,}00$

   4.2 R$22\,629{,}05$

   4.3 £$1\,164{,}38$

12.13 Exercise 13

1.1 72%

1.2 15%

1.3 54%

1.4 40%

1.5 90%

1.6 148%

1.7 80%

1.8 10%

2. Completed table:

Percentage	Common Fraction	Decimal Fraction
19%	$\frac{19}{100}$	0,19
150%	$\frac{3}{2}$	1,5
66%	$\frac{33}{50}$	0,66
25%	$\frac{1}{4}$	0,25
120%	$\frac{6}{5}$	1,2
4%	$\frac{1}{25}$	0,04
18%	$\frac{9}{50}$	0,18
45%	$\frac{9}{20}$	0,45
380%	$\frac{19}{5}$	3,8

3.1 30

3.2 7 500 c

3.3 3 000 ml

3.4 0,4 mm

3.5 5 500 g

3.6 6 000 kg

3.7 1 089 c

3.8 43,2 min

3.9 R393,75

3.10 1 500 ml

3.11 3 dozen

3.12 186 120 l

4.1 20%

4.2 30%

4.3 12,5%

4.4 14,5%

4.5 1,17%

4.6 200%

5 Completed table:

350 cm is increased by 20%	420 cm
R36 is decreased by 10%	R$32{,}40$
Decrease 120 minutes by 25%	5400 s
Increase R150 by $0{,}2\%$	R$150{,}30$

6. Completed table:

25 kg increases with 5 kg	20%
12,5 litres increases with $2{,}5$ litres	20%
300 metres decreases by 30 cm	$0{,}1\%$
R2 800 decreases to R2100	25%

7. 96 Vodacom users, 84 MTN users, 60 Cell C users

8. 448

9. Girls $= 37{,}5\%$
   Boys $= 62{,}5\%$

10.1 720 took part

10.2 432 girls

10.3 54 in grade 8

11. $12{,}14\%$ Bones

12. 16%

13. $26{,}04\%$

14. $12{,}90\%$