CHAPTER 12

Euclidean Geometry

CONTENTS

August 26, 2021

1 EUCLIDEAN GEOMETRY

Geometry (from the Greek “geo” = earth and “metria” = measure) arose as the field of knowledge dealing with spatial relationships. Geometry can be split into Euclidean geometry and analytical geometry. Analytical geometry deals with space and shape using algebra and a coordinate system. Euclidean geometry deals with space and shape using a system of logical deductions.

Proofs and conjectures

We will now apply what we have learnt about geometry and the properties of polygons (in particular triangles and quadrilaterals) to prove some of these properties. We will also look at how we can prove a particular quadrilateral is one of the special quadrilaterals.

Visit

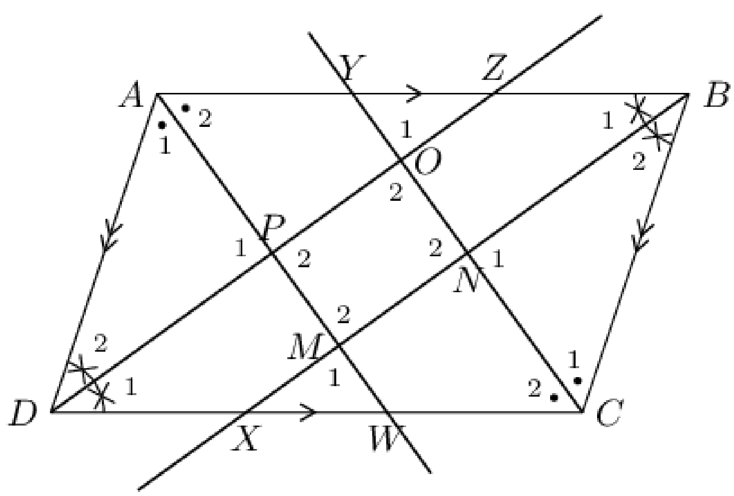
This video shows how to prove that the the diagonals of a rhombus are perpendicular.

Video: https://www.youtube.com/embed/GDcVdBAnBdU?list=PL26812DF9846578C3

WORKED EXAMPLE 1: PROVING A QUADRILATERAL IS A PARALLELOGRAM

QUESTON

In parallelogram $ABCD$, the bisectors of the angles (AW, BX, CY and DZ) have been constructed. You are also given $AB = CD$, $AD = BC$, $AB||CD$, $AD||BC$, $\hat{A} = \hat{C}$, $\hat{B} = \hat{D}$. Prove that $MNOP$ is a parallelogram.



SOLUTION

Step 1: Use properties of the parallelogram $ABCD$ to fill in on the diagram all equal sides and angles.

Step 2: Prove that $\hat{M}_2 = \hat{O}_2$

In $\triangle CDZ$ and $\triangle ABX$

$D\hat{C}Z = B\hat{A}X \qquad (given)$

$\hat{D}_1 = \hat{B}_1 \qquad (given)$

$DC = AB \qquad (given)$

$\therefore \triangle CDZ = \triangle ABX \qquad (AAS)$

$\therefore CZ = AX$

and $C\hat{Z}D = A\hat{X}B$

In $\triangle XAM$ and $\triangle ZCO$

$X\hat{A}M = Z\hat{C}O \qquad (given : \triangle CDZ = \triangle ABX)$

$A\hat{X}M = C\hat{Z}O \qquad (proved\ above)$

$AX = CZ \qquad (proved\ above)$

$\therefore \triangle XAM = \triangle ZCO \qquad (AAS)$

$\therefore \hat{M}_1 = \hat{O}_1$

but $\hat{M}_1 = \hat{M}_2 \qquad (vert\ opp\ \angle s =)$

and $\hat{O}_1 = \hat{O}_2 \qquad (vert\ opp\ \angle s =)$

$\therefore \hat{M}_2 = \hat{O}_2$

Step 3: Similarly, we can show that $\hat{N}_2 = \hat{P}_2$

First show $\triangle ADW \equiv \triangle CBY$. Then show $\triangle PDW \equiv \triangle NBY$.

