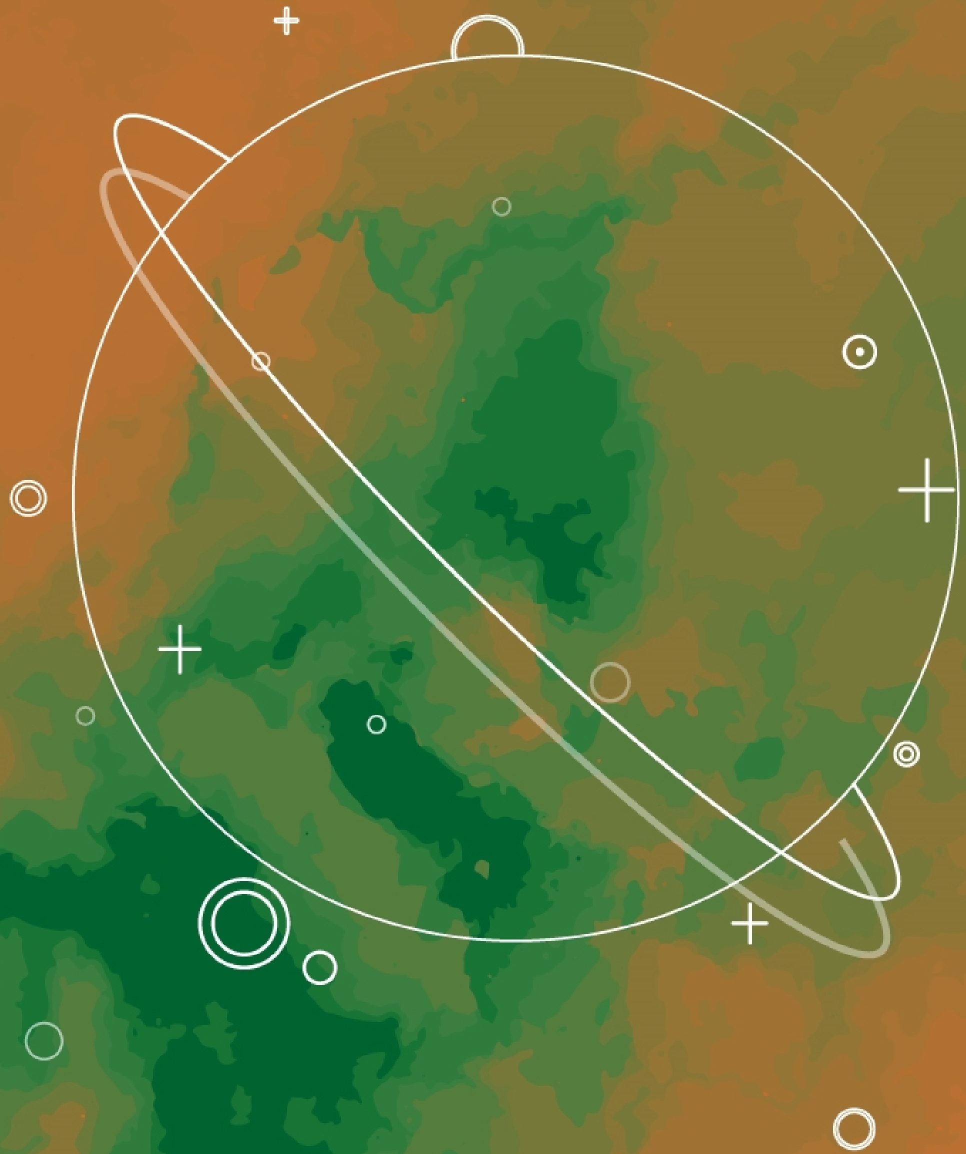


CHAPTER 4

Equations And Inequalities

MathU Teaching® Emporium

CONTENTS

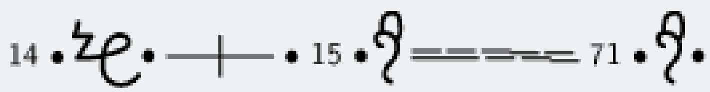
April 20, 2021

1 INTRODUCTION

Equations are widely used to describe the world around us. In science equations are used to describe everything from how a ball rolls down a slope to how the planets move around the sun.

