



# CHAPTER 7

*Probability*

---

# CONTENTS

|          |  |           |
|----------|--|-----------|
| <b>1</b> | <b>Expression of probability</b>                         | <b>1</b>  |
| 1.1      | Exercise 54: Probability Expressions . . . . .           | 1         |
| <b>2</b> | <b>Prediction</b>  | <b>2</b>  |
| 2.1      | Exercise 55 . . . . .                                    | 2         |
| <b>3</b> | <b>Representations for Determining Possible Outcomes</b> | <b>3</b>  |
| 3.1      | Exercise 56: Tree Diagram . . . . .                      | 5         |
| <b>4</b> | <b>Grids</b>   | <b>7</b>  |
| 4.1      | Exercise 57 . . . . .                                    | 8         |
| <b>5</b> | <b>Gambling</b>  | <b>9</b>  |
| 5.1      | Exercise 58: Gambling . . . . .                          | 10        |
| <b>6</b> | <b>Appendix of Units</b>                                 | <b>13</b> |
| <b>7</b> | <b>Exercise Answers</b>                                  | <b>17</b> |
| 7.1      | Exercise 54 . . . . .                                    | 17        |
| 7.2      | Exercise 55 . . . . .                                    | 17        |
| 7.3      | Exercise 56 . . . . .                                    | 17        |
| 7.4      | Exercise 57 . . . . .                                    | 18        |
| 7.5      | Exercise 58 . . . . .                                    | 18        |

August 26, 2021

---

# 1 EXPRESSION OF PROBABILITY

## Scale that shows certain, uncertain and impossible events:

From this section you can see that there are three main ways in which we can choose to describe the probability of an event:

- As a fraction: a value that lies between 0 and 1
- As a decimal: the result of computing the fraction.
- As a percentage.

| <b>Certain</b>  |                | <b>Very likely</b> | <b>likely</b>  |                | <b>Unsure</b>  |                |                | <b>Unlikely</b> | <b>Highly unlikely</b> | <b>Impossible</b> |
|-----------------|----------------|--------------------|----------------|----------------|----------------|----------------|----------------|-----------------|------------------------|-------------------|
| $\frac{10}{10}$ | $\frac{9}{10}$ | $\frac{8}{10}$     | $\frac{7}{10}$ | $\frac{6}{10}$ | $\frac{5}{10}$ | $\frac{4}{10}$ | $\frac{3}{10}$ | $\frac{2}{10}$  | $\frac{1}{10}$         | $\frac{0}{10}$    |
| 1               | 0,9            | 0,8                | 0,7            | 0,6            | 0,5            | 0,4            | 0,3            | 0,2             | 0,1                    | 0                 |
| 100%            | 90%            | 80%                | 70%            | 60%            | 50%            | 40%            | 30%            | 20%             | 10%                    | 0%                |

## 1.1 Exercise 54: Probability Expressions

- 1.1 What is the probability that a pregnant lady expects a boy?
- 1.2 What is the probability, if you study hard, that you will get a distinction?
- 1.3 What is the probability that South Africa will host the Olympics?

| May |    |    |    |    |    |    |
|-----|----|----|----|----|----|----|
| M   | T  | W  | T  | F  | S  | S  |
|     | 1  | 2  | 3  | 4  | 5  | 6  |
| 7   | 8  | 9  | 10 | 11 | 12 | 13 |
| 14  | 15 | 16 | 17 | 18 | 19 | 20 |
| 21  | 22 | 23 | 24 | 25 | 26 | 27 |
| 28  | 29 | 30 | 31 |    |    |    |

- 1.4 If 1 May is a Tuesday, what is the probability that 2 May will be a Wednesday?
- 1.5 If 8 May is a Tuesday what is the probability that 31 May will be a Friday?
- 1.6 If you were to guess what the weather is going to be on the 6<sup>th</sup> of May, what would you guess? Give a reason for your answer.

---

## 2 PREDICTION

$$Probability = \frac{\text{Number of successful outcomes (NSO)}}{\text{Total number of possible outcomes (TNPO)}}$$

For example, Two dice are thrown. What is the probability ...

to throw a four?

to throw an uneven number?

