CHAPTER 1

Algebraic Expressions

CONTENTS

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1 INTRODUCTION

Over human history, all people and cultures have contributed to the field of Mathematics. Topics like algebra may seem obvious now, but for many centuries mathematicians had to make do without it. Over the next three grades, you will explore more advanced and abstract mathematics. It may not be obvious how this applies to everyday life, but the truth is, mathematics is required for nearly everything you will do one day. Enjoy your mathematical journey. Remember, there is no such thing as a "maths person" or "not a maths person". We can all do mathematics, it just takes practice.

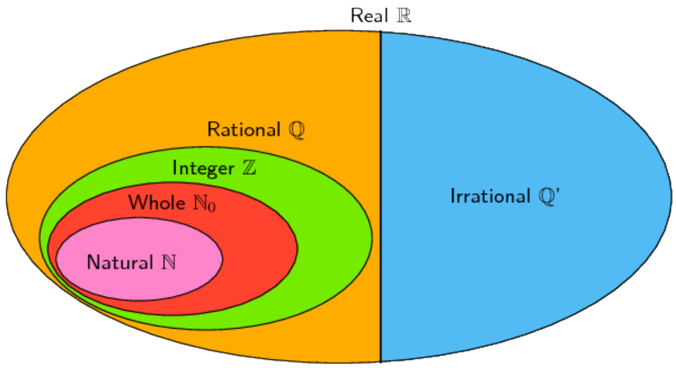
Figure 1: Some examples of early tally sticks. These were used to help people count things such as the number of days between events or the number of livestock they had.

In this chapter, we will begin by revising the real number system and then learn about estimating surds and rounding real numbers. We will also be expanding on prior knowledge of factorisation and delve into more complex calculations involving binomial and trinomial expressions.

2 THE REAL NUMBER SYSTEM

We use the following definitions:

- N: natural numbers are $\{1; 2; 3; \ldots\}$
- $\mathbb{N}_0$: whole numbers are $\{0; 1; 2; 3; \ldots\}$
- $\mathbb{Z}$: integers are $\{\ldots; -3; -2; -1; 0; 1; 2; 3; \ldots\}$



NOTE

Not all numbers are real numbers. The square root of a negative number is called a non-real or imaginary number. For example $\sqrt{-1}$, $\sqrt{-28}$ and $\sqrt{-5}$ are all non-real numbers.

3 RATIONAL AND IRRATIONAL NUMBERS

Definition: Rational number

A rational number ($\mathbb{Q}$) is any number which can be written as:

$$\frac{a}{b}$$

where a and b are integers and $b \neq 0$.

The following numbers are all rational numbers:

$$\frac{10}{1}; \frac{21}{7}; \frac{-1}{-3}; \frac{10}{20}; \frac{-3}{6}$$

We see that all numerators and all denominators are integers.

This means that all integers are rational numbers, because they can be written with a denominator of 1.

Definition: Irrational numbers

Irrational numbers ($\mathbb{Q}'$) are numbers that cannot be written as a fraction with the numerator and denominator as integers.

Examples of irrational numbers:

$$\sqrt{2}; \sqrt{3}; \sqrt[3]{4}; \pi; \frac{1+\sqrt{5}}{2}$$

These are not rational numbers, because either the numerator or the denominator is not an integer.

3.1 Decimal numbers

All integers and fractions with integer numerators and non-zero integer denominators are rational numbers. Remember that when the denominator of a fraction is zero then the fraction is undefined.

You can write any rational number as a decimal number but not all decimal numbers are rational numbers. These types of decimal numbers are rational numbers:

- Decimal numbers that end (or terminate). For example, the fraction $\frac{4}{10}$ can be written as $0,4$.
- Decimal numbers that have a repeating single digit. For example, the fraction $\frac{1}{3}$ can be written as $0,\dot{3}$ or $0,\overline{3}$. The dot and bar notations are equivalent and both represent recurring 3's, i.e. $0,\dot{3} = 0,\overline{3} = 0,333\ldots$
- Decimal numbers that have a recurring pattern of multiple digits. For example, the fraction $\frac{2}{11}$ can also be written as $0,\overline{18}$. The bar represents a recurring pattern of 1 's and 8 's, i.e. $0,\overline{18} = 0,181818\ldots$

NOTE

You may see a full stop instead of a comma used to indicate a decimal number. So the number $0,4$ can also be written as 0.4.

Notation: You can use a dot or a bar over the repeated digits to indicate that the decimal is a recurring decimal. If the bar covers more than one digit, then all numbers beneath the bar are recurring.

If you are asked to identify whether a number is rational or irrational, first write the number in decimal form. If the number terminates then it is rational. If it goes on forever, then look for a repeated pattern of digits. If there is no repeated pattern, then the number is irrational.

When you write irrational numbers in decimal form, you may continue writing them for many, many decimal places. However, this is not convenient and it is often necessary to round off.

NOTE

Rounding off an irrational number makes the number a rational number that approximates the irrational number.

WORDKED EXAMPLE 1: RATIONAL AND IRRATIONAL NUMBERS

Question

1. $\pi = 3,14159265358979323846264338327950288419716939937510\ldots$
2. $1,4$
3. $1,618033989\ldots$
4. 100
5. $1,7373737373\ldots$
6. $0,\overline{02}$

Solution

1. Irrational, decimal does not terminate and has no repeated pattern.
2. Rational, decimal terminates.
3. Irrational, decimal does not terminate and has no repeated pattern.
4. Rational, all integers are rational.
5. Rational, decimal has repeated pattern.
6. Rational, decimal has repeated pattern.

3.2 Converting terminating decimals into rational numbers

A decimal number has an integer part and a fractional part. For example, $10,589$ has an integer part of 10 and a fractional part of $0,589$ because $10 + 0,589 = 10,589$.

Each digit after the decimal point is a fraction with a denominator in increasing powers of 10.

For Example:

- $0,1$ is $\frac{1}{10}$
- $0,01$ is $\frac{1}{100}$
- $0,001$ is $\frac{1}{1000}$

This means that

$$\begin{aligned} 10,589 &= 10 + \frac{5}{10} + \frac{8}{100} + \frac{9}{1000} \\ &= \frac{10000}{1000} + \frac{500}{1000} + \frac{80}{1000} + \frac{9}{1000} \\ &= \frac{10589}{1000} \end{aligned}$$

3.3 Converting recurring decimals into rational numbers

When the decimal is a recurring decimal, a bit more work is needed to write the fractional part of the decimal number as a fraction.

WORKED EXAMPLE 2: CONVERTING DECIMALS INTO FRACTIONS

Question

Write $0,\dot{3}$ in the form $\frac{a}{b}$ (where a and b are integers).

Solution

Step 1: Define an equation

$$\text{Let } x = 0,33333\ldots$$

Step 2: Multiply by 10 on both sides

$$10x = 3,33333\ldots$$

Step 3: Subtract the first equation from the second equation

$$9x = 3$$

Step 4: Simplify

$$x = \frac{3}{9} = \frac{1}{3}$$

WORKED EXAMPLE 3: CONVERTING DECIMAL NUMBERS INTO FRACTIONS

Question

Write $5,\dot{4}3\dot{2}$ as a rational fraction.

Solution

Step 1: Define an equation

$$x = 5,432432432\ldots$$

Step 2: Multiply by $1\,000$ on both sides

$$1000x = 5\,432,432432432\ldots$$

Step 3: Subtract the first equation from the second equation

$$999x = 5\,427$$

Step 4: Simplify

$$x = \frac{5\,427}{999} = \frac{201}{37} = 5\frac{16}{37}$$

In the first example, the decimal was multiplied by 10 and in the second example, the decimal was multiplied by $1\,000$. This is because there was only one digit recurring (i.e. 3) in the first example, while there were three digits recurring (i.e. 432) in the second example.

In general, if you have one digit recurring, then multiply by 10. If you have two digits recurring, then multiply by 100. If you have three digits recurring, then multiply by $1\,000$ and so on.

Not all decimal numbers can be written as rational numbers. Why? Irrational decimal numbers like $\sqrt{2} = 1,4142135\ldots$ cannot be written with an integer numerator and denominator, because they do not have a pattern of recurring digits and they do not terminate.

4 ROUNDING OFF

Rounding off a decimal number to a given number of decimal places is the quickest way to approximate a number. For example, if you wanted to round off $2,6525272$ to three decimal places, you would:

- count three places after the decimal and place a $|$ between the third and fourth numbers;
- round up the third digit if the fourth digit is greater than or equal to 5;
- leave the third digit unchanged if the fourth digit is less than 5;
- if the third digit is 9 and needs to be rounded up, then the 9 becomes a 0 and the second digit is rounded up.

So, since the first digit after the $|$ is a 5, we must round up the digit in the third decimal place to a 3 and the final answer of $2,6525272$ rounded to three decimal places is $2,653$.

WORKED EXAMPLE 4: ROUNDING OFF

Question

Round off the following numbers to the indicated number of decimal places:

1. $\frac{120}{99} = 1,\dot{1}\dot{2}$ to 3 decimal places.
2. $\pi = 3,141592653\ldots$ to 4 decimal places.
3. $\sqrt{3} = 1,7320508\ldots$ to 4 decimal places.
4. $2,78974526$ to 3 decimal places.

Solution

Step 1: Mark off the required number of decimal places

If the number is not a decimal you first need to write the number as a decimal.

1. $\frac{120}{99} = 1,212 \mid 121212\ldots$
2. $\pi = 3,1415 \mid 92653\ldots$
3. $\sqrt{3} = 1,7320 \mid 508\ldots$
4. $2,789 \mid 74526$

Step 2: Check the next digit to see if you must round up or round down

1. The last digit of $\frac{120}{99} = 1,212 \mid 1212121\dot{1}\dot{2}$ must be rounded down.
2. The last digit of $\pi = 3,1415 \mid 92653\ldots$ must be rounded up.
3. The last digit of $\sqrt{3} = 1,7320 \mid 508\ldots$ must be rounded up.
4. The last digit of $2,789 \mid 74526$ must be rounded up. Since this is a 9 we replace it with a 0 and round up the second last digit.