In this chapter we will explore different types of equations as well as looking at how these can be used to solve problems in the real world. We will also look at linear inequalities.

DID YOU KNOW?



The first use of an "equals" sign from *The Whetstone of Witte* by Robert Recorde 1557. This equation represents $14x + 15 = 71$. Recorde is also responsible for introducing the pre-existing "plus" sign(+) to the English-speaking world.

2 SOLVING LINEAR EQUATIONS

The simplest equation to solve is a linear equation. A linear equation is an equation where the highest exponent of the variable is 1. The following are examples of linear equations:

$$2x + 2 = 1$$
$$\frac{2 - x}{3x + 1} = 2$$
$$4(2x - 9) - 4x = 4 - 6x$$
$$\frac{2a - 3}{3} - 3a = \frac{a}{3}$$

Solving an equation means finding the value of the variable that makes the equation true. For example, to solve the simple equation $x + 1 = 1$, we need to determine the value of x that will make the left hand side equal to the right hand side. The solution is $x = 0$.

The solution, also called the root of an equation, is the value of the variable that satisfies the equation. For linear equations, there is at most one solution for the equation.

To solve equations we use algebraic methods that include expanding expressions, grouping terms and factorising.

For example:

$$\begin{aligned} 2x + 2 &= 1 \\ 2x &= 1 - 2 \text{ (rearrange)} \\ 2x &= -1 \text{ (simplify)} \\ x &= -\frac{1}{2} \text{ (divide both sides by 2)} \end{aligned}$$

Check the answer by substituting $x = -\frac{1}{2}$.

$$\begin{aligned} \text{LHS} &= 2x + 2 \\ &= 2(-\frac{1}{2}) + 2 \\ &= -1 + 2 \\ &= 1 \\ \text{RHS} &= 1 \end{aligned}$$

Therefore $x = -\frac{1}{2}$

2.1 Method for solving linear equations

The general steps for solving linear equations are:

1. Expand all brackets.
2. Rearrange the terms so that all terms containing the variable are on one side of the equation and all constant terms are on the other side.
3. Group like terms together and simplify.
4. Factorise if necessary.
5. Find the solution and write down the answer.
6. Check the answer by substituting the solution back into the original equation.

IMPORTANT

An equation must always be balanced, whatever you do to the left-hand side, you must also do to the right-hand side.

WORKED EXAMPLE 1: SOLVING LINEAR EQUATIONS

QUESTION

Solve for x:

$$4(2x - 9) - 4x = 4 - 6x$$

SOLUTION

Step 1: Expand the brackets and simplify

$$\begin{aligned} 4(2x - 9) - 4x &= 4 - 6x \\ 8x - 36 - 4x &= 4 - 6x \\ 8x - 4x + 6x &= 4 + 36 \\ 10x &= 40 \end{aligned}$$

Step 2: Divide both sides by 10

$$x = 4$$

Step 3: Check the answer by substituting the solution back into the original equation

$$\begin{aligned} \text{LHS} &= 4[2(4) - 9] - 4(4) \\ &= 4(8 - 9) - 16 \\ &= 4(-1) - 16 \\ &= -4 - 16 \\ &= -20 \\ \text{RHS} &= 4 - 6(4) \\ &= 4 - 24 \\ &= -29 \\ \therefore \text{LHS} &= \text{RHS} \end{aligned}$$

Since both sides are equal, the answer is correct.

