CHAPTER 11

Trigonometry

CONTENTS

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Trigonometry was developed in ancient civilisations to solve practical problems such as building construction and navigating by the stars. We will show that trigonometry can also be used to solve some other practical problems. We can use the trigonometric ratios to solve problems in two dimensions that involve right-angled triangles.

As revision the three trigonometric ratios can be defined for right-angled triangles as:

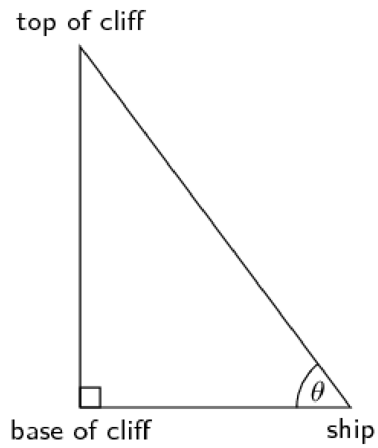
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

We will use these three ratios and the theorem of Pythagoras to help us solve two-dimensional problems.

1 TWO DIMENSIONAL PROBLEMS

In two-dimensional problems we will often refer to the angle of elevation and the angle of depression. To understand these two angles let us consider a person sailing alongside some cliffs. The person looks up and sees the top of the cliffs as shown below:

In this diagram θ is the angle of elevation

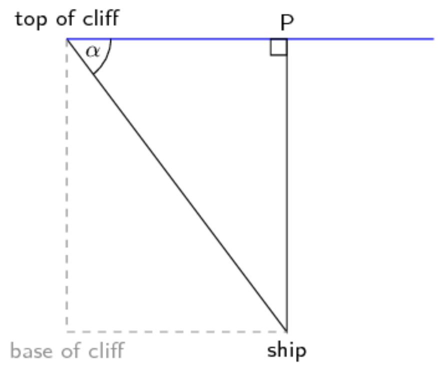


Definition : Angle of elevation

The angle of elevation is the angle formed by the line of sight and the horizontal plane for an object above the horizontal plane.

In our diagram the line of sight is from the ship to the top of the cliffs. The horizontal plane is from the ship to the base of the cliffs. Also note the we can consider the cliffs to be a straight vertical line and so we have a right-angled triangle.

To understand the angle of depression let us now consider the same situation as above but instead our observer is standing on top of the cliffs looking down at the ship.



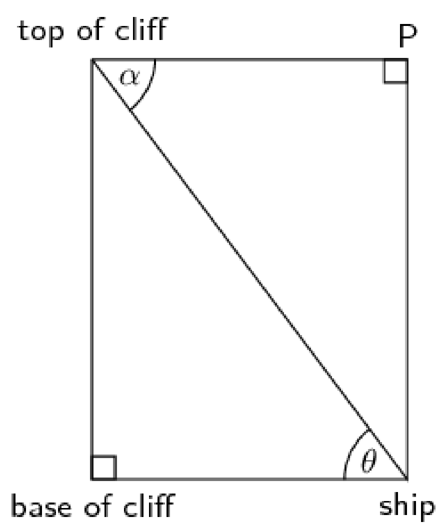
In this diagram α is the angle of depression.