Step 3: Write the final answer

1. $\frac{120}{99} = 1,212$ rounded to 3 decimal places.
2. $\pi = 3,1416$ rounded to 4 decimal places.
3. $\sqrt{3} = 1,7321$ rounded to 4 decimal places.
4. $2,790$

5 ESTIMATING SURDS

If the n^{th} root of a number cannot be simplified to a rational number, we call it a surd. For example, $\sqrt{2}$ and $\sqrt[3]{6}$ are surds, but $\sqrt{4}$ is not a surd because it can be simplified to the rational number 2.

In this chapter we will look at surds of the form $\sqrt[n]{a}$ where a is any positive number, for example, $\sqrt{7}$ or $\sqrt[3]{5}$. It is very common for n to be 2, so we usually do not write $\sqrt[2]{a}$. Instead we write the surd as just $\sqrt{a}$.

It is sometimes useful to know the approximate value of a surd without having to use a calculator. For example, we want to be able to estimate where a surd like $\sqrt{3}$ is on the number line. From a calculator we know that $\sqrt{3}$ is equal to $1,73205\ldots$ It is easy to see that $\sqrt{3}$ is above 1 and below 2. But to see this for other surds like $\sqrt{18}$, without using a calculator you must first understand the following:
If a and b are positive whole numbers, and $a < b$, then $\sqrt[n]{a} < \sqrt[n]{b}$.

A perfect square is the number obtained when an integer is squared. For example, 9 is a perfect square since $3^2 = 9$.

Similarly, a perfect cube is a number which is the cube of an integer. For example, 27 is a perfect cube, because $3^3 = 27$.

Consider the surd $\sqrt[3]{52}$. It lies somewhere between 3 and 4, because $\sqrt[3]{27} = 3$ and $\sqrt[3]{64} = 4$ and 52 is between 27 and 64.

WORKED EXAMPLE 5: ESTIMATING SURDS

Question

Find two consecutive integers such that $\sqrt{26}$ lies between them. (Remember that consecutive integers are two integers that follow one another on the number line, for example, 5 and 6 or 8 and 9).

Solution

Step 1: Use perfect squares to estimate the lower integer

$5^2 = 25$. Therefore $5 < \sqrt{26}$.

Step 2: Use perfect squares to estimate the upper integer

$6^2 = 36$. Therefore $\sqrt{26} < 6$.

Step 3: Write the final answer

$5 < \sqrt{26} < 6$

WORKED EXAMPLE 6: ESTIMATING SURDS

Question

Find two consecutive integers such that $\sqrt[3]{49}$ lies between them.

Solution

Step 1: Use perfect cubes to estimate the lower integer

$3^3 = 27$. Therefore $3 < \sqrt[3]{49}$.

Step 2: Use perfect cubes to estimate the upper integer

$4^3 = 64$. Therefore $\sqrt[3]{49} < 4$.

Step 3: Write the final answer

$3 < \sqrt[3]{49} < 4$

Step 4: Check the answer by cubing all terms in the inequality and then simplify

$27 < 49 < 64$. This is true, so $\sqrt[3]{49}$ lies between 3 and 4.

6 PRODUCTS

Mathematical expressions are just like sentences and their parts have special names. You should be familiar with the following words used to describe the parts of mathematical expressions.

$$3x^2 + 7xy - 5^3$$

Name	Examples
term	$3x^2; 7xy; -5^3$
expression	$3x^2 + 7xy - 5^3$
coefficient	$3; 7$
exponent	$2; 1; 3$
base	$x; y; 5$
constant	$3; 7; 5$
variable	$x; y$
equation	$3x^2 + 7xy - 5^3 = 0$

6.1 Multiplying a monomial and a binomial

A monomial is an expression with one term, for example, $3x$ or y^2. A binomial is an expression with two terms, for example, $ax + b$ or $cx + d$.

WORKED EXAMPLE 7: SIMPLIFYING BRACKETS

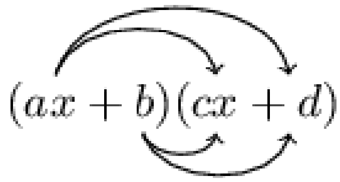
Question

Simplify:

$$2a(a-1) - 3(a^2 - 1)$$

Solution

$$\begin{aligned} 2a(a-1) - 3\left(a^2 - 1\right) &= 2a(a) + 2a(-1) + (-3)\left(a^2\right) + (-3)(-1) \\ &= 2a^2 - 2a - 3a^2 + 3 \\ &= -a^2 - 2a + 3 \end{aligned}$$



6.2 Multiplying two binomials

Here we multiply (or expand) two linear binomials:

WORKED EXAMPLE 8: MULTIPLYING TWO BINOMIALS

Question

Find the product: $(3x-2)(5x+8)$

Solution

$$\begin{aligned}(3x-2)(5x+8) &= (3x)(5x)+(3x)(8)+(-2)(5x)+(-2)(8)\\ &= 15x^2+24x-10x-16\\ &= 15x^2+14x-16\end{aligned}$$

The product of two identical binomials is known as the square of the binomial and is written as:

$$(ax+b)^2 = a^2x^2+2abx+b^2$$

If the two terms are of the form $ax+b$ and $ax-b$ then their product is:

$$(ax+b)(ax-b) = a^2x^2-b^2$$

This product yields the difference of two squares.

6.3 Multiplying a binomial and a trinomial

A trinomial is an expression with three terms, for example, ax^2+bx+c. Now we can learn how to multiply a binomial and a trinomial.

To find the product of a binomial and a trinomial, multiply out the brackets:

$$(A+B)(C+D+E) = A(C+D+E)+B(C+D+E)$$

WORKED EXAMPLE 9: MULTIPLYING A BINOMIAL AND A TRINOMIAL

Question

Find the product: $(x-1)(x^2-2x+1)$

Solution

Step 1: Expand the bracket

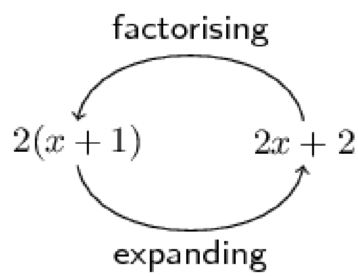
$$\begin{aligned}(3x-2)(5x+8) &= (3x)(5x)+(3x)(8)+(-2)(5x)+(-2)(8)\\ &= 15x^2+24x-10x-16\\ &= 15x^2+14x-16\end{aligned}$$

Step 2: Simplify

$$(x-1)\left(x^2-2x+1\right) = x^3-3x^2+3x-1$$

7 FACTORISATION

Factorisation is the opposite process of expanding brackets. For example, expanding brackets would require $2(x+1)$ to be written as $2x+2$. Factorisation would be to start with $2x+2$ and end up with $2(x+1)$.



The two expressions $2(x+1)$ and $2x+2$ are equivalent; they have the same value for all values of x.

In previous grades, we factorised by taking out a common factor and using difference of squares.

7.1 Common factors

Factorising based on common factors relies on there being factors common to all the terms.

For example, $2x-6x^2$ can be factorised as follows:

$$2x-6x^2=2x(1-3x)$$

And $2(x-1)-a(x-1)$ can be factorised as follows:

$$(x-1)(2-a)$$

WORKED EXAMPLE 10: FACTORISING USING A SWITCH AROUND IN BRACKETS

Question

Factorise:

$$5(a-2)-b(2-a)$$

Solution

Use a “switch around” strategy to find the common factor.

Notice that $2-a=-(a-2)$

$$\begin{aligned}5(a-2)-b(2-a)&=5(a-2)-[-b(a-2)]\\&=5(a-2)+b(a-2)\\&=(a-2)(5+b)\end{aligned}$$

7.2 Difference of two squares

We have seen that $(ax+b)(ax-b)$ can be expanded to $a^2x^2-b^2$.

Therefore $a^2x^2-b^2$ can be factorised as $(ax+b)(ax-b)$. For example, x^2-16 can be written as x^2-4^2 which is a difference of two squares. Therefore, the factors of x^2-16 are $(x-4)$ and $(x+4)$.

To spot a difference of two squares, look for expressions:

- consisting of two terms;
- with terms that have different signs (one positive, one negative);
- with each term a perfect square.

For example: $a^2-1; 4x^2-y^2; -49+p^4$

WORKED EXAMPLE 11: THE DIFFERENCE OF TWO SQUARES

Question

Factorise: $3a(a^2 - 4) - 7(a^2 - 4)$

Solution

Step 1: Take out the common factor $(a^2 - 4)$

$$3a(a^2 - 4) - 7(a^2 - 4) = (a^2 - 4)(3a - 7)$$

Step 2: Factorise the difference of two squares $(a^2 - 4)$

$$(a^2 - 4)(3a - 7) = (a - 2)(a + 2)(3a - 7)$$

7.3 Factorising by grouping in pairs

The taking out of common factors is the starting point in all factorisation problems. We know that the factors of $3x + 3$ are 3 and $(x + 1)$. Similarly, the factors of $2x^2 + 2x$ are $2x$ and $(x + 1)$. Therefore, if we have an expression:

$$2x^2 + 2x + 3x + 3$$

there is no common factor to all four terms, but we can factorise as follows:

$$\left(2x^2 + 2x\right) + (3x + 3) = 2x(x + 1) + 3(x + 1)$$

We can see that there is another common factor $(x + 1)$. Therefore, we can write:

$$(x + 1)(2x + 3)$$

We get this by taking out the $(x + 1)$ and seeing what is left over. We have $2x$ from the first group and $+3$ from the second group. This is called factorising by grouping.