WORKED EXAMPLE 2: SOLVING LINEAR EQUATIONS

QUESTION

Solve for x:

$$\frac{2-x}{3x+1}=2$$

SOLUTION

Step 1: Multiply both sides of the equation by $(3x+1)$

Division by 0 is undefined so there must be a restriction: ($x \neq -\frac{1}{3}$).

$$\frac{2-x}{3x+1}=2$$
$$(2-x)=2(3x+1)$$

Step 2: Expand the brackets and simplify

$$2-x=6x+2$$
$$-x-6x=2-2$$
$$-7x=0$$

Step 3: Divide both sides by -7

$$x=\frac{0}{-7}$$
$$x=0$$

Step 4: Check the answer by substituting the solution back into the original equation

$$\text{LHS}=\frac{2-(0)}{3(0)+1}$$
$$=2$$
$$=\text{RHS}$$

Since both sides are equal, the answer is correct.

WORKED EXAMPLE 3: SOLVING LINEAR EQUATIONS

QUESTION

Solve for a:

$$\frac{2a-3}{3} - 3a = \frac{a}{3}$$

SOLUTION

Step 1: Multiply the equation by the common denominator 3 and simplify

$$\begin{aligned} 2a - 3 - 9a &= a \\ -7a - 3 &= a \end{aligned}$$

Step 2: Rearrange the terms and simplify

$$\begin{aligned} -7a - a &= 3 \\ -8a &= 3 \end{aligned}$$

Step 3: Divide both sides by -8

$$a = -\frac{3}{8}$$

Step 4: Check the answer by substituting the solution back into the original equation

$$\begin{aligned} \text{LHS} &= \frac{2(-\frac{3}{8}) - 3}{3} - 3(-\frac{3}{8}) \\ &= \frac{(-\frac{3}{4}) - \frac{12}{4}}{3} + \frac{9}{8} \\ &= [-\frac{15}{4} \times \frac{1}{3}] + \frac{9}{8} \\ &= -\frac{5}{4} + \frac{9}{8} \\ &= -\frac{10}{8} + \frac{9}{8} \\ &= -\frac{1}{8} \\ \text{RHS} &= \frac{-\frac{3}{8}}{3} \\ &= -\frac{3}{8} \times \frac{1}{3} \\ &= -\frac{1}{8} \\ \therefore \text{LHS} &= \text{RHS} \end{aligned}$$

Since both sides are equal, the answer is correct.

3 SOLVING QUADRATIC EQUATIONS

A quadratic equation is an equation where the exponent of the variable is at most 2. The following are examples of quadratic equations:

$$2x^2 + 2x = 1$$
$$3x^2 + 2x - 1 = 0$$
$$0 = -2x^2 + 4x - 2$$

Quadratic equations differ from linear equations in that a linear equation has only one solution, while a quadratic equation has at most two solutions. There are some special situations, however, in which a quadratic equation has either one solution or no solutions.

We solve quadratic equations using factorisation. For example, in order to solve $2x^2 - x - 3 = 0$, we need to write it in its equivalent factorised form as $(x+1)(2x-3) = 0$. Note that if $a \times b = 0$ then $a = 0$ or $b = 0$.

3.1 Method for solving quadratic equations

1. Rewrite the equation in the required form, $ax^2 + bx + c = 0$.
2. Divide the entire equation by any common factor of the coefficients to obtain an equation of the form $ax^2 + bx + c = 0$, where a,b and c have no common factors. For example $2x^2 + 4x + 2 = 0$ can be written as $x^2 + 2x + 1 = 0$.
3. Factorise $ax^2 + bx + c = 0$ to be of the form $(rx + s)(ux + v) = 0$.
4. The two solutions are $(rx + s) = 0$ or $(ux + v) = 0$ so $x = -\frac{s}{r}$ or $x = -\frac{v}{u}$, respectively.
5. Check the answer by substituting it back into the original equation.