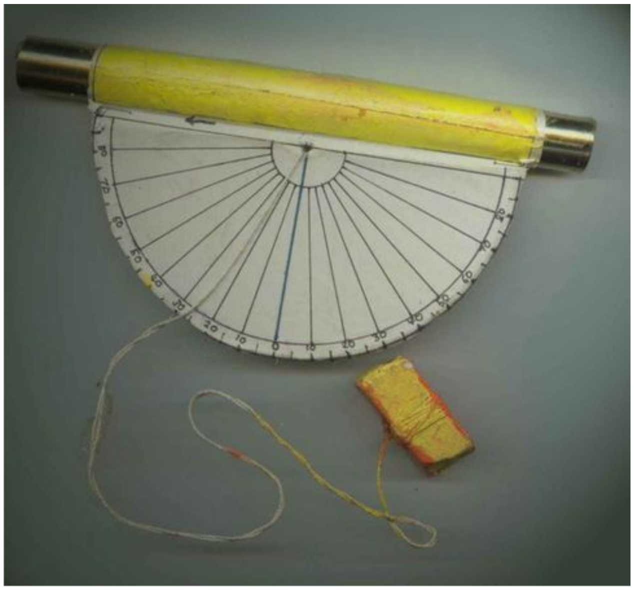
Definition : Angle of depression

The angle of depression is the angle formed by the line of sight and the horizontal plane for an object below the horizontal plane.

In our diagram the line of sight is from the top of the cliffs to the ship. The horizontal plane is from the top of the cliffs through P. Note that this is parallel to the line between the base of the cliffs and the ship. P lies directly above the ship. We can construct a vertical, perpendicular line to the horizontal plane at the point P.

Finally we can compare the angle of elevation and the angle of depression. In the following diagram the line from the base of the cliffs to the ship is parallel to the line from the top of the cliffs to P. The angle of elevation and the angle of depression are indicated. Notice that α = 0





An inclinometer. Inclinometers can be used to measure angles of inclination and so can be used to determine the height of an object.

NOTE

In trigonometry the angle of inclination is the same as the angle of elevation.

Example 1 : Flying a kite

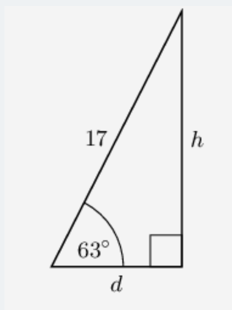
QUESTION

Mandla flies a kite on a 17 m string at an inclination of 63°.

1. What is the height, h, of the kite above the ground?
2. If Mandla's friend Sipho stands directly below the kite, calculate the distance, d, between the two friends.

SOLUTION

Step 1: Make a sketch and identify the opposite and adjacent sides and the hypotenuse



Step 2: Use given information and appropriate ratio to solve for h and d

1.

$$\begin{aligned} \sin 63° &= \frac{\text{opposite}}{\text{hypotenuse}} \\ \sin 63° &= \frac{h}{17} \\ \therefore h &= 17 \sin 63° \\ &= 15{,}14711\ldots \\ &\approx 15{,}15 \text{ m} \end{aligned}$$

2.

$$\begin{aligned} \cos 63° &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ \cos 63° &= \frac{d}{17} \\ \therefore d &= 17 \cos 63° \\ &= 7{,}7178\ldots \\ &\approx 7{,}72 \text{ m} \end{aligned}$$

Step 3: Write the final answers

1. The kite is 15,15 m above the ground.
2. Mandla and Sipho are 7,72 m apart.

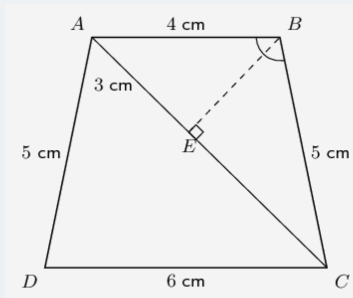
Example 2 : Calculating angles

QUESTION

$ABCD$ is a trapezium with $AB = 4$ cm, $CD = 6$ cm, $BC = 5$ cm and $AD = 5$ cm. Point E on diagonal AC divides the diagonal such that $AE = 3$ cm. $B\hat{E}C = 90°$. Find $A\hat{B}C$.