WORKED EXAMPLE 12: FACTORISING BY GROUPING IN PAIRS

Question

Find the factors of $7x + 14y + bx + 2by$.

Solution

Step 1: There are no factors common to all terms

Step 2: Group terms with common factors together

7 is a common factor of the first two terms and b is a common factor of the second two terms. We see that the ratio of the coefficients $7 : 14$ is the same as $b : 2b$.

$$\begin{aligned} 7x + 14y + bx + 2by &= (7x + 14y) + (bx + 2by) \\ &= 7(x + 2y) + b(x + 2y) \end{aligned}$$

Step 3: Take out the common factor $(x + 2y)$

$$(x + 2y) + b(x + 2y) = (x + 2y)(7 + b)$$

OR

Step 4: Group terms with common factors together

x is a common factor of the first and third terms and $2y$ is a common factor of the second and fourth terms ($7 : b = 14 : 2b$).

Step 5: Rearrange the equation with grouped terms together

$$\begin{aligned} 7x + 14y + bx + 2by &= (7x + bx) + (14y + 2by) \\ &= x(7 + b) + 2y(7 + b) \end{aligned}$$

Step 6: Take out the common factor$(7 + b)$

$$x(7 + b) + 2y(7 + b) = (7 + b)(x + 2y)$$

Step 7: Write the final answer

The factors of $7x + 14y + bx + 2by$ are $(7 + b)$ and $(x + 2y)$.

7.4 Factorising a quadratic trinomial

Factorising is the reverse of calculating the product of factors. In order to factorise a quadratic, we need to find the factors which, when multiplied together, equal the original quadratic.

Consider a quadratic expression of the form $ax^2 + bx$. We see here that x is a common factor in both terms. Therefore $ax^2 + bx$ factorises as $x(ax + b)$. For example, $8y^2 + 4y$ factorises as $4y(2y + 1)$.

Another type of quadratic is made up of the difference of squares. We know that:

$$(a+b)(a-b) = a^2 - b^2$$

So $a^2 - b^2$ can be written in factorised form as $(a+b)(a-b)$.

This means that if we ever come across a quadratic that is made up of a difference of squares, we can immediately write down the factors. These types of quadratics are very simple to factorise. However, many quadratics do not fall into these categories and we need a more general method to factorise quadratics.

We can learn about factorising quadratics by looking at the opposite process, where two binomials are multiplied to get a quadratic. For example:

$$\begin{aligned}(x+2)(x+3) &= x^2 + 3x + 2x + 6 \\ &= x^2 + 5x + 6\end{aligned}$$

We see that the x^2 term in the quadratic is the product of the x -terms in each bracket. Similarly, the 6 in the quadratic is the product of the 2 and 3 in the brackets. Finally, the middle term is the sum of two terms.

So, how do we use this information to factorise the quadratic?

Let us start with factorising $x^2 + 5x + 6$ and see if we can decide upon some general rules. Firstly, write down the two brackets with an x in each bracket and space for the remaining terms.

$$(x \quad)(x \quad)$$

Next, decide upon the factors of 6. Since the 6 is positive, possible combinations are: 1 and 6,2 and 3,-1 and -6 or -2 and -3. Therefore, we have four possibilities:

Option 1	Option 2	Option 3	Option 4
$(x+1)(x+6)$	$(x-1)(x-6)$	$(x+2)(x+3)$	$(x-2)(x-3)$

Next, we expand each set of brackets to see which option gives us the correct middle term.

Option 1	Option 2	Option 3	Option 4
$(x+1)(x+6)$	$(x-1)(x-6)$	$(x+2)(x+3)$	$(x-2)(x-3)$
x^2+7x+6	x^2-7x+6	x^2+5x+6	x^2-5x+6

We see that Option 3, $(x+2)(x+3)$, is the correct solution.

The process of factorising a quadratic is mostly trial and error but there are some strategies that can be used to ease the process.

7.5 General procedure for factorising a trinomial

1. Take out any common factor in the coefficients of the terms of the expression to obtain an expression of the form $ax^2 + bx + c$ where a, b and c have no common factors and a is positive.
2. Write down two brackets with an x in each bracket and space for the remaining terms:

$$(x \quad)(x \quad)$$

3. Write down a set of factors for a and c.
4. Write down a set of options for the possible factors for the quadratic using the factors of a and c.
5. Expand all options to see which one gives you the correct middle term bx.

IMPORTANT

If c is positive, then the factors of c must be either both positive or both negative. If c is negative, it means only one of the factors of c is negative, the other one being positive. Once you get an answer, always multiply out your brackets again just to make sure it really works.

WORKED EXAMPLE 13: FACTORISING A QUADRATIC TRINOMIAL

Question

Factorise: $3x^2 + 2x - 1$.

Solution

Step 1: Check that the quadratic is in required form $ax^2 + bx + c$

Step 2: Write down a set of factors for a and c

$$(x \quad)(x \quad)$$

The possible factors for a are: 1 and 3

The possible factors for c are: -1 and 1

Write down a set of options for the possible factors of the quadratic using the factors of a and c. Therefore, there are two possible options.

Option 1	Option 2
$(x-1)(3x+1)$	$(x+1)(3x-1)$
$3x^2 - 2x - 1$	$3x^2 + 2x - 1$

Step 3: Check that the solution is correct by multiplying the factors

$$\begin{aligned}(x+1)(3x-1) &= 3x^2 - x + 3x - 1 \\ &= 3x^2 + 2x - 1\end{aligned}$$

Step 4: Write the final answer

$$3x^2 + 2x - 1 = (x+1)(3x-1)$$

7.6 Sum and difference of two cubes

We now look at two special results obtained from multiplying a binomial and a trinomial:

Sum of two cubes:

$$\begin{aligned}(x+y)\left(x^2 - xy + y^2\right) &= x\left(x^2 - xy + y^2\right) + y\left(x^2 - xy + y^2\right) \\ &= \left[x\left(x^2\right) + x(-xy) + x\left(y^2\right)\right] + \left[y\left(x^2\right) + y(-xy) + y\left(y^2\right)\right] \\ &= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3 \\ &= x^3 + y^3\end{aligned}$$

Difference of two cubes:

$$\begin{aligned}(x-y)\left(x^2+xy+y^2\right) &= x\left(x^2+xy+y^2\right)-y\left(x^2+xy+y^2\right)\\ &= \left[x\left(x^2\right)+x(xy)+x\left(y^2\right)\right]-\left[y\left(x^2\right)+y(xy)+y\left(y^2\right)\right]\\ &= x^3+x^2y+xy^2-x^2y-xy^2-y^3\\ &= x^3-y^3\end{aligned}$$

So we have seen that:

$$\begin{aligned}x^3+y^3 &= (x+y)\left(x^2-xy+y^2\right)\\ x^3-y^3 &= (x-y)\left(x^2+xy+y^2\right)\end{aligned}$$

We use these two basic identities to factorise more complex examples.

WORKED EXAMPLE 14: FACTORISING A DIFFERENCE OF TWO CUBES

Question

Factorise: a^3-1.

Solution

Step 1: Take the cube root of terms that are perfect cubes

We are working with the difference of two cubes. We know that $x^3-y^3=(x-y)\left(x^2+xy+y^2\right)$, so we need to identify x and y.

We start by noting that $\sqrt[3]{a^3}=a$ and $\sqrt[3]{1}=1$. These give the terms in the first bracket. This also tells us that $x=a$ and $y=1$.

Step 2: Find the three terms in the second bracket

We can replace x and y in the factorised form of the expression for the difference of two cubes with a and 1. Doing so we get the second bracket:

$$(a^3-1)=(a-1)(a^2+a+1)$$

Step 3: Expand the brackets to check that the expression has been correctly factorised

$$\begin{aligned}(a-1)\left(a^2+a+1\right) &= a\left(a^2+a+1\right)-1\left(a^2+a+1\right)\\ &= a^3+a^2+a-a^2-a-1\\ &= a^3-1\end{aligned}$$

WORKED EXAMPLE 15: FACTORISING A SUM OF TWO CUBES

Question

Factorise: $x^3 + 8$.

Solution

Step 1: Take the cube root of terms that are perfect cubes

We are working with the sum of two cubes. We know that $x^3 + y^3 = (x + y)\left(x^2 - xy + y^2\right)$, so we need to identify x and y. We start by noting that $\sqrt[3]{x^3} = x$ and $\sqrt[3]{8} = 2$. These give the terms in the first bracket. This also tells us that $x = x$ and $y = 2$.

Step 2: Find the three terms in the second bracket

We can replace x and y in the factorised form of the expression for the difference of two cubes with x and 2. Doing so we get the second bracket:

$$(x^3 + 8) = (x + 2)(x^2 - 2x + 4)$$

Step 3: Expand the brackets to check that the expression has been correctly factorised

$$\begin{aligned}(x + 2)\left(x^2 - 2x + 4\right) &= x\left(x^2 - 2x + 4\right) + 2\left(x^2 - 2x + 4\right) \\ &= x^3 - 2x^2 + 4x + 2x^2 - 4x + 8 \\ &= x^3 + 8\end{aligned}$$

WORKED EXAMPLE 16: FACTORISING A DIFFERENCE OF TWO CUBES

Question

Factorise: $16y^3 - 432$.

Solution

Step 1: Take out the common factor 16

$$16y^3 - 432 = 16(y^3 - 27)$$

Step 2: Take the cube root of terms that are perfect cubes

We are working with the difference of two cubes. We know that $x^3 - y^3 = (x - y)\left(x^2 + xy + y^2\right)$, so we need to identify x and y. We start by noting that $\sqrt[3]{y^3} = y$ and $\sqrt[3]{27} = 3$. These give the terms in the first bracket. This also tells us that $x = y$ and $y = 3$.