WORKED EXAMPLE 4: SOLVING QUADRATIC EQUATIONS

QUESTION

Solve for x:

$$3x^2 + 2x - 1 = 0$$

SOLUTION

Step 1: The equation is already in the required form

$ax^2 + bx + c = 0$

Step 2: Factorise

$$(x + 1)(3x - 1) = 0$$

Step 3: Solve for both factors

We have

$$x + 1 = 0$$
$$\therefore x = -1$$

OR

$$3x - 1 = 0$$
$$\therefore x = \frac{1}{3}$$

Step 4: Check both answers by substituting back into the original equation

Step 5: Write the final answer The solution to $3x^2 + 2x - 1 = 0$ is $x = -1$ or $x = \frac{1}{3}$.

WORKED EXAMPLE 5: SOLVING QUADRATIC EQUATIONS

QUESTION

Find the roots:

$$0 = -2x^2 + 4x - 2$$

SOLUTION

Step 1: The equation is already in the required form

$ax^2 + bx + c = 0$

Step 2: Divide the equation by common factor -2

$$-2x^2 + 4x - 2 = 0$$
$$x^2 - 2x + 1 =$$

Step 3: Factorise

$$(x-1)(x-1) = 0$$
$$(x-1)^2 = 0$$

Step 4: The quadratic is a perfect square

This is an example of a special situation in which there is only one solution to the quadratic equation because both factors are the same.

$$x - 1 = 0$$
$$\therefore x = 1$$

Step 5: Check the answer by substituting back into the original equation

Step 6: Write the final answer

The solution to $0 = -2x^2 + 4x - 2$ is $x = 1$.

4 SOLVING SIMULTANEOUS EQUATIONS

Up to now we have solved equations with only one unknown variable. When solving for two unknown variables, two equations are required and these equations are known as simultaneous equations. The solutions are the values of the unknown variables which satisfy both equations simultaneously. In general, if there are n unknown variables, then n independent equations are required to obtain a value for each of the n variables.

An example of a system of simultaneous equations is:

$$\begin{aligned} x + y &= -1 \\ 3 &= y - 2x \end{aligned}$$

We have two independent equations to solve for two unknown variables. We can solve simultaneous equations algebraically using substitution and elimination methods. We will also show that a system of simultaneous equations can be solved graphically.

4.1 Solving by substitution

- Use the simplest of the two given equations to express one of the variables in terms of the other.
- Substitute into the second equation. By doing this we reduce the number of equations and the number of variables by one.
- We now have one equation with one unknown variable which can be solved.
- Use the solution to substitute back into the first equation to find the value of the other unknown variable.

WORKED EXAMPLE 6: SIMULTANEOUS EQUATIONS

QUESTION

Solve for x and y:

$$x - y = 1 \qquad \ldots(1)$$
$$3 = y - 2x \qquad \ldots(2)$$

SOLUTION

Step 1: Use equation (1) to express x in terms of y

$$x = y + 1$$

Step 2: Substitute x into equation (2) and solve for y

$$3 = y - 2(y + 1)$$
$$3 = y - 2y - 2$$
$$5 = -y$$
$$\therefore y = -5$$

Step 3: Substitute y back into equation (1) and solve for x

$$x = (-5) + 1$$
$$\therefore x = -4$$

Step 4: Check the solution by substituting the answers back into both original equations

Step 5: Write the final answer

$$x = -4$$
$$y = -5$$

WORKED EXAMPLE 7: SIMULTANEOUS EQUATIONS

QUESTION

Solve the following system of equations:

$$4y + 3x = 100 \qquad \ldots(1)$$
$$4y - 19x = 12 \qquad \ldots(2)$$

SOLUTION

Step 1: Use either equation to express x in terms of y

$$\begin{aligned} 4y + 3x &= 100 \\ 3x &= 100 - 4y \\ x &= \frac{100 - 4y}{3} \end{aligned}$$

Step 2: Substitute x into equation (2) and solve for y

$$\begin{aligned} 4y - 19\left(\frac{100 - 4y}{3}\right) &= 12 \\ 12y - 19(100 - 4y) &= 36 \\ 12y - 1\ 900 + 76y &= 36 \\ 88y &= 1\ 936 \\ \therefore y &= 22 \end{aligned}$$