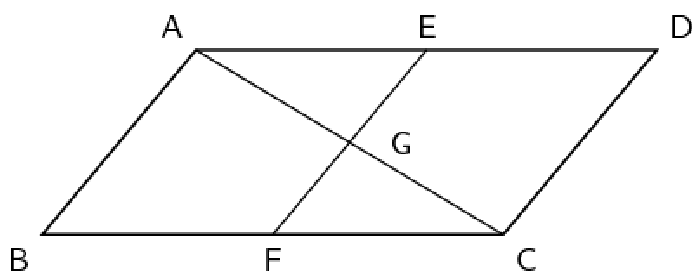
Step 4: Conclusion

Both pairs of opposite angles of $MNOP$ are equal. Therefore $MNOP$ is a parallelogram.

2 EXERCISES

2.1 Exercise 1

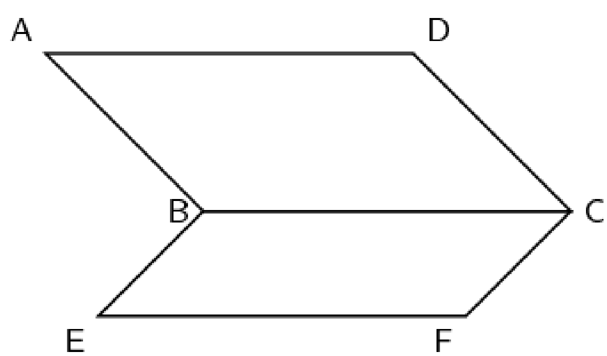
1. In the diagram below, AC and EF bisect each other at G. E is the midpoint of AD, and F is the midpoint of BC.



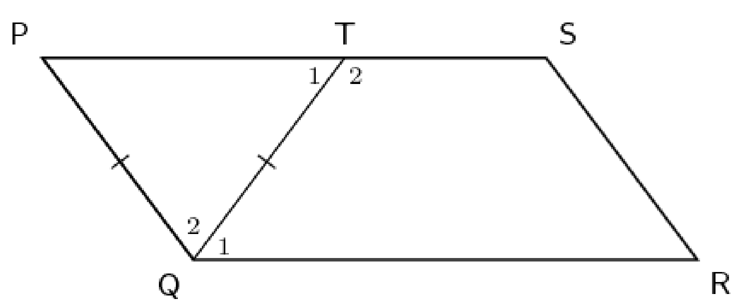
   1.1 Is $AECF$ a parallelogram?

   1.2 Is $ABCD$ a parallelogram?

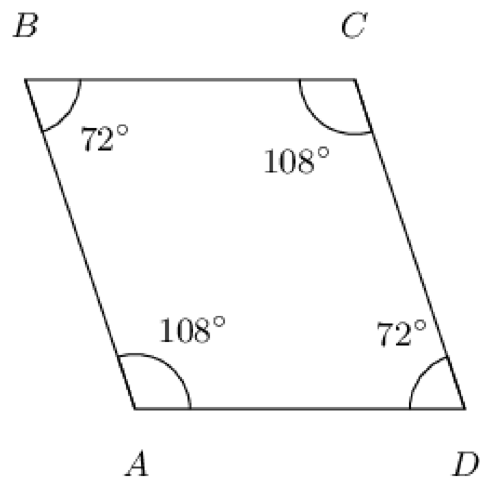
2. Parallelogram $ABCD$ and EFC are shown below. Give the steps to determine that $AB = EF$.



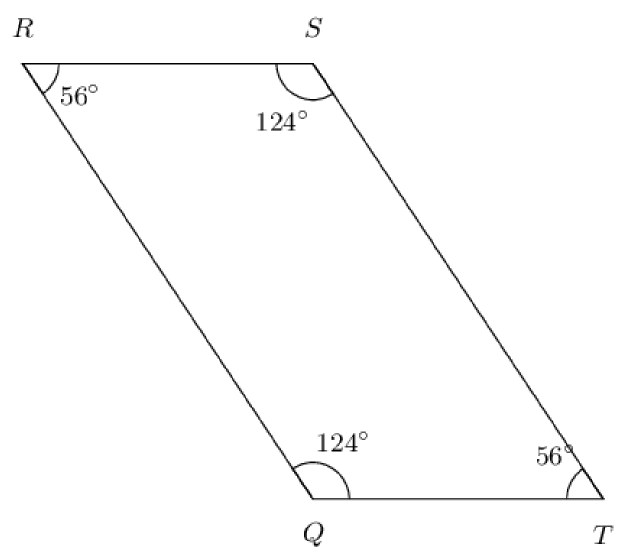
3. $PQRS$ is a parallelogram. $PQ = TQ$. Prove $\hat{Q}_1 = \hat{R}$