Step 3: Find the three terms in the second bracket We can replace x and y in the factorised form of the expression for the difference of two cubes with y and 3. Doing so we get the second bracket:

$$16(y^3 - 27) = 16(y - 3)(y^2 + 3y + 9)$$

Step 4: Expand the brackets to check that the expression has been correctly factorised

$$\begin{aligned} 16(y-3)\left(y^2+3y+9\right) &= 16\left[\left(y\left(y^2+3y+9\right) - 3\left(y^2+3y+9\right)\right]\right. \\ &= 16\left[y^3+3y^2+9y-3y^2-9y-27\right] \\ &= 16y^3 - 432 \end{aligned}$$

WORKED EXAMPLE 17: FACTORISING A SUM OF TWO CUBES

Question

Factorise: $8t^3 + 125p^3$.

Solution

Step 1: Take the cube root of terms that are perfect cubes

We are working with the sum of two cubes. We know that $x^3 + y^3 = (x+y)\left(x^2 - xy + y^2\right)$, so we need to identify x and y. We start by noting that $\sqrt[3]{8t^3} = 2t$ and $\sqrt[3]{125p^3} = 5p$. These give the terms in the first bracket. This also tells us that $x = 2t$ and $y = 5p$.

Step 2: Find the three terms in the second bracket We can replace x and y in the factorised form of the expression for the difference of two cubes with $2t$ and $5p$. Doing so we get the second bracket:

$$\begin{aligned}\left(8t^3 + 125p^3\right) &= (2t+5p)\left[(2t)^2 - (2t)(5p) + (5p)^2\right] \\ &= (2t+5p)\left(4t^2 - 10tp + 25p^2\right)\end{aligned}$$

Step 3: Expand the brackets to check that the expression has been correctly factorised

$$\begin{aligned}(2t+5p)\left(4t^2 - 10tp + 25p^2\right) &= 2t\left(4t^2 - 10tp + 25p^2\right) + 5p\left(4t^2 - 10tp + 25p^2\right) \\ &= 8t^3 - 20pt^2 + 50p^2t + 20pt^2 - 50p^2t + 125p^3 \\ &= 8t^3 + 125p^3\end{aligned}$$

8 SIMPLIFICATION OF FRACTIONS

We have studied procedures for working with fractions in earlier grades.

1. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \quad (b \neq 0; d \neq 0)$
2. $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad (b \neq 0)$
3. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc} \quad (b \neq 0; d \neq 0)$

Note: dividing by a fraction is the same as multiplying by the reciprocal of the fraction.

In some cases of simplifying an algebraic expression, the expression will be a fraction. For example,

$$\frac{x^2 + 3x}{x + 3}$$

has a quadratic binomial in the numerator and a linear binomial in the denominator. We have to apply the different factorisation methods in order to factorise the numerator and the denominator before we can simplify the expression.

$$\frac{x^2 + 3x}{x + 3} = \frac{x(x+3)}{x+3} = x \quad (x \neq -3)$$

If $x = -3$ then the denominator, $x + 3 = 0$ and the fraction is undefined.

WORKED EXAMPLE 18: SIMPLIFYING FRACTIONS

Question

Simplify:

$$\frac{ax - b + x - ab}{ax^2 - abx}, \quad (x \neq 0; x \neq b)$$

Solution

Step 1: Use grouping to factorise the numerator and take out the common factor ax in the denominator

$$\frac{(ax - ab) + (x - b)}{ax^2 - abx} = \frac{a(x-b) + (x-b)}{ax(x-b)}$$

Step 2: Take out common factor ($x - b$) in the numerator

$$= \frac{(x-b)(a+1)}{ax(x-b)}$$

Step 3: Cancel the common factor in the numerator and the denominator to give the final answer

$$= \frac{a+1}{ax}$$

WORKED EXAMPLE 19: SIMPLIFYING FRACTIONS

Question

Simplify:

$$\frac{x^2 - x - 2}{x^2 - 4} \div \frac{x^2 + x}{x^2 + 2x}, \quad (x \neq 0; x \neq \pm 2)$$

Solution

Step 1: Factorise the numerator and denominator

$$= \frac{(x+1)(x-2)}{(x+2)(x-2)} \div \frac{x(x+1)}{x(x+2)}$$

Step 2: Change the division sign and multiply by the reciprocal

$$= \frac{(x+1)(x-2)}{(x+2)(x-2)} \times \frac{x(x+2)}{x(x+1)}$$

Step 3: Cancel the terms and write the final answer

$$= 1$$

WORKED EXAMPLE 20: SIMPLIFYING FRACTIONS

Question

Simplify:

$$\frac{x-2}{x^2-4}+\frac{x^2}{x-2}-\frac{x^3+x-4}{x^2-4}, \quad (x \neq \pm 2)$$

Solution

Step 1: Factorise the denominators

$$=\frac{x-2}{(x+2)(x-2)}+\frac{x^2}{x-2}-\frac{x^3+x-4}{(x+2)(x-2)}$$

Step 2: Make all denominators the same so that we can add or subtract the fractions

The lowest common denominator is $(x-2)(x+2)$.

$$=\frac{x-2}{(x+2)(x-2)}+\frac{\left(x^2\right)(x+2)}{(x+2)(x-2)}-\frac{x^3+x-4}{(x+2)(x-2)}$$

Step 3: Write as one fraction

$$=\frac{x-2+\left(x^2\right)(x+2)-\left(x^3+x-4\right)}{(x+2)(x-2)}$$

Step 3: Simplify

$$\frac{x-2+x^3+2x^2-x^3-x+4}{(x+2)(x-2)}=\frac{2x^2+2}{(x+2)(x-2)}$$

Step 4: Take out the common factor and write the final answer

$$\frac{2\left(x^2+1\right)}{(x+2)(x-2)}$$

WORKED EXAMPLE 21: SIMPLIFYING FRACTIONS

Question

Simplify:

$$\frac{2}{x^2 - x} + \frac{x^2 + x + 1}{x^3 - 1} - \frac{x}{x^2 - 1}, \quad (x \neq 0; x \neq \pm 1)$$

Solution

Step 1: Factorise the numerator and denominator

$$= \frac{2}{x(x-1)} + \frac{\left(x^2 + x + 1\right)}{(x-1)\left(x^2 + x + 1\right)} - \frac{x}{(x-1)(x+1)}$$

Step 2: Simplify and find the common denominator

$$= \frac{2(x+1) + x(x+1) - x^2}{x(x-1)(x+1)}$$

Step 3: Write the final answer

$$= \frac{2x + 2 + x^2 + x - x^2}{x(x-1)(x+1)} = \frac{3x + 2}{x(x-1)(x+1)}$$

9 CHAPTER SUMMARY

-
  - $\mathbb{N}$: natural numbers are $\{1; 2; 3; \ldots\}$
  - $\mathbb{N}_0$: whole numbers are $\{0; 1; 2; 3; \ldots\}$
  - $\mathbb{Z}$: integers are $\{\ldots; -3; -2; -1; 0; 1; 2; 3; \ldots\}$
- A rational number is any number that can be written as $\frac{a}{b}$ where a and b are integers and $b \neq 0$.
- The following are rational numbers:
  - Fractions with both numerator and denominator as integers
  - Integers
  - Decimal numbers that terminate
  - Decimal numbers that recur (repeat)
- Irrational numbers are numbers that cannot be written as a fraction with the numerator and denominator as integers.
- If the n^{th} root of a number cannot be simplified to a rational number, it is called a surd.
- If a and b are positive whole numbers, and $a < b$, then $\sqrt[n]{a} < \sqrt[n]{b}$.
- A binomial is an expression with two terms.
- The product of two identical binomials is known as the square of the binomial.
- We get the difference of two squares when we multiply $(ax + b)(ax - b)$.
- Factorising is the opposite process of expanding the brackets.
- The product of a binomial and a trinomial is:

$$(A + B)(C + D + E) = A(C + D + E) + B(C + D + E)$$

- Taking out a common factor is the basic factorisation method.
- We often need to use grouping to factorise polynomials.
- To factorise a quadratic we find the two binomials that were multiplied together to give the quadratic.
- The sum of two cubes can be factorised as:

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

- The difference of two cubes can be factorised as:

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

- We can simplify fractions by incorporating the methods we have learnt to factorise expressions.
- Only factors can be cancelled out in fractions, never terms.
- To add or subtract fractions, the denominators of all the fractions must be the same.

10 EXERCISES

10.1 Exercise 1

1. The figure here shows the Venn diagram for the special sets $\mathbb{N}, \mathbb{N}_0$ and $\mathbb{Z}$.

   1.1 Where does the number $-\frac{12}{3}$ belong in the diagram?

   1.2 In the following list, there is one false statements and two true statements. Which of the statements is true?

      1. Every integer is a natural number.
      2. Every natural number is a whole number.
      3. There are no decimals in the whole numbers.

2. The figure here shows the Venn diagram for the special sets $\mathbb{N}$, $\mathbb{N}_0$ and $\mathbb{Z}$

   2.1 Where does the number $-\frac{1}{2}$ belong in the diagram?

   2.2 In the following list, there are two false statements and one true statement. Which of the statements is true?

      1. Every integer is a natural number.
      2. Every whole number is an integer.
      3. There are decimals in the whole numbers.

3. State whether the following numbers are real, non-real or undefined.

   3.1 $-\sqrt{3}$

   3.2 $\frac{0}{\sqrt{2}}$

   3.3 $\sqrt{-9}$

   3.4 $\frac{-\sqrt{7}}{0}$

   3.5 $-\sqrt{-16}$

   3.6 $\sqrt{2}$

4. State whether the following numbers are rational or irrational. If the number is rational, state whether it is a natural number, whole number or an integer.