Step 3: Substitute y back into equation (1) and solve for x

$$\begin{aligned} x &= \frac{100 - 4(22)}{3} \\ &= \frac{100 - 88}{3} \\ &= \frac{12}{3} \\ \therefore x &= 4 \end{aligned}$$

Step 4: Check the solution by substituting the answers back into both original equations

Step 5: Write the final answer

$$x = 4$$
$$y = 22$$

4.2 Solving by elimination

WORKED EXAMPLE 8: SIMULTANEOUS EQUATIONS

QUESTION

Solve the following system of equations:

$$3x + y = 2 \qquad \ldots (1)$$
$$6x - y = 25 \qquad \ldots (2)$$

SOLUTION

Step 1: Make the coefficients of one of the variables the same in both equations

The coefficients of y in the given equations are 1 and –1. Eliminate the variable y by adding equation (1) and equation (2) together:

$$\begin{array}{rrcr} & 3x + y & = & 2 \\ + & 6x - y & = & 25 \\ \hline & 9x + 0 & = & 27 \end{array}$$

Step 2: Simplify and solve for x

$$9x = 27$$
$$\therefore x = 3$$

Step 3: Substitute x back into either original equation and solve for y

$$3\,(3) + y = 2$$
$$y = 2 - 9$$
$$\therefore y = -7$$

Step 4: Check that the solution $x = 3$ and $y = $ –7 satisfies both original equations

Step 5: Write the final answer

$$x = 3$$
$$y = -7$$

WORKED EXAMPLE 9: SIMULTANEOUS EQUATIONS

QUESTION

Solve the following system of equations:

$$2a - 3b = 5 \qquad \ldots(1)$$

$$3a - 2b = 20 \qquad \ldots(2)$$

SOLUTION

Step 1: Make the coefficients of one of the variables the same in both equations

By multiplying equation (1) by 3 and equation (2) by 2, both coefficients of a will be 6.

$$\begin{array}{rrcr} & 6a - 9b & = & 15 \\ - & (6a - 4b & = & 40) \\ \hline & 0 - 5b & = & -25 \end{array}$$

(When subtracting two equations, be careful of the signs.)

Step 2: Simplify and solve for b

$$b = \frac{-25}{-5}$$

$$\therefore b = 5$$

Step 3: Substitute b back into either original equation and solve for a

$$2a - 3(5) = 5$$

$$2a - 15 = 5$$

$$2a = 20$$

$$\therefore a = 10$$

Step 4: Check that the solution $a = 10$ and $b = 5$ satisfies both original equations

Step 5: Write the final answer

$$a = 10$$

$$b = 5$$

4.3 Solving graphically

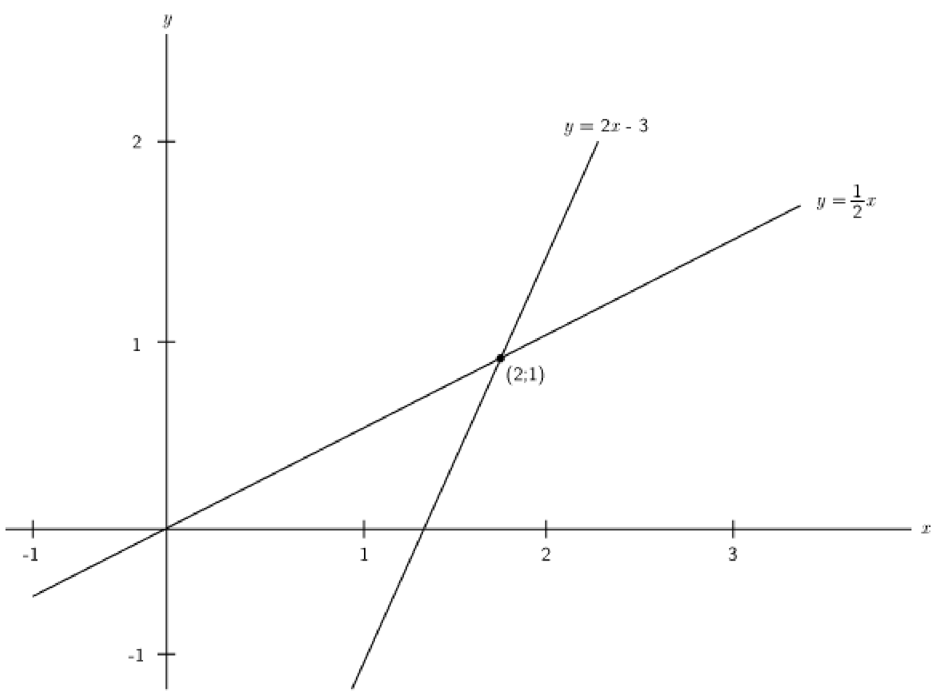
Simultaneous equations can also be solved graphically. If the graphs of each linear equation are drawn, then the solution to the system of simultaneous equations is the coordinates of the point at which the two graphs intersect.