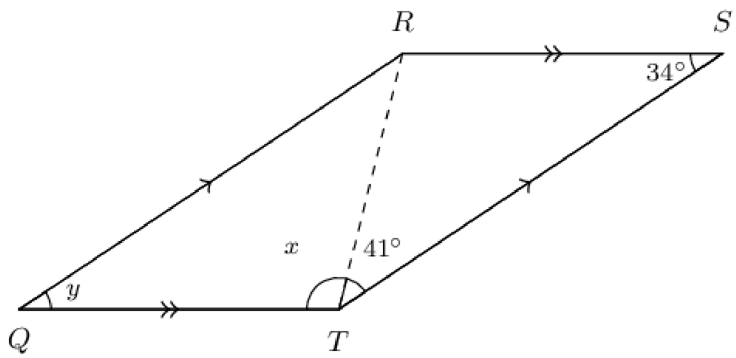
4. Study the quadrilateral $ABCD$ with opposite angles $\hat{A} = \hat{C} = 108°$ and angles $\hat{B} = \hat{D} = 72°$ carefully. Prove that the quadrilateral $ABCD$ is a parallelogram.



5. Study the quadrilateral $QRST$ with opposite angles $Q = S = 124°$ and angles $R = T = 56°$ carefully. Fill in the missing reasons and steps to prove that the quadrilateral $QRST$ is a parallelogram.



6. Quadrilateral $QRST$ with sides $QR \| TS$ and $QT \| RS$ is given. You are also given that: $\hat{Q} = y$ and $\hat{S} = 34°; Q\hat{T}R = x$ and $R\hat{T}S = 41°$.

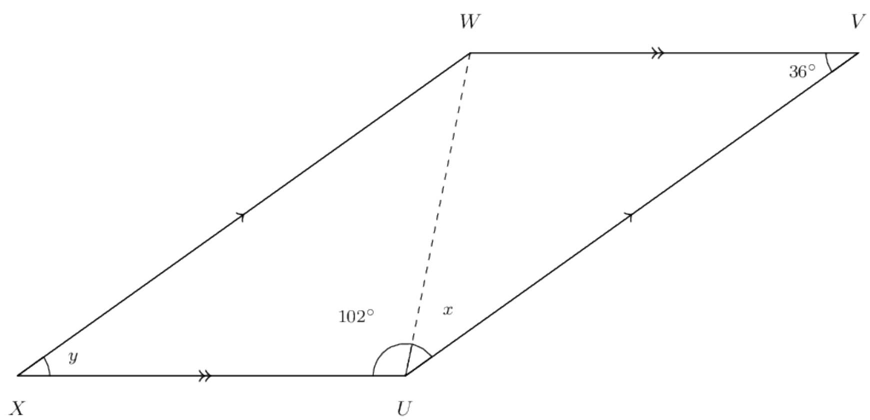


6.1 Prove that $QRST$ is a parallelogram.

6.2 Find the value of y

6.3 Find the value of x

7. Quadrilateral $XWVU$ with sides $XW \| UV$ and $XU \| WV$ is given. Also given is $\hat{X} = y$ and $\hat{V} = 36°$; $X\hat{U}W = 102°$ and $W\hat{U}V = x$.