   4.1 $-\frac{1}{3}$

   4.2 $0,651268962154862\ldots$

   4.3 $\frac{\sqrt{9}}{3}$

   4.4 π^2

   4.5 π^4

   4.6 $\sqrt[3]{19}$

   4.7 $(\sqrt[3]{1})^7$

   4.8 $\pi + 3$

   4.9 $\pi + 0,858408346$

5. If a is an integer, b is an integer and c is irrational, which of the following are rational numbers?

   5.1 $\frac{5}{6}$

   5.2 $\frac{a}{3}$

   5.3 $\frac{-2}{b}$

   5.4 $\frac{1}{c}$

6. For each of the following values of a state whether $\frac{a}{14}$ is rational or irrational.

   6.1 1

   6.2 -10

   6.3 $\sqrt{2}$

   6.4 $2,1$

7. Consider the following list of numbers:
   $-3; 0; \sqrt{-1}; -8\frac{4}{5}; -\sqrt{8}; \frac{22}{7}; \frac{14}{0}; 7; 1,\overline{34}; 3,3231089\ldots; 3+\sqrt{2}; 9\frac{7}{10}; \pi; 11$
   Which of the numbers are:

   7.1 Natural numbers

   7.2 Irrational numbers

   7.3 Non-real numbers

   7.4 Rational numbers

   7.5 Integers

   7.6 Undefined

8. For each of the following numbers:
   1. Write the next three digits and
   2. State whether the number is rational or irrational.

   8.1 $1,1\dot{5}$

   8.2 $2,121314\ldots$

   8.3 $1,242244246\ldots$

   8.4 $3,324354\ldots$

   8.5 $3,32435\dot{4}$

10.2 Exercise 2

1. Write the following as fractions:

   1.1 $0,1$

   1.2 $0,12$

   1.3 $0,58$

   1.4 $0,2589$

2. Write the following using the recurring decimal notation:

   2.1 $0,1111111\ldots$

   2.2 $0,1212121212\ldots$

   2.3 $0,123123123123\ldots$

   2.4 $0,11414541454145\ldots$

3. Write the following in decimal form, using the recurring decimal notation:

   3.1 $\frac{25}{45}$

   3.2 $\frac{10}{18}$

   3.3 $\frac{7}{33}$

   3.4 $\frac{2}{3}$

   3.5 $1\frac{3}{11}$

   3.6 $4\frac{5}{6}$

   3.7 $2\frac{1}{9}$

4. Write the following decimals in fractional form:

   4.1 $0,\dot{5}$

   4.2 $0,6\dot{3}$

   4.3 $0,\dot{4}$

   4.4 $5,\overline{31}$

   4.5 $4,\overline{93}$

   4.6 $3,\overline{93}$

10.3 Exercise 3

1. Round off the following to 3 decimal places:

   1.1 $12,56637061\ldots$

   1.2 $3,31662479\ldots$

   1.3 $0,2666666\ldots$

   1.4 $1,912931183\ldots$

   1.5 $6,32455532\ldots$

   1.6 $0,05555555\ldots$

2. Round off each of the following to the indicated number of decimal places:

   2.1 $345,04399906$ to 4 decimal places

   2.2 $1361,72980445$ to 2 decimal places

   2.3 $728,00905239$ to 6 decimal places

   2.4 $\frac{1}{27}$ to 4 decimal places

   2.5 $\frac{45}{99}$ to 5 decimal places

   2.6 $\frac{1}{12}$ to 2 decimal places

3. Study the diagram above:

   3.1 Calculate the area of $ABDE$ to 2 decimal places.

   3.2 Calculate the area of BCD to 2 decimal places.

   3.3 Using your answers in (a) and (b) calculate the areas of $ABCDE$

   3.4 Without rounding off, what is the area of $ABCDE$?

4. Given $i = \frac{r}{600}$; $r = 7$; $n = 96$; $P = 200000$.

    4.1 Calculate i correct to 2 decimal places

    4.2 Using your answer from (a), calculate A in $A = P(1+i)^n$

    4.3 Calculate A without rounding off your answer in (a), compare this answer with your answer in (b)

5. If it takes 1 person to carry 3 boxes, how many people are needed to carry 31 boxes?

6. If 7 tickets cost $R35,20$, how much does one ticket cost?

10.4 Exercise 4

1. Determine between which two consecutive integers the following lie, without using a calculator:

    1.1 $\sqrt{18}$

    1.2 $\sqrt[3]{81}$

    1.3 $\sqrt{29}$

    1.4 $\sqrt[3]{5}$

    1.5 $\sqrt[3]{79}$

    1.6 $\sqrt{155}$

    1.7 $\sqrt{57}$

    1.8 $\sqrt{71}$

    1.9 $\sqrt[3]{123}$

    1.10 $\sqrt[3]{90}$

2. Estimate the following surds to the nearest 1 decimal place, without using a calculator.

    2.1 $\sqrt{10}$

    2.2 $\sqrt{82}$

    2.3 $\sqrt{15}$

    2.4 $\sqrt{90}$

3. Consider the following list of numbers:

    $\frac{27}{7}$; $\sqrt{19}$; 2π; $0,45$; $0,\overline{45}$; $-\sqrt{\frac{9}{4}}$; 6; $-\sqrt{8}$; $\sqrt{51}$

    Without using a calculator, rank all the numbers in ascending order.

10.5 Exercise 5

1. Expand the following products:

   1.1 $-(7-x)(7+x)$

   1.2 $2y(y+4)$

   1.3 $(3x-1)(3x+1)$

   1.4 $(7k+2)(3-2k)$

   1.5 $(1-4x)^2$

   1.6 $(-3-y)(5-y)$

   1.7 $(8-x)(8+x)$

   1.8 $(9+x)^2$

   1.9 $(-7y+11)(-12y+3)$

   1.10 $(g-5)^2$

   1.11 $(d+9)^2$

   1.12 $(y+5)(y+2)$

   1.13 $(6d+7)(6d-7)$

   1.14 $(5z+1)(5z-1)$

   1.15 $(1-3h)(1+3h)$

   1.16 $(2p+3)(2p+2)$

   1.17 $(8a+4)(a+7)$

   1.18 $(5r+4)(2r+4)$

   1.19 $(w+1)(w-1)$

   1.20 $(2-t)(1-2t)$

   1.21 $(x-4)(x+4)$

   1.22 $-(4-x)(x+4)$

   1.23 $-(a+b)(b-a)$

   1.24 $(2p+9)(3p+1)$

   1.25 $(3k-2)(k+6)$

   1.26 $(s+6)^2$

2. Expand the following products:

   2.1 $(g+11)(g-11)$

   2.2 $(4b-2)(2b-4)$

   2.3 $(4b-3)(2b-1)$

   2.4 $(6x-4)(3x+6)$

   2.5 $(3w-2)(2w+7)$

   2.6 $(2t-3)^2$

   2.7 $(5p-8)^2$

   2.8 $(4y+5)^2$

   2.9 $(2y^6+3y^5)(-5y-12)$

10.6 Exercise 6

1. Expand the following products:

   1.1 $9(8y^2-2y+3)$

   1.2 $3m(9m^2+2)+5m^2(5m+6)$

   1.3 $4x^2(10x^3+4)+4x^3(2x^2+6)$

   1.4 $3k^3(k^2+3)+2k^2(6k^3+7)$

   1.5 $(3x+2)(3x-2)(9x^2-4)$

   1.6 $(-6y^4+11y^2+3y)(y+4)(y-4)$

   1.7 $(x+2)(x-3)(x^2+2x-3)$

   1.8 $(a+2)^2-(2a-4)^2$

   1.9 $(-2y^2-4y+11)(5y-12)$

   1.10 $(7y^2-6y-8)(-2y+2)$

   1.11 $(10y+3)(-2y^2-11y+2)$

   1.12 $(-12y-3)(2y^2-11y+3)$

   1.13 $(-10)(2y^2+8y+3)$

   1.14 $(7y+3)(7y^2+3y+10)$

   1.15 $(a+2b)(a^2+b^2+2ab)$

   1.16 $(x+y)(x^2-xy+y^2)$

2. Expand the following products:

   2.1 $(2x+3)^2-(x-2)^2$

   2.2 $\left(x-\frac{2}{x}\right)\left(\frac{x}{3}+\frac{4}{x}\right)$

   2.3 $\frac{1}{2}(10x-12y)+\frac{1}{3}(15x-18y)$

   2.4 $\frac{1}{2}a(4a+6b)+\frac{1}{4}(8a+12b)$

   2.5 $\left(2a^2-a-1\right)\left(a^2+3a+2\right)$

   2.6 $\left(y^2+4y-1\right)\left(1-4y-y^2\right)$

   2.7 $2(x-2y)\left(x^2+xy+y^2\right)$

   2.8 $3(a-3b)\left(a^2+3ab-b^2\right)$

   2.9 $(2a-b)(2a+b)\left(2a^2-3ab+b^2\right)$

   2.10 $2(3x+y)(3x-y)-(3x-y)^2$

   2.11 $(x+y)(x-3y)+(2x-y)^2$

   2.12 $\left(\frac{x}{3}-\frac{3}{x}\right)\left(\frac{x}{4}+\frac{4}{x}\right)$

10.7 Exercise 7

1. What is the value of b, in $(x+b)(x-1)=x^2+3x-4$?

2. What is the value of g, in $(x-2)(x+g)=x^2-6x+8$?

3. In $(x-4)(x+k)=x^2+bx+c$:

   3.1 For which of these values of k will b be positive?
   $-3; -1; 0; 3; 5$

   3.2 For which of these values of k will c be positive?
   $-3; -1; 0; 3; 5$

   3.3 For what real values of k will c be positive?

   3.4 For what real values of k will b be positive?

4. Answer the following:

   4.1 Expand: $\left(x+\frac{4}{x}\right)^2$

   4.2 Given that $\left(x+\frac{4}{x}\right)^2=14$, determine the value of $x^2+\frac{16}{x^2}$ without solving for x.

5. Answer the following:

   5.1 Expand: $\left(a+\frac{1}{a}\right)^2$

   5.2 Given that $\left(a+\frac{1}{a}\right)=3$, determine the value of $\left(a+\frac{1}{a}\right)^2$ without solving for a.