1.  $P = \frac{NSO}{TNPO} = \frac{1}{6}$  ∴ The chance to throw a four is 1 out of 6, because a die has only one four.

2.  $P = \frac{NSO}{TNPO} = \frac{1}{2}$  ∴ The chance to throw an uneven number is 1 out of 2 because there are three uneven numbers on a die.

### 2.1 Exercise 55

Answer the following questions:

1. A die is thrown. What is the probability to:
  - 1.1 throw a factor of six? P( a factor of 6)
  - 1.2 throw a six? P(6)?
  - 1.3 throw a zero? P(0) ?
  - 1.4 throw a prime number? P (prime number)
2. On a farm there are 60 cows, 3 bulls and 37 calves. All the cattle are gathered to be weighed. What is the probability that the first one weighed is:
  - 2.1 a cow? P( a cow )
  - 2.2 not a bull? P (not a bull)?
  - 2.3 a calf? P (a calf)

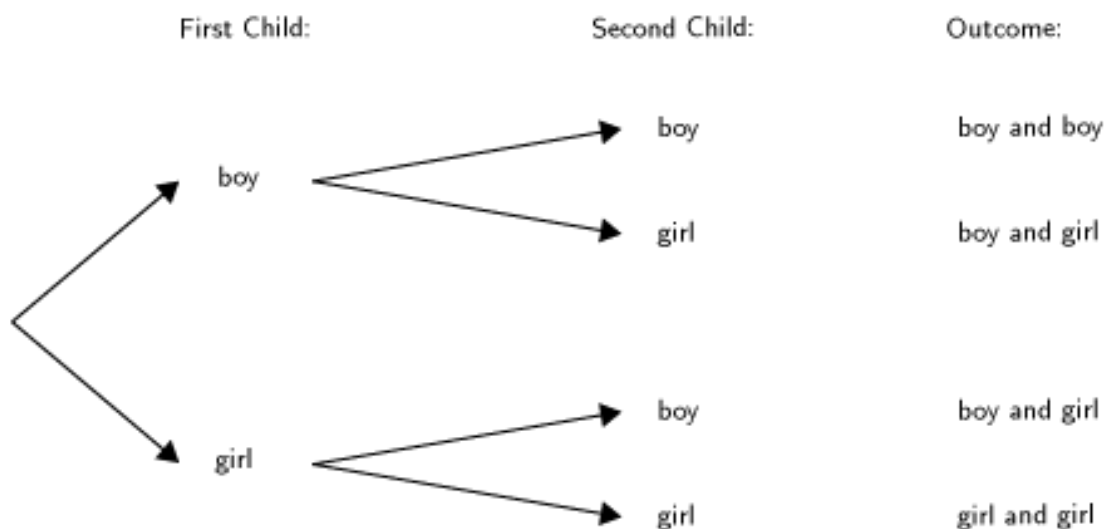
---

### 3 REPRESENTATIONS FOR DETERMINING POSSIBLE OUTCOMES

A **Tree diagram** is useful if more than one action must be carried out.