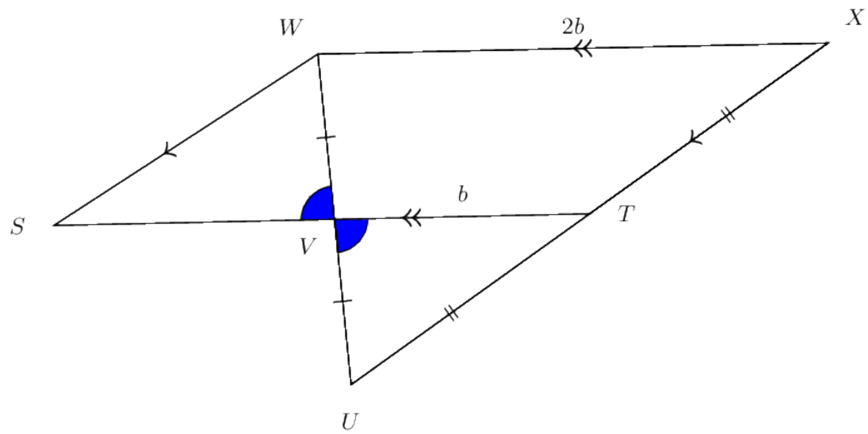


7.1 Prove that $XWVU$ is a parallelogram.

7.2 Find the value of y

7.3 Find the value of x

8. Study the diagram below; it is not necessarily drawn to scale. Quadrilateral $XWST$ is a parallelogram and TV and XW have lengths b and $2b$, respectively, as shown. You need to prove that $\triangle TVU \equiv \triangle SVW$.



3 ANSWERS FOR EXERCISES

3.1 Exercise 1

1.1 Yes, diagonals bisect each other

1.2 Yes, ABCD is a parallelogram (two sides are parallel and equal)

2. AD $=$ BC (opp sides of $\parallel$ m)
 BC $=$ EF (opp sides of $\parallel$ m)
 $\therefore$ AD $=$ EF

3. $\hat{P} = \hat{T}_1$ ($\angle$s opp equal sides)
   $\hat{T}_1 = \hat{Q}_1$ (alt $\angle$s; $(PS \| QR)$)
   $\hat{P} = \hat{Q}_1$

   $\hat{P} = \hat{R}$ (opp $\angle$s of $\|$m)
   $\therefore \hat{Q}_1 = \hat{R}$

4. .

Steps	Reasons
B D=B D	given both $\angle'$ s $= 108°$
A C=A C	given both $\angle'$ s $= 72°$
+++=360°	sum of $\angle$s in quad
B D+A C=180°	given $108° + 72° = 180°$
$\therefore AB \| DC$	co-int $\angle$s; $AB \| DC$
$\therefore BC \| AD$	co-int $\angle$s; $BC \| AD$
$\therefore ABCD$ is a parallelogram	opp sides of quad $\|$

5. .

Steps	Reasons
R T=R T	given both $\angle$s $= 124°$
Q S=Q S	given both $\angle$s $= 56°$
+++=360°	sum of $\angle$s in quad
R T+Q S=180°	given $124° + 56° = 180°$
$\therefore QR \| TS$	co-int $\angle$s; $QR \| TS$
$\therefore RS \| QT$	co-int $\angle$s; $RS \| QT$
$\therefore QRST$ is a parallelogram	opp sides of quad $\|$

6.1 .

Steps	Reasons
Q R=T S	alt $\angle$s$QT \| RS$
S R=Q T	alt $\angle$s$QR \| TS$
In $\triangle QRT$ and $\triangle RST$ side $RT = RT$	common side
$\therefore \triangle QRT \equiv \triangle STR$	congruent (AAS)
=	congruent triangles (AAS)
Q R=T S and R S=Q T	congruent triangles (AAS)
$\therefore QRST$ is a parallelogram	opp. sides of quad are =

6.2 $34°$

6.3 $105°$

7.1 .

Steps	Reasons
X W=U V	alt $\angle$s; $XU \| WV$
V W=X U	alt $\angle$s; $XW \| UV$
In $\triangle XWU$ and $\triangle WVU$ side $WU = WU$	common side
$\therefore \triangle XWU \equiv \triangle VUW$	congruent (AAS)
$\therefore XW = U$ Vand $XU = WV$	congruent triangles (AAS)
=	congruent triangles (AAS)
$\therefore XWVU$ is a parallelogram	opp sides of quad are =

7.2 $36°$

7.3 $42°$

8. T and V are mid-points

Steps	Reasons
W V=V U	definition of mid-point
T U=S W	vert opp $\angle$s $=$
T V+V S=X W	opp sides parm are equal
b+V S=2 b	substitute given values: T V=b and X W=2 b
V S=b=V T	solve for V S ; note that it is equal to V T
$\triangle TVU \equiv \triangle SVW$	SAS