5.3 Given that: $\left(a - \frac{1}{a}\right) = 3$ determine the value of $\left(a + \frac{1}{a}\right)^2$ without solving for a.

6. Answer the following:

   6.1 Expand: $\left(3y + \frac{1}{2y}\right)^2$

   6.2 Given that $3y + \frac{1}{2y} = 4$, determine the value of $\left(3y + \frac{1}{2y}\right)^2$ without solving for y.

7. Answer the following:

   7.1 Expand: $\left(a + \frac{1}{3a}\right)^2$

   7.2 Expand: $\left(a + \frac{1}{3a}\right)\left(a^2 - \frac{1}{3} + \frac{1}{9a^2}\right)$

   7.3 Given that $a + \frac{1}{3a} = 2$, determine the value of $a^3 + \frac{1}{27a^3}$ without solving for a.

10.8 Exercise 8

1. Factorise:

   1.1 $12x + 32y$

   1.2 $125x^6 - 5y$

   1.3 $6x^2 + 2x + 10x^3$

   1.4 $2xy^2 + xy^2z + 3xy$

   1.5 $12k^2j + 24k^2j^2$

   1.6 $3a^2 + 6a - 18$

   1.7 $7a + 4$

   1.8 $-2ab^2 - 4a^2b$

   1.9 $18ab - 3bc$

   1.10 $12kj + 18kq$

   1.11 $-12a + 24a^3$

   1.12 $-2ab - 8a$

   1.13 $24kj - 16k^2j$

   1.14 $-a^2b - b^2a$

   1.15 $72b^2q - 18b^3q^2$

10.9 Exercise 9

1. Factorise:

   1.1 $4(y-3)+k(3-y)$

   1.2 $4x^2-(4x-3y)^2$

   1.3 $16a^2-(3b+4c)^2$

   1.4 $(b-4)^2-9(b-5)^2$

   1.5 $4(a-3)^2-49(4a-5)^2$

   1.6 $16k^2-4$

   1.7 $a^2b^2c^2-1$

   1.8 $\frac{1}{y}a^2-4b^2$

   1.9 $\frac{1}{2}x^2-2$

   1.10 y^2-8

   1.11 y^2-13

   1.12 $a^2(a-1)-25(a-1)$

   1.13 $a^2\left(a-2ab-15b^2\right)-9b^2\left(a^2-2ab-15b^2\right)$

   1.14 $bm(b+4)-6m(b+4)$

   1.15 $a^2(a+7)+9(a+7)$

   1.16 $3b(b-4)-7(4-b)$

   1.17 $3g(z+6)+2(6+z)$

   1.18 $4b(y+2)+5(2+y)$

   1.19 $3d(r+5)+14(5+r)$

   1.20 $6x+y)^2-9$

10.10 Exercise 10

1. Factorise the following:

   1.1 $6d-9r+2t^5d-3t^5r$

   1.2 $14m-4n+7jm-2jn$

   1.3 $28r-20x+7gr-5gx$

   1.4 $25d-15m+5yd-3ym$

   1.5 $45q-18z+5cq-2cz$

1.6 $6j - 15v + 2yj - 5yv$

1.7 $16a - 40k + 2za - 5zk$

1.8 $ax - bx + ay - by + 2a - 2b$

1.9 $3ax + bx - 3ay - by - 9a - 3b$

1.10 $9z - 18m + b^3z - 2b^3m$

1.11 $35z - 10y + 7c^5z - 2c^5y$

1.12 $6x + a + 2ax + 3$

1.13 $x^2 - 6x + 5x - 30$

1.14 $5x + 10y - ax - 2ay$

1.15 $a^2 - 2a - ax + 2x$

1.16 $5xy - 3y + 10x - 6$

1.17 $ab - a^2 - a + b$

10.11 Exercise 11

1. Factorise the following:

1.1 $x^2 + 8x + 15$

1.2 $x^2 - x - 20$

1.3 $2x^2 - 22x + 20$

1.4 $6a^2 + 14a + 8$

1.5 $6v^2 - 27v + 27$

1.6 $6g^2 - 15g - 9$

1.7 $3x^2 + 19x + 6$

1.8 $3x^2 + 17x - 6$

1.9 $7x^2 - 6x - 1$

1.10 $6x^2 - 15x - 9$

1.11 $a^2 - 7ab + 12b^2$

1.12 $x^2 + 9x + 8$

1.13 $3a^2 + 5ab - 12b^2$

1.14 $98x^4 + 14x^2 - 4$

1.15 $(x - 2)^2 - 7(x - 2) + 12$

1.16 $(a - 2)^2 - 4(a - 2) - 5$

1.17 $(y+3)^2-3(y+3)-18$

1.18 $3\left(b^2+5b\right)+12$

1.19 $6\left(a^2+3a\right)-168$

1.20 $x^2+12x+36$

1.21 $2h^2+5h-3$

1.22 $3x^2+4x+1$

1.23 $3s^2+s-10$

1.24 $x^2-2x-15$

1.25 x^2+2x-3

1.26 x^2+x-20

10.12 Exercise 12

1. Factorise:

1.1 w^3-8

1.2 $125x^3+1$

1.3 $25x^3+1$

1.4 $z-125z^4$

1.5 $8m^6+n^9$

1.6 $216n^3-k^3$

1.7 $125s^3+d^3$

1.8 $8k^3+r^3$

1.9 $8j^3k^3l^3-b^3$

1.10 $27x^3y^3+w^3$

1.11 $128m^3+2f^3$

1.12 g^3+64

1.13 $p^{15}-\frac{1}{8}y^{12}$

1.14 $\frac{27}{t^3}-s^3$

1.15 $\frac{1}{64q^3}-h^3$

1.16 $72g^3+\frac{1}{3}v^3$

1.17 $1-(x-y)^3$

1.18 $h^4\left(8g^6+h^3\right)-\left(8g^6+h^3\right)$

1.19 $x\left(125w^3 - h^3\right) + y\left(125w^3 - h^3\right)$

1.20 $x^2\left(27p^3 + w^3\right) - 5x\left(27p^3 + w^3\right) - 6\left(27p^3 + w^3\right)$

1.21 $h^3 + 1$

1.22 $x^3 + 8$

1.23 $27 - m^3$

1.24 $2x^3 - 2y^3$

1.25 $3k^3 + 81q^3$

1.26 $64t^3 - 1$

1.27 $64x^3 - 1$

10.13 Exercise 13

1. Simplify (assume all denominators are non-zero):

1.1 $\frac{3a}{15}$

1.2 $\frac{9x^2-16}{6x-8}$

1.3 $\frac{b^2-81a^2}{18a-2b}$

1.4 $\frac{t^2-8^2}{s^2-2at+t^2}$

1.5 $\frac{x^2-2x-15}{5x-25}$

1.6 $\frac{x^2+2x-15}{x^2+8x+15}$

1.7 $\frac{x^2-x-6}{x^3-27}$

1.8 $\frac{a^2+6a-16}{a^3-8}$

1.9 $\frac{a^2-4ab-12b^2}{a^2+4ab+4b^2}$

1.10 $\frac{6a^2-7a-3}{3ab+b}$

1.11 $\frac{2x^2-x-1}{x^3-x}$

1.12 $\frac{2a+10}{4}$

1.13 $\frac{qz+qr+16z+16r}{z+r}$

1.14 $\frac{pz-pq+5z-5q}{z-q}$

1.15 $\frac{hx-hg+13x-13g}{x-g}$

1.16 $\frac{f^2a-fa^2}{f-a}$

1.17 $\frac{5a+20}{a+4}$

1.18 $\frac{a^2-4a}{a-4}$

1.19 $\frac{3a^2-9a}{2a-6}$

1.20 $\frac{9a+27}{9a+18}$

1.21 $\frac{6ab+2a}{2b}$

1.22 $\frac{16x^2y-8xy}{12x-6}$

1.23 $\frac{4xyp-8xp}{12xy}$

2. Simplify (assume all denominators are non-zero):

2.1 $\frac{b^2+10b+21}{3(b^2-9)} \div \frac{2b^2+14b}{30b^2-90b}$

2.2 $\frac{5ab-15b}{4a-12} \div \frac{6b^2}{a+b}$

2.3 $\frac{16-x^2}{x^2-x-12} \times \frac{x+3}{x+4}$

2.4 $\frac{a^3+b^3}{a^3} \times \frac{5a+5b}{a^2+2ab+b^2}$

2.5 $\frac{a-4}{a+5a+4} \times \frac{a^2+2a+1}{a^2-3a-4}$

2.6 $\frac{3x+2}{x^2-6x+8} \times \frac{x-2}{3x^2+8x+4}$

2.7 $\frac{a^2-2a-8}{a^2+6a+8} \times \frac{a^2+a-12}{3} - \frac{3}{2}$

2.8 $\frac{4x^2-1}{3x^2+10x+3} \div \frac{6x^2+5x+1}{4x^2-7x+3} \times \frac{9x^2+6x+1}{8x^2-6x+1}$