SOLUTION

Step 1: Draw trapezium and label all given lengths on diagram. Indicate that $B\hat{E}C = 90°$



We will use $\triangle ABE$ and $\triangle CBE$ to find $A\hat{B}E$ and $C\hat{B}E$. We can then add these two angles together to find $A\hat{B}C$

Step 2: Find the first angle, $A\hat{B}E$

The hypotenuse and opposite side are given for both triangles, therefore use the sin **function.**

In $\triangle ABE$:

$$\begin{aligned}\sin A\hat{B}E &= \frac{\textbf{opposite}}{\textbf{hypotenuse}} \\ &= \frac{3}{4} \\ A\hat{B}E &= \textbf{48,5903...} \\ &\approx \textbf{48,6°}\end{aligned}$$

Step 3: Use the theorem of Pythagoras to determine BE

In $\triangle ABE$:

$$\begin{aligned}BE^2 &= AB^2 - AE^2 \\ &= 4^2 - 3^2 \\ &= 7 \\ \therefore BE &= \sqrt{7}\text{cm}\end{aligned}$$

Step 4: Find the second angle $C\hat{B}E$ In $\triangle CBE$

Example 2 : Calculating angles

$$
\begin{aligned}
\cos C\hat{B}E &= \frac{\text{adjacent}}{\text{hypotenuse}} \\
&= \frac{\sqrt{7}}{5} \\
&= 0{,}52915\ldots \\
C\hat{B}E &= 58{,}0519\ldots \\
&\approx 58{,}1^\circ
\end{aligned}
$$

Step 5: Calculate the sum of the angles

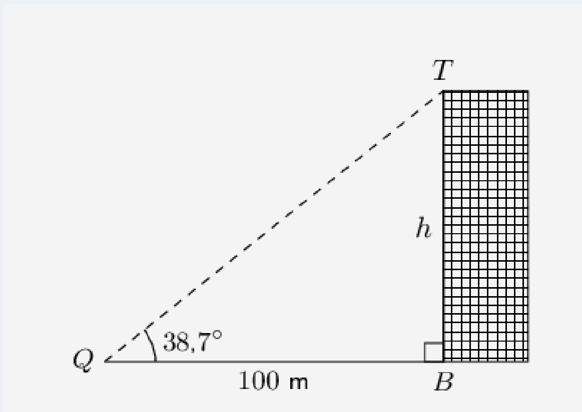
$$A\hat{B}C = 48{,}6^\circ + 58{,}1^\circ = 106{,}7^\circ$$

Another application is using trigonometry to find the height of a building. We could use a tape measure lowered from the roof, but this is impractical (and dangerous) for tall buildings. It is much more sensible to use trigonometry.

Example 3 : Finding height of a building

QUESTION

The given diagram shows a building of unknown height h. We start at point B and walk 100 m away from the building to point Q. Next we measure the angle of elevation from the ground to the top of the building, T, and find that the angle is 38,7°. Calculate the height of the building, correct to the nearest metre.



SOLUTION

Step 1: Identify the opposite and adjacent sides and the hypotenuse

We have a right-angled triangle and know the length of one side and an angle. We can therefore calculate the height of the building.

Step 2: In $\triangle QTB$:

$$\tan 38{,}7° = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{h}{100}$$

Step 3: Rearrange and solve for h

$$h = 100 \times \tan 38{,}7°$$
$$= 80{,}1151...$$
$$\approx 80$$

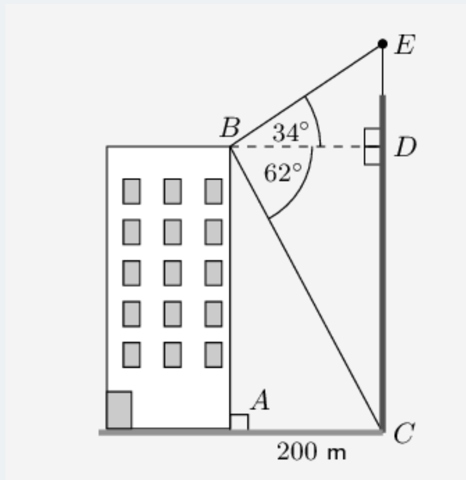
Step 4: Write final answer The height of the building is 80 m.