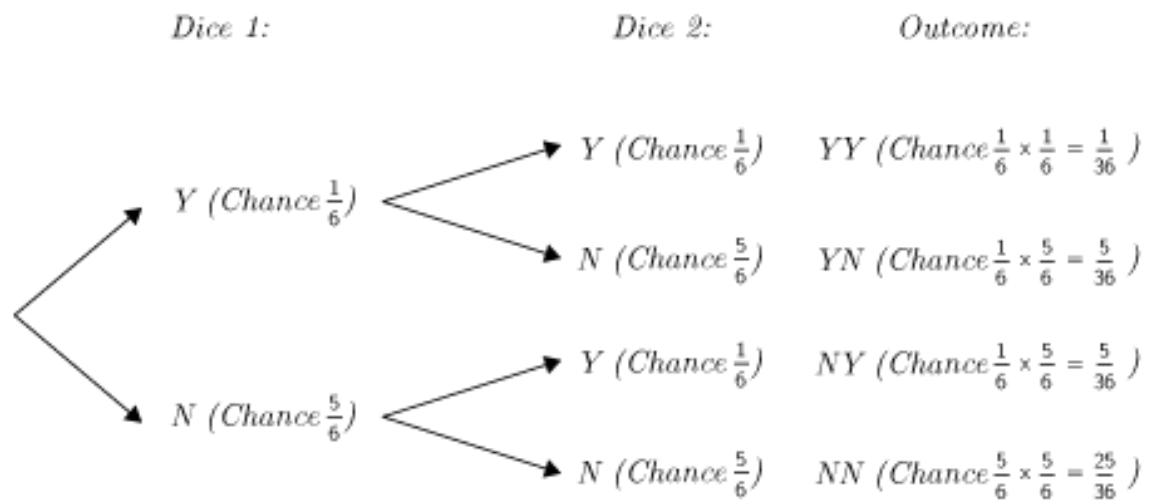
For example, A family has two children. What is the probability that the family has one girl and one boy?

Outcome:



E.g. Two dice are thrown. Investigate the probability of each of the following:

1. to throw a four on both the dice.
2. to throw a four on only one of the dice.
3. to throw no four at all.



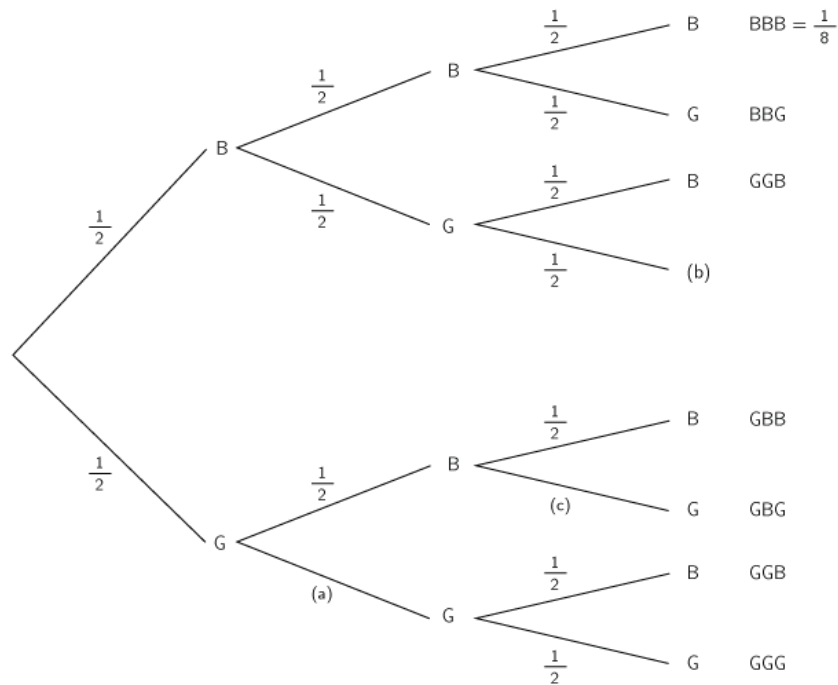
Key:

Throw a 4  $\longrightarrow$   $Y$

Not showing a 4  $\longrightarrow$   $N$

### 3.1 Exercise 56: Tree Diagram

1. A married couple wants to have 3 children.



1.1 Complete the tree diagram by naming **a**, **b** and **c**.

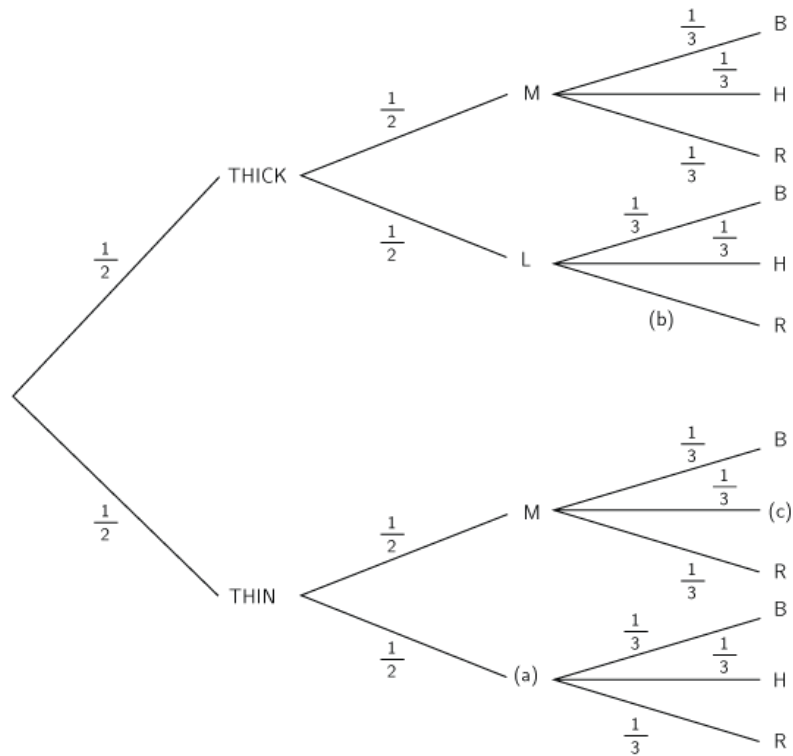
1.2 What is the probability that all three of the babies will be boys?

2. A tree diagram of the choices below is given.

You and your friends want to order pizza. You can choose from the following:

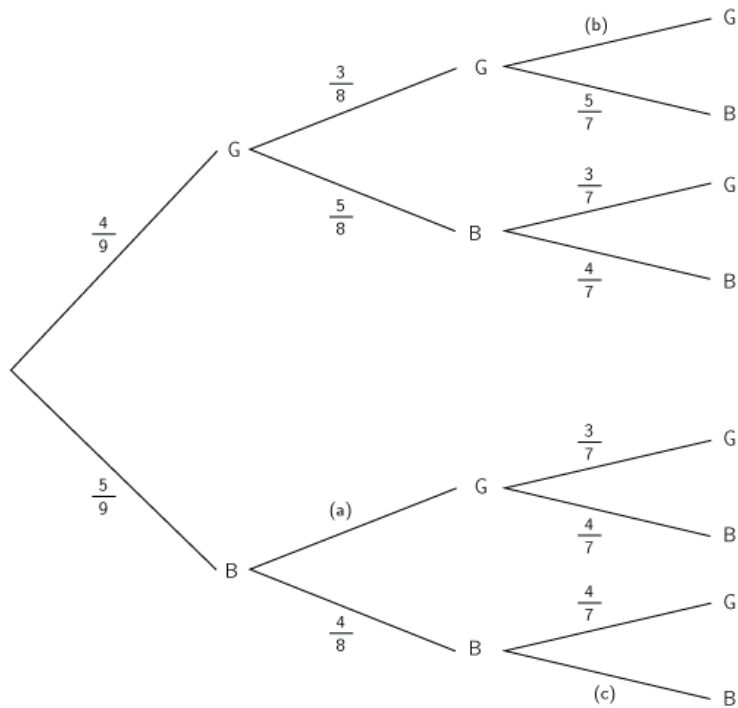
You can order a thick (THICK) or a thin (THIN) base. You can order a medium (M) or a large (L) pizza.

There are 3 toppings you can choose from: bacon (B), ham (H) and rib (R).



- 2.1 Complete the tree diagram by giving **a**, **b** and **c**.
- 2.2 How many different choices are available from the restaurant?
- 2.3 What is the probability for a client to choose a thick base, a large pizza with a bacon topping?
3. An orator's team consists of THREE members. In a certain school Four girls and FIVE boys qualify for such a team.