For example:

$$x = 2y \qquad \ldots (1)$$

$$y = 2x - 3 \qquad \ldots (2)$$

The graphs of the two equations are shown below.



The intersection of the two graphs is **(2;1)**. So the solution to the system of simultaneous equations is $x = 2$ and $y = 1$. We can also check the solution using algebraic methods.

Substitute equation (1) into (2):

$$x = 2y$$
$$\therefore y = 2(2y) - 3$$

Then solve for y:

$$y - 4y = -3$$
$$-3y = -3$$
$$\therefore y = 1$$

Substitute the value of y back into equation (1):

$$x = 2(1)$$
$$\therefore x = 2$$

Notice that both methods give the same solution.

VISIT

You can use an online tool such as graphsketch to draw the graphs and check your solution.

WORKED EXAMPLE 10: SIMULTANEOUS EQUATIONS

QUESTION

Solve the following system of equations graphically:

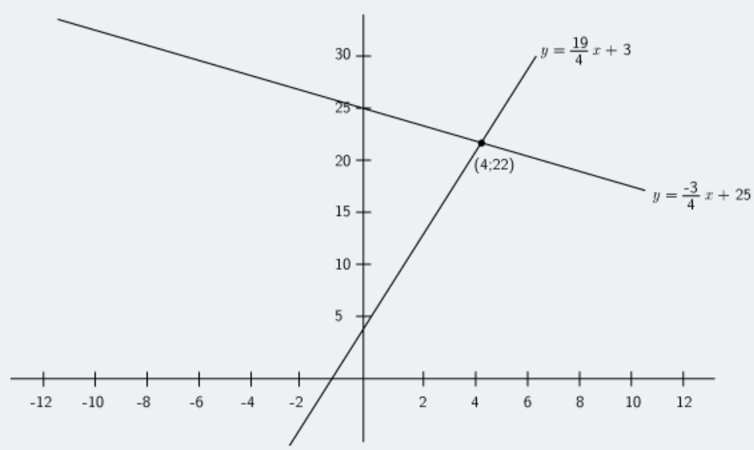
$$4y + 3x = 100 \qquad \ldots(1)$$
$$4y - 19x = 12 \qquad \ldots(2)$$

SOLUTION **Step 1: Write both equations in form $y = mx + c$**

$$\begin{aligned} 4y + 3x &= 100 \\ 4y &= 100 - 3x \\ y &= -\frac{3}{4}x + 25 \end{aligned}$$

$$\begin{aligned} 4y - 19x &= 12 \\ 4y &= 19x + 12 \\ y &= \frac{19}{4}x + 3 \end{aligned}$$

Step 2: Sketch the graphs on the same set of axes



Step 3: Find the coordinates of the point of intersection

The two graphs intersect at **(4;22)**

Step 4: Write the final answer

$$x = 4$$
$$y = 22$$

5 WORD PROBLEMS

To solve word problems we need to write a set of equations that represent the problem mathematically. The solution of the equations is then the solution to the problem.