2.9 $\frac{x+4}{3} - \frac{x-2}{2}$

2.10 $\frac{p^3+q^3}{p^2} \times \frac{3p-3q}{p^2-q^2}$

2.11 $\frac{x^2+17x+70}{5(x^2-100)} \div \frac{3x^2+21x}{45x^2-450x}$

2.12 $\frac{z^2+17z+66}{3(z^2-121)} \div \frac{2z^2+12z}{24z^2-264z}$

2.13 $\frac{3a+9}{14} \div \frac{7a+21}{a+3}$

2.14 $\frac{a^2-5a}{2a+10} \times \frac{4a}{3a+15}$

2.15 $\frac{3xp+4p}{8p} \div \frac{12p^2}{3x+4}$

2.16 $\frac{24a-8}{12} \div \frac{9a-3}{6}$

2.17 $\frac{a^2+2a}{5} \div \frac{2a+4}{20}$

2.18 $\frac{p^2+pq}{7p} \times \frac{21q}{8p+8q}$

10.14 Exercise 14

1. Simplify (assume all denominators are non-zero):

1.1 $\frac{x-3}{3} - \frac{x+5}{4}$

1.2 $\frac{t+2}{3q} + \frac{t+1}{2q}$

1.3 $\frac{3}{p^2-4} + \frac{2}{(p-2)^2}$

1.4 $\frac{x}{x+y} + \frac{x^2}{y^2-x^2}$

1.5 $\frac{1}{m+n} + \frac{3mn}{m^3+n^3}$

1.6 $\frac{h}{h^3-f^3} - \frac{1}{h^2+hf+f^2}$

1.7 $\frac{x^2-1}{3} \times \frac{1}{x-1} - \frac{1}{2}$

1.8 $\frac{x^2-2x+1}{(x-1)^3} - \frac{x^2+x+1}{x^3-1}$

1.9 $\frac{1}{(x-1)^2} - \frac{2x}{x^3-1}$

1.10 $\frac{t^2+2t-8}{t^2+t-6} + \frac{1}{t^2-9} + \frac{t+1}{t-3}$

1.11 $\frac{x^2-3x+9}{x^3+27} + \frac{x-2}{x^2+4x+3} - \frac{1}{x-2}$

1.12 $\frac{2x-4}{9} - \frac{x-3}{4} + 1$

1.13 $\frac{1}{a^2-4ab+4b^2} + \frac{a^2+2ab+b^2}{a^3-8b^3} - \frac{1}{a^2-4b^2}$

1.14 $1 + \frac{3x-4}{4} - \frac{x+2}{3}$

1.15 $\frac{11}{a+11} + \frac{8}{a-8}$

1.16 $\frac{12}{x-12} - \frac{6}{x-6}$

1.17 $\frac{12}{r+12} + \frac{8}{r-8}$

1.18 $\frac{2}{xy} + \frac{4}{xz} + \frac{3}{yz}$

1.19 $\frac{5}{t-2} - \frac{1}{t-3}$

1.20 $\frac{k+2}{k^2+2} - \frac{1}{k+2}$

2. What are the restrictions in the following:

2.1 $\frac{1}{x-2}$

2.2 $\frac{3x-9}{4x+4}$

2.3 $\frac{3}{x} - \frac{1}{x^2-1}$

11 ANSWERS FOR EXERCISES

11.1 Exercise 1

1.1 $\mathbb{Z}$

1.2 (2) and (3)

2.1 It is in the space between the rectangle and $\mathbb{Z}$

2.2 Only (2) is true.

3.1 Real

3.2 Real

3.3 Non-real

3.4 Undefined

3.5 Non-real

3.6 Real

4.1 Only a rational number

4.2 Irrational number

4.3 Rational, Integer, Whole number and a Natural number

4.4 Irrational

4.5 Irrational

4.6 Irrational

4.7 Rational, Integer, Whole number and a Natural number

4.8 Irrational

4.9 Irrational

5.1 Rational

5.2 Rational

5.3 Rational

5.4 Irrational

6.1 Rational

6.2 Rational

6.3 Irrational

6.4 Rational

7.1 7 and 11 are natural numbers.

7.2 $-\sqrt{8}$; 3,3231089 . . . ; $3+\sqrt{2}$; π are all irrational

7.3 $-3; 0; -8\frac{4}{5}; \frac{22}{7}; 7; 1,\overline{34}; 9\frac{7}{10}; 11$ are all rational numbers

7.4 Only $\sqrt{-1}$ is non-real

7.5 $-3; 7; 11$ are integers

7.6 Only $\frac{14}{0}$ is undefined

8.1 The next three digits are: 555 - Rational

8.2 Irrational there is no repeating pattern

8.3 Irrational there is no repeating pattern

8.4 Irrational there is no repeating pattern

8.5 Rational

11.2 Exercise 2

1.1 $0,1 = \frac{1}{10}$

1.2 $\frac{3}{25}$

1.3 $\frac{29}{50}$

1.4 $\frac{2589}{10000}$

2.1 $0,\dot{1}$

2.2 $0,\overline{12}$

2.3 $0,\overline{123}$

2.4 $0,11\overline{4145}$

3.1 $0,\dot{5}$

3.2 $0,\dot{5}$

3.3 $0,2\dot{2}$

3.4 $0,\dot{6}$

3.5 $1,\overline{27}$

3.6 $4,8\dot{3}$

3.7 $2,\dot{1}$

4.1 $x = \frac{5}{9}$

4.2 $x = \frac{57}{90}$

4.3 $x = \frac{4}{9}$

4.4 $x = \frac{526}{99}$

4.5 $x = \frac{163}{33}$

4.6 $x = \frac{130}{33}$

11.3 Exercise 3

1.1 $12,566$

1.2 $3,317$

1.3 $0,267$

1.4 $1,913$

1.5 $6,325$

1.6 $0,056$

2.1 $345,0440$

2.2 $1361,73$

2.3 $728,009052$

2.4 $0,0370$

2.5 $0,45455$

2.6 $0,08$

3.1 $9,87$

3.2 $4,93$

3.3 $14,80$

3.4 $14,8044\ldots$

4.1 $0,01$

4.2 $519854,59$

4.3 $648768,22$; There is a $170913,63$ difference.

5 11 people

6 One ticket costs R$5,03$

11.4 Exercise 4

1.1 4 and 5

1.2 4 and 5

1.3 5 and 6

1.4 1 and 2

1.5 4 and 5

1.6 12 and 13

1.7 7 and 8

1.8 8 and 9

1.9 4 and 5

1.10 4 and 5

2.1 $3,1$ or $3,2$

2.2 $9,1$

2.3 $3,9$

2.4 $9,5$

3 $-\sqrt{8}; -\sqrt{\frac{9}{4}}; 0,45; 0,\overline{45}; \frac{27}{7}; \sqrt{19}; 6; 2\pi; \sqrt{51}$