3.1 Give **a**, **b** and **c** from the diagram.

3.2 What is the probability that the three team members will be girls if all of them have an even chance?

## 4 GRIDS

This grid shows that there are a total of 12 possible results that could occur. However, there is only one possible way that the result could be "Heads & 6". This means that the probability that the coin will land on "Heads" and the die on "6" is  $\frac{1}{12}$

In other words, to calculate the probability of two events happening together you multiply the probability of each event occurring:  $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

|      |   | COIN     |          |
|------|---|----------|----------|
|      |   | Heads    | Tails    |
| DICE | 1 | H1       | <b>a</b> |
|      | 2 | H2       | T2       |
|      | 3 | <b>b</b> | T3       |
|      | 4 | H4       | T4       |
|      | 5 | H5       | <b>c</b> |
|      | 6 | H6       | T6       |

#### 4.1 Exercise 57

1. Study the same Grid above:

1.1 Complete the grid by giving **a** to **c**

1.2 What is the probability that the coin will land on "Heads" and the die on "6"?

1.3 What is the probability that the coin will land on "Tail" and the dice on "6"?

1.4 What is the probability that the coin will land on "Tail" and the dice on an even number?

1.5 What is the probability that the coin will land on "Tail" and the dice on "7"?

2. The grid below shows the outcomes if two coins are tossed

|   |          |          |
|---|----------|----------|
|   | H        | T        |
| H | HH       | <b>a</b> |
| T | <b>b</b> | <b>c</b> |

2.1 Complete the grid by giving **a** to **c**.

2.2 What is the probability that both land on heads

3. The two sight values if two dice are thrown can be summarised as in the table. Complete the table by giving values of **a** to **d**.

|       |   | Die 1 |          |          |          |          |     |
|-------|---|-------|----------|----------|----------|----------|-----|
|       |   | 1     | 2        | 3        | 4        | 5        | 6   |
| Die 2 | 1 | 1;1   | 1;2      | 1;3      | 1;4      | <b>a</b> | 1;6 |
|       | 2 | 2;1   | <b>b</b> | 2;3      | 2;4      | 2;5      | 2;6 |
|       | 3 | 3;1   | 3;2      | 3;3      | 3;4      | 3;5      | 3;6 |
|       | 4 | 4;1   | 4;2      | 4;3      | <b>c</b> | 4;5      | 4;6 |
|       | 5 | 5;1   | 5;2      | 5;3      | 5;4      | 5;5      | 5;6 |
|       | 6 | 6;1   | 6;2      | <b>d</b> | 6;4      | 6;5      | 6;6 |

4. The **sum** of two sight values if two dice are thrown can be summarised as in the table.

|       |   | Die 1    |   |          |    |          |          |
|-------|---|----------|---|----------|----|----------|----------|
|       |   | 1        | 2 | 3        | 4  | 5        | 6        |
| Die 2 | 1 | 2        | 3 | 4        | 5  | <b>a</b> | 7        |
|       | 2 | <b>b</b> | 4 | 5        | 6  | 7        | 8        |
|       | 3 | 4        | 5 | <b>c</b> | 7  | 8        | 9        |
|       | 4 | 5        | 6 | 7        | 8  | 9        | 10       |
|       | 5 | 6        | 7 | 8        | 9  | <b>d</b> | 11       |
|       | 6 | 7        | 8 | 9        | 10 | 11       | <b>e</b> |

4.1 Complete the table by giving the values of **a** to **e**.

4.2 How many possible combinations of throws are there?

4.3 After one throw of two dice, what is the probability that the sum of the sight value is 12?

4.4 After one throw of two dice, what is the probability that the sum of the sight value is more than 8?

4.5 After one throw of two dice, what is the probability that the sum of the sight value is a double (they have the same sight value)?

## 5 GAMBLING

While statistics are difficult to find since much of what happens online is unregulated, one estimate suggests revenues of \$27 billion worldwide in 2009, rising to \$36 billion by 2012.

So much money is involved that governments around the world have been trying to find ways of getting their share, legitimising gambling so as to apply taxation. Every day, some two million people deposit additional funds into their online gambling account.

The US State of Iowa, keen to capitalise on gambling revenue estimated at \$80 million per year. Several commentators have noted that governments seem to be addicted to gambling revenue, one newspaper going as far as accusing the Canadian government of "fueling the spread of a destructive disease."