Example 4 : Angles of elevation and depression

QUESTION

A block of flats is 200 m away from a cellphone tower. Someone stands at B. They measure the angle from B to the top of the tower (E) to be 34° (the angle of elevation). They then measure the angle from B to the bottom of the tower (C) to be 62° (the angle of depression).

What is the height of the cellphone tower (correct to the nearest metre)?



SOLUTION

Step 1: To determine height CE, first calculate lengths DE and CD

$\triangle BDE$ and $\triangle BDC$ are both right-angled triangles. In each of the triangles, the length BD is known. Therefore we can calculate the sides of the triangles.

Step 2: Calculate CD

The length AC is given. $CABD$ is a rectangle so $BD = AC = 200$ m.

In $\triangle CBD$:

$$\tan C\hat{B}D = \frac{CD}{BD}$$
$$\therefore CD = BD \times \tan C\hat{B}D$$
$$= 200 \times \tan 62°$$
$$= 376{,}1452...$$
$$\approx 376 \text{ m}$$

Step 3: Calculate DE In $\triangle DBE$:

$$\tan D\hat{B}E = \frac{DE}{BD}$$

Example 4 : Angles of elevation and depression

$$\therefore DE = BD \times \tan D\hat{B}E$$
$$= 200 \times \tan 34°$$
$$= 134{,}9017...$$
$$\approx 135 \text{ m}$$

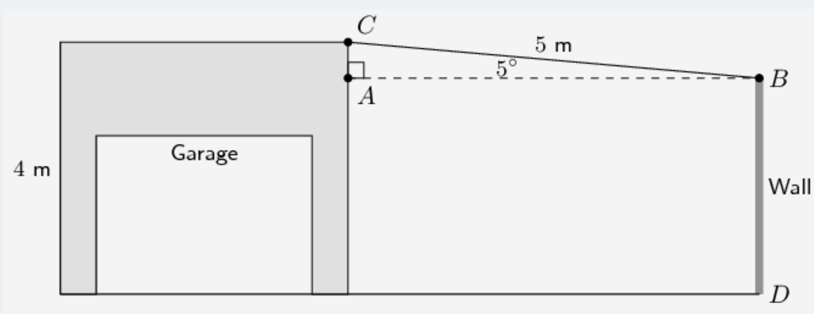
Step 4: Add the two heights to get the final answer

The height of the tower is: $CE = CD + DE = 135 \text{ m} + 376 \text{ m} = 511 \text{ m}$

Example 5 : Building plan

QUESTION

Mr Nkosi has a garage at his house and he decides to add a corrugated iron roof to the side of the garage. The garage is 4 m high, and his sheet for the roof is 5 m long. If the angle of the roof is 5°, how high must he build the wall BD? Give the answer correct to 1 decimal place



SOLUTION

Step 1: Identify opposite and adjacent sides and hypotenuse

$\triangle ABC$ is right-angled. The hypotenuse and an angle are known therefore we can calculate AC. The height of the wall BD is then the height of the garage minus AC.

Example 5 : Building plan

$$\sin A\hat{B}C = \frac{AC}{BC}$$
$$\therefore AC = BC \times \sin A\hat{B}C$$
$$= 5 \sin 5°$$
$$= 0{,}43577...$$
$$\approx 0{,}4 \text{ m}$$

$$\therefore BD = 4 \text{ m} - 0{,}4 \text{ m}$$
$$= 3{,}6 \text{ m}$$

Step 2: Write the final answer

Mr Nkosi must build his wall to be 3,6 m high.

2 CHAPTER SUMMARY

- We can define three trigonometric ratios for right-angled triangles: sine (sin), cosine (cos) and tangent (tan).
- Trigonometry is used to help us solve problems in two dimensions that involve right-angled triangles, such as finding the height of a building.
- The angle of elevation is the angle formed by the line of sight and the horizontal plane for an object above the horizontal plane.
- The angle of depression is the angle formed by the line of sight and the horizontal plane for an object below the horizontal plane.