11.5 Exercise 5

1.1 $x^2 - 49$

1.2 $2y^2 + 8y$

1.3 $9x^2 - 1$

1.4 $-14k^2 + 17k + 6$

1.5 $16x^2 - 8x + 1$

1.6 $y^2 - 2y - 15$

1.7 $-x^2 + 64$

1.8 $x^2 + 18x + 81$

1.9 $84y^2 - 153y + 33$

1.10 $g^2 - 10g + 25$

1.11 $d^2 + 18d + 81$

1.12 $y^2 + 7y + 10$

1.13 $36d^2 - 49$

1.14 $25z^2 - 1$

1.15 $1 - 9h^2$

1.16 $4p^2 + 10p + 6$

1.17 $8a^2 + 60a + 28$

1.18 $10r^2 + 28r + 16$

1.19 $w^2 - 1$

1.20 $2t^2 - 5t + 2$

1.21 $x^2 - 16$

1.22 $x^2 - 16$

1.23 $a^2 - b^2$

1.24 $6p^2 + 29p + 9$

1.25 $3k^2 + 16k - 12$

1.26 $s^2 + 12s + 36$

2.1 $g^2 - 121$

2.2 $8b^2 - 20b + 8$

2.3 $8b^2 - 10b + 3$

2.4 $18x^2 + 24x - 24$

2.5 $6w^2 + 17w - 14$

2.6 $4t^2 - 12t + 9$

2.7 $25p^2 - 80p + 64$

2.8 $16y^2 + 40y + 25$

2.9 $-10y^7 - 39y^6 - 36y^5$

11.6 Exercise 6

1.1 $72y^2 - 18y + 27$

1.2 $52m^3 + 6m + 30m^2$

1.3 $48x^5 + 16x^2 + 24x^3$

1.4 $15k^5 + 9k^3 + 14k^2$

1.5 $81x^4 - 72x^2 + 16$

1.6 $-6y^6 + 107y^4 + 3y^3 - 176y^2 - 48y$

1.7 $x^4 + x^3 - 11x^2 - 9x + 18$

1.8 $-3a^2 + 20a - 12$

1.9 $-10y^3 + 4y^2 + 103y - 132$

1.10 $-14y^3 + 26y^2 + 4y - 16$

1.11 $-20y^3 - 116y^2 - 13y + 6$

1.12 $-24y^3 + 126y^2 - 3y - 9$

1.13 $-20y^2 - 80y - 30$

1.14 $49y^3 + 42y^2 + 79y + 30$

1.15 $a^3 + 4a^2b + 5ab^2 + 2b^3$

1.16 $x^3 + y^3$

2.1 $3x^2 + 16x + 5$

2.2 $\frac{x^2}{3} + \frac{10}{3} - \frac{8}{x^2}$

2.3 $10x - 12y$

2.4 $2a^2 + 3ab + 2a + 3b$

2.5 $2a^4 + 5a^3 - 5a - 2$

2.6 $-y^4 - 8y^3 - 14y^2 + 8y - 1$

2.7 $2x^3 - 2x^2y - 2xy^2 - 4y^3$

2.8 $3a^3 - 30ab^2 + 9b^3$

2.9 $8a^4 - 12a^3b + 2a^2b^2 + 3ab^3 - b^4$

2.10 $9x^2 + 6xy - 3y^2$

2.11 $5x^2 - 6xy - 2y^2$

2.12 $\frac{x^2}{12} + \frac{7}{12} - \frac{12}{x^2}$

11.7 Exercise 7

1 $-x + 4x = 3x$

2 -4

31 $k = 5$

32 $k = -3$ or $k = -1$

33 $k < 0$

34 $k > 4$

4.1 $x^2 + 8 + \frac{16}{x^2}$

4.2 $6 = x^2 + \frac{16}{x^2}$

5.1 $a^2 + 2 + \frac{1}{a^2}$

5.2 9

5.3 13

6.1 $9y^2 + 3 + \frac{1}{4y^2}$

6.2 16

7.1 $a^2 + \frac{2}{3} + \frac{1}{9a^2}$

7.2 $a^3 + \frac{1}{27a^3}$

7.3 6

11.8 Exercise 8

1.1 $4(3x + 8y)$

1.2 $5(5x^3 - y)(5x^3 + y)$

1.3 $2x(3x + 1 + 5x^2)$

1.4 $xy(2y + yz + 3)$

1.5 $12k^2j(1 + 2j)$

1.6 $3(a^2 + 2a - 6)$

1.7 $7a + 4$

1.8 $-2ab(b + 2a)$

1.9 $3b(6a - c)$

1.10 $6k(2j + 3q)$

1.11 $12a(-1 + 2a^2)$

1.12 $-2a(b + 4)$

1.13 $8kj(3 - 2k)$

1.14 $-ab(a + b)$

1.15 $18b^2q(4 - bq)$

11.9 Exercise 9

1.1 $(y - 3)(4 - k)$

1.2 $3(2x - y)(3y - 2x)$

1.3 $(4a + 3b + 4c)(4a - 3b - 4c)$

1.4 $(-2b+11)(4b-19)$

1.5 $(29-26a)(30a-41)$

1.6 $(4k-2)(4k+2)$

1.7 $(abc-1)(abc+1)$

1.8 $\left(\frac{1}{3}a-2b\right)\left(\frac{1}{3}a+2b\right)$

1.9 $2\left(\frac{1}{2}x+1\right)\left(\frac{1}{2}x-1\right)$

1.10 $(y-\sqrt{8})(y+\sqrt{8})$

1.11 $(y-\sqrt{13})(y+\sqrt{13})$

1.12 $(a-1)(a-5)(a+5)$

1.13 $(a-3b)(a-5b)(a+3b)^2$

1.14 $m(b+4)(b-6)$

1.15 $(a+7)(a^2+9)$

1.16 $(b-4)(3b+7)$

1.17 $(z+6)(3g+2)$

1.18 $(y+2)(4b+5)$

1.19 $(r+5)(3d+14)$

1.20 $(6x+y-3)(6x+y+3)$

11.10 Exercise 10

1.1 $(2d-3r)(3+t^5)$

1.2 $(7m-2n)(2+j)$

1.3 $(7r-5x)(4+g)$

1.4 $(5d-3m)(5+y)$

1.5 $(5q-2z)(9+c)$

1.6 $(2j-5v)(3+y)$

1.7 $(2a-5k)(8+z)$

1.8 $(a-b)(x+y+2)$

1.9 $(3a+b)(x-y-3)$

1.10 $(z-2m)(9+b^3)$

1.11 $(7z-2y)(5+c^5)$

1.12 $(3+a)(2x+1)$

1.13 $(x+5)(x-6)$

1.14 $(5-a)(x+2y)$

1.15 $(a-x)(a-2)$

1.16 $(y+2)(5x-3)$

1.17 $(-a+b)(a+1)$

11.11 Exercise 11

1.1 $(x+5)(x+3)$

1.2 $(x-5)(x+4)$

1.3 $2(x+1)(x+10)$

1.4 $2(a+1)(3a+4)$

1.5 $3(2v-3)(v-3)$

1.6 $3(g-3)(2g+1)$

1.7 $(3x+1)(x+6)$

1.8 $(3x-1)(x+6)$

1.9 $(7x+1)(x-1)$

1.10 $3(2x+1)(x-3)$

1.11 $(a-4b)(a-3b)$

1.12 $(x+8)(x+1)$

1.13 $(3a-4b)(a+3b)$

1.14 $2\left(7x^2+2\right)\left(7x^2-1\right)$

1.15 $(x-6)(x-5)$

1.16 $(a-7)(a-1)$

1.17 $(y-3)(y+6)$

1.18 $3(b+4)(b+1)$

1.19 $6(a+7)(a-4)$

1.20 $(x+6)^2$

1.21 $(h+3)(2h-1)$

1.22 $(x+1)(3x+1)$

1.23 $(s+2)(3s-5)$

1.24 $(x+3)(x-5)$

1.25 $(x+3)(x-1)$

1.26 $(x+5)(x-4)$

11.12 Exercise 12

1.1 $(w-2)(w^2+2w+4)$

1.2 $(5x+1)(25x^2-5x+1)$

1.3 $(\sqrt[3]{25}x+1)(\sqrt[3]{25})^2x^2-\sqrt[3]{25}x+1)$

1.4 $(z)(1-5z)(1+5z+25z^2)$

1.5 $(2m^2+n^3)(4m^4-2m^2n^3+n^6)$

1.6 $(6n-k)(36n^2+6nk+k^2)$

1.7 $(5s+d)(25s^2-5sd+d^2)$

1.8 $(2k+r)(4k^2-2kr+r^2)$

1.9 $(2jkl-b)(4j^2k^2l^2+2jklb+b^2)$

1.10 $(3xy+w)(9x^2y^2-3xyw+w^2)$

1.11 $2(4m+f)(16m^2-4mf+f^2)$

1.12 $(g+4)(g^2-4g+16)$

1.13 $\left(p^5-\frac{1}{2}y^4\right)\left(p^{10}+\frac{1}{2}p^5y^4+\frac{1}{4}y^8\right)$

1.14 $(\frac{3}{t}-s)(\frac{9}{t^2}+\frac{3s}{t}+s^2)$

1.15 $(\frac{1}{4q}-h)(\frac{1}{16q^2}+\frac{h}{4q}+h^2)$

1.16 $\frac{1}{3}(6g+v)(36g^2-6gv+v^2)$

1.17 $(1-x+y)(1+x-y+x^3-2xy+y^2)$

1.18 $(h-1)(h+1)(h^2+1)(2g^2+h)(4g^4-2g^2h+h^2)$

1.19 $(x+y)(5w-h)(25w^2+5wh+h^2)$

1.20 $(x-6)(x+1)(3p+w)(9p^2-3pw+w^2)$

1.21 $(h+1)(h^2-h+1)$

1.22 $(x+2)(x^2-2x+4)$

1.23 $(3-m)(9+3m+m^2)$

1.24 $2(x-y)(x^2+xy+y^2)$

1.25 $3(k+3q)(k^2-3kq+9q^2)$

1.26 $(4t-1)(16t^2+4t+1)$

1.27 $(8x-1)(8x+1)$

11.13 Exercise 13

1.1 $\frac{a}{5}$

1.2 $\frac{3x+4}{2}$

1.3 $-\frac{b+9a}{2}$

1.4 $\frac{t+s}{t-s}$

1.5 $\frac{x+3}{5}$

1.6 $\frac{x-3}{x+3}$

1.7 $\frac{x+2}{x^2+3x+9}$

1.8 $\frac{a+8}{a^2+2a+4}$

1.9 $\frac{a-6b}{a+2b}$

1.10 $\frac{2a-3}{b}$

1.11 $\frac{2x+1}{x(x+1)}$

1.12 $\frac{a+5}{2}$

1.13 $q+16$

1.14 $p+5$

1.15 $h+13$

1.16 af

1.17 5

1.18 a

1.19 $\frac{3a}{2}$

1.20 $\frac{a+3}{a+2}$

1.21 $\frac{a(3b+1)}{b}$

1.22 $\frac{4xy}{3}$

1.23 $\frac{p(y-2)}{3y}$

2.1 5

2.2 $\frac{5(a+b)}{24b}$

2.3 -1

2.4 $\frac{5(a^2-ab+b^2)}{a^3}$

2.5 $\frac{1}{a+4}$

2.6 $\frac{1}{(x-4)(x+2)}$

2.7 $\frac{2a^2-14a+15}{6}$

2.8 $\frac{4x^2-7x+3}{4x^2+11x-3}$

2.9 $\frac{14-x}{6}$

2.10 $\frac{3(p^2-pq+q^2)}{p^2}$

2.11 3

2.12 4

2.13 $\frac{3(a+3)}{98}$

2.14 $\frac{4a^2(a-5)}{6(a+5)^2}$

2.15 $\frac{(3x+4)^2}{96p^2}$

2.16 $\frac{4}{3}$

2.17 $2a$

2.18 $\frac{3q}{8}$

11.14 Exercise 14

1.1 $\frac{x-27}{12}$

1.2 $\frac{5t+7}{6q}$

1.3 $\frac{5p-2}{(p+2)(p-2)^2}$

1.4 $\frac{xy}{(x+y)(y-x)}$

1.5 $\frac{m+n}{m^2-mn+n^2}$

1.6 $\frac{f}{(h-f)(h^2+hf+f^2)}$

1.7 $\frac{2x-1}{6}$

1.8 0

1.9 $\frac{-x^2+3x+1}{(x-1)^2(x^2+x+1)}$

1.10 $\frac{2t^2+5t-8}{t^2-9}$

1.11 $\frac{x^2-9x-1}{(x+3)(x+1)(x-2)}$

1.12 $\frac{47-x}{36}$

1.13 $\frac{a^2+4b-4b^2}{(a-2b)^2(a+2b)}$

1.14 $\frac{5x-8}{12}$

1.15 $\frac{19a}{(a+11)(a-8)}$

1.16 $\frac{6x}{(x-12)(x-6)}$

1.17 $\frac{20r}{(r+12)(r-8)}$

1.18 $\frac{2z+4y+3x}{xyz}$

1.19 $\frac{4t-13}{(t-2)(t-3)}$

1.20 $\frac{2(k+2)}{(k^2+2)(k+2)}$