In the light of the reforms brought about by President Obama, one organisation which ironically calls itself "the safe and secure Internet gambling initiative," claims that taxation from regulated Internet gambling would raise nearly \$42 billion in new revenue, which would help Congress "pay for health care reform and other critical programs"!

---

What is your chance to win some money?

While some people gamble out of desperation and others see it as a form of entertainment, it is not unreasonable to suggest that people gamble, whether online or not, because they have an expectation of winning, however slight. But the odds of winning are always against the gambler and in favour of the "house".

The likelihood of winning the British Jackpot lottery is estimated at around one in fourteen million.

$$\frac{1}{14\,000\,000}$$

In contrast the chances of being struck by lightning is one in two million.

$$\frac{1}{2\,000\,000}$$

The chance of a woman giving birth to quadruplets is one in 705 000

$$\frac{1}{705\,000}$$

The chance of being killed in a car crash is one in five thousand.

$$\frac{1}{5\,000}$$

Professor Tyler Jarvis, chair of the Department of Mathematics at Brigham Young University in Utah, has contributed to several academic papers on the odds at gambling. He writes: "Almost every one has trouble understanding the huge and tiny numbers involved in gambling odds. But learning about these odds has convinced many people that gambling is not the harmless pastime they thought it was. The main thing to understand is that the odds always favour the house. For example, the house's take on a slot machine can be as high as 35%. This means if you bet \$10, you can expect to walk away with only \$6,50 and so forth. The more you play, the more you lose. Although some gamblers are ahead temporarily, in the long run the odds will prevail, and the gambler will lose."

Reference: 'The Banner' January 2011

Author: David Clark

## 5.1 Exercise 58: Gambling

1. Answer the following questions:

1.1 What were the estimated suggested revenues from gambling worldwide in 2009?

1.2 According to statistics what will it be in 2012?

- 1.3 By how much \$ will it increase?
- 1.4 How much Rand is this If the current exchange rate is 1\$=R15,12 ?
- 1.5 According to President Obama how much money (\$) will the "safe and secure Gambling" bring in for health care?
- 1.6 Why are the odds of winning always against the gambler and in favour of the "house"?
- 1.7 What is your likelihood of winning the British Jackpot lottery?
- 1.8 What is your chance of being struck by lightning?
- 1.9 What is your bigger chance, to win the British Jackpot lottery, or being killed by lightning?
- 1.10 The "house" keeps 35% of a gambling machine. How much money will you receive if you gamble with \$100?
- 1.11 It is estimated that there are four times more gamblers since internet gambling is available. There were 80000 gamblers, how many gamblers are there now?
- 1.12 What is the age restriction on gambling in South Africa?

The Lottery is a form of gambling where cash prizes can be won. The Lotto is a game of chance which involves picking numbers from a lottery slip and paying a certain fee per entry. In the Lotto, 6 numbers are randomly selected from a field of 49 numbers. If these 6 numbers match the drawn numbers, the winner wins the main prize. When the numbers are drawn, an additional bonus number is also drawn. The winning amount paid out to winners is 45% of the total sales.

- 2.1 Liam bought a Lotto ticket on which he predicted the following six numbers:



Determine the probability that the first number drawn is '1'.

- 2.2 Determine the probability that the first two numbers drawn are '1' and '15'.
- 2.3 Determine the probability that the first three numbers drawn are '1', '15' and '21'.
- 2.4 If six balls are drawn in the same order as Liam's numbers, calculate the denominator of the probability fractions.
- 2.5 Because the order of the numbers drawn does not matter in our national lottery, the value of 10068347520 must be divided by  $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ . Determine the probability of drawing all six numbers on Liam's lottery slip as a probability, as well as in the ratio 1 :
- 2.6 How many Lotto tickets must Liam buy if he wants to place a bet for every possible outcome?

- 
- 2.7 How much would it have cost Liam if he placed a bet for every possible outcome? Assume that one Lotto ticket costs R3, 50.
- 2.8 How much of his money would have been paid out to the winners after the numbers were drawn?
- 2.9 Will the probability of winning increase if Liam buys two Lottery tickets instead of one?

## 6 APPENDIX OF UNITS

|                  | SI Units                            |                 |
|------------------|-------------------------------------|-----------------|
| Length           | Area                                | Volume          |
| km               | km <sup>2</sup>                     | km <sup>3</sup> |
| cm               | cm <sup>2</sup>                     | cm <sup>3</sup> |
| mm               | mm <sup>2</sup>                     | mm <sup>3</sup> |
|                  | Ha (hectare=10 000 m <sup>2</sup> ) |                 |
| Capacity         | mass                                | velocity        |
| litre,millilitre | tonne                               | km/h            |

|             | International system of units |                |
|-------------|-------------------------------|----------------|
| Quantity    | Units                         | Symbols        |
| Length      | metre                         | m              |
| mass        | kilogram                      | kg             |
| Area        | square metre                  | m <sup>2</sup> |
| Volume      | cubic metre                   | m <sup>3</sup> |
| Velocity    | metre per second              | m/s            |
| Time        | seconds                       | s              |
| Temperature | degree Celsius,Fahrenheit     | °C,°F          |

| Useful symbols |                  |
|----------------|------------------|
| =              | is equal to      |
| ≈              | is approximately |
| ⊥              | perpendicular    |
| ≠              | is not equal to  |
|                | parallel         |

---

| <b>Metrical Units</b>   | <b>Imperial Units</b>        |
|-------------------------|------------------------------|
| <b>Weights</b>          |                              |
| 907 <i>kg</i>           | 1 <i>ton</i>                 |
| 453 <i>g</i>            | 1 <i>pound(lb)</i>           |
| 28 <i>g</i>             | 1 <i>ounce(oz)</i>           |
| <b>Distance(Length)</b> |                              |
| 1.6 <i>km</i>           | 1 <i>mile(mi.)</i>           |
| 0.91 <i>m</i>           | 1 <i>yard</i>                |
| 30 <i>cm</i>            | 1 <i>feet</i>                |
| 2.54 <i>cm</i>          | 1 <i>inch</i>                |
| <b>Capacity</b>         |                              |
| 3.8 <i>l</i>            | 1 <i>gallon</i>              |
| 473 <i>ml</i>           | 1 <i>pint(pt)</i>            |
| 29.6 <i>ml</i>          | 1 <i>fluid ounce(fl.oz.)</i> |



---

### Use of the calculator:... Casio FX-82ZA Plus

You can also visit the website: [www.casiocalcs.co.za](http://www.casiocalcs.co.za)

1. How to clear (Initialise) your calculator.

**Shift; 9; 3; = AC**

2. Tips:

- ONLY use **ON** when switching the scientific calculator on.
- To clear your screen, rather use **AC**. This saves your calculator's temporary memory.
- See the  $\Delta$  in the top right corner of the screen.
- Use the up and down arrows to review previous calculations.

3. How to set your calculator to round off to 2 decimal places:

**Shift; Mode; 6; now select how many decimal places.**

4. How to clear your calculator from rounding off to 2 decimal places.

**Shift; Mode; 8; choose 2.**

5. To give answers in scientific notation.

**Shift; Mode; 8**

Norm 1 is the default setting and gives answers in scientific notation.

Norm 2 is generally preferred as answers are only expressed in scientific notation when they are too big to fit on the screen.

6. Exponents:

$x^2; x$

7. Surds:

$\sqrt{\quad}$  and shift  $\sqrt[\square]{\square}$

8. Percentages:

- Writing a fraction as a percentage:

Example: Write  $\frac{126}{150}$  as a percentage.

**Press: 126;  $\frac{\square}{\square}$ ; 150;  $\text{>}; \times; 100; =$**

- Finding the percentage of an amount:

Example: 15% of 1 250

**Press: 15;  $\frac{\square}{\square}$ ; 100;  $\text{>}; \times; 1250; =; \text{SD}$**

- Increase 2 000 by 15% **Press: 2000;  $\times; 1,15; =$**

- 
- Decrease 2000 by 15%

**Press:** 2000 - 15; =; 100; |>; ×; 2000; =

9. Converting from a decimal to hours, minutes and seconds

Example: How long will it take to travel a distance of 534 km, if your average speed is 90 km/h?

$$\text{Time} = \frac{534}{90} = 5,9333$$

**Press: FACT**

10. Converting from hours, minutes and seconds to a decimal.

Example: At what speed are you travelling if 150 km takes 1 hour 16 minutes and 17 seconds?

$$\text{Speed} = \frac{150}{1^{\circ}16^{\circ}17^{\circ}} = 117,98 \text{ km/h}$$

**Press:** 150;  $\frac{\square}{\square}$ ; 1; **fact**; 16; **fact**; 17; **fact**; |>; =

---

## 7 EXERCISE ANSWERS

### 7.1 Exercise 54

- 1.1 50%, 0.5,  $\frac{1}{2}$ , Unsure
- 1.2 0.8, 80%, 0.8, Very likely
- 1.3 10%, 0.1, Highly unlikely
- 1.4 100%, 1, Certain
- 1.5 0%, 0, Impossible
- 1.6 50%, Unsure, the weather can change everyday

### 7.2 Exercise 55

- 1.1  $\frac{4}{6}, \frac{2}{3}$ , factor: 1, 2, 3, 6
- 1.2  $\frac{1}{6}$
- 1.3  $\frac{0}{6}$ , 0
- 1.4  $\frac{3}{6}, \frac{1}{2}$ , Prime numbers: 2, 3, 5
- 2.1  $\frac{60}{100}, \frac{3}{5}$
- 2.2  $\frac{3}{100}$
- 2.3  $\frac{37}{100}$

### 7.3 Exercise 56

- 1.1  $\mathbf{a}=\frac{1}{2}, \mathbf{b}=\mathbf{G}, \mathbf{c}=\frac{1}{2}$
- 1.2  $\frac{1}{8}$
- 2.1  $\mathbf{a}=\mathbf{L}, \mathbf{b}=\frac{1}{3}, \mathbf{c}=\mathbf{H}$
- 2.2 12
- 2.3  $\frac{1}{12}$
- 3.1  $\mathbf{a}=\frac{4}{8}, \mathbf{b}=\frac{2}{7}, \mathbf{c}=\frac{3}{7}$
- 3.2  $\frac{1}{21}$

---

## 7.4 Exercise 57

1.1  $a=T1, b=H3, c=T5$

1.2  $\frac{1}{12}$

1.3  $\frac{1}{12}$

1.4  $\frac{3}{12}, \frac{1}{4}$

1.5 0

2.1  $a=HT, b=TH, c=TT$

2.2  $\frac{1}{4}$

3.  $a=1;5, b=2;2, c=4;4, d=6;3$

4.1  $a=6, b=3, c=6, d=10, e=12$

4.2 36

4.3  $\frac{1}{36}$

4.4  $\frac{10}{36}$

4.5  $\frac{6}{36}, \frac{1}{6}$

## 7.5 Exercise 58

1.1 R27 billion

1.2 R36 billion and \$12 Trillion

1.3 \$9 billion

1.4 R1351,08 billion

1.5 \$42 billion

1.6 The house makes the profit

1.7  $\frac{1}{14\,000\,000}$

1.8  $\frac{1}{2\,000\,000}$

1.9 Killed by lightning

---

1.10 \$ 65

1.11 320 000

1.12 18 years

2.1  $P(1) = \frac{1}{49}$

2.2  $P(1 \text{ and } 15) = \frac{1}{2352}$

2.3  $P(1; 15; 21) = \frac{1}{110544}$

2.4 10 068 447 520

2.5  $\frac{1}{13983816}$

Ratio 1 : 13 983 816

2.6 13 983 816

2.7 R48 943 356

2.8 Only R 22 024 510.20

2.9 Probability will increase by a small percentage Liam will have to pay more to increase the probability to win.