5.1 Problem solving strategy

1. Read the whole question.
2. What are we asked to solve for?
3. Assign a variable to the unknown quantity, for example, x.
4. Translate the words into algebraic expressions by rewriting the given information in terms of the variable.
5. Set up an equation or system of equations to solve for the variable.
6. Solve the equation algebraically using substitution.
7. Check the solution.

WORKED EXAMPLE 11: SOLVING WORD PROBLEMS

QUESTION

A shop sells bicycles and tricycles. In total there are 7 cycles (cycles include both bicycles and tricycles) and 19 wheels. Determine how many of each there are, if a bicycle has two wheels and a tricycle has three wheels.

SOLUTION

Step 1: Assign variables to the unknown quantities

Let b be the number of bicycles and let t be the number of tricycles.

Step 2: Set up the equations

$$\begin{aligned} b + t &= 7 \qquad \ldots (1) \\ 2b + 3t &= 19 \qquad \ldots (2) \end{aligned}$$

Step 3: Rearrange equation (1) and substitute into equation (2)

$$\begin{aligned} t &= 7 - b \\ \therefore 2b + 21 - 3b &= 19 \\ -b &= -2 \\ \therefore b &= 2 \end{aligned}$$

Step 4: Calculate the number of tricycles t

$$\begin{aligned} t &= 7 - b \\ &= 7 - 2 \\ &= 5 \end{aligned}$$

Step 5: Write the final answer

There are 5 tricycles and 2 bicycles.

WORKED EXAMPLE 12: SOLVING WORD PROBLEMS

QUESTION

Bongani and Jane are friends. Bongani takes Jane's maths test paper and will not tell her what her mark is. He knows that Jane dislikes word problems so he decides to tease her. Bongani says: "I have 2 marks more than you do and the sum of both our marks is equal to 14. What are our marks?"

SOLUTION

Step 1: Assign variables to the unknown quantities

We have two unknown quantities, Bongani's mark and Jane's mark. Let Bongani's mark be b and Jane's mark be j.

Step 2: Set up a system of equations

Bongani has 2 more marks than Jane.

$$b = j + 2 \qquad \ldots (1)$$

Both marks add up to 14.

$$b + j = 14 \qquad \ldots (2)$$

Step 3: Use equation (1) to express b in terms of j

$$b = j + 2$$

Step 4: Substitute into equation (2)

$$\begin{aligned} b + j &= 14 \\ (j + 2) + j &= 14 \end{aligned}$$

Step 5: Rearrange and solve for j

$$\begin{aligned} 2j &= 14 - 2 \\ &= 12 \\ \therefore j &= 6 \end{aligned}$$

Step 6: Substitute the value for j back into equation (1) and solve for b

$$\begin{aligned} b &= j + 2 \\ &= 6 + 2 \\ &= 8 \end{aligned}$$

Step 7: Check that the solution satisfies both original equations

Step 8: Write the final answer

Bongani got 8 for his test and Jane got 6.

WORKED EXAMPLE 13: SOLVING WORD PROBLEMS

QUESTION

A fruitshake costs $R2,00$ more than a chocolate milkshake. If 3 fruitshakes and 5 chocolate milkshakes cost $R78,00$, determine the individual prices.

SOLUTION

Step 1: Assign variables to the unknown quantities

Let the price of a chocolate milkshake be x and let the price of a fruitshake be y.

Step 2: Set up a system of equations

$$\begin{aligned} y &= x + 2 \qquad \ldots(1) \\ 3y + 5x &= 78 \qquad \ldots(2) \end{aligned}$$

Step 3: Substitute equation (1) into (2)

$$3(x+2) + 5x = 78$$

Step 4: Rearrange and solve for x

$$\begin{aligned} 3x + 6 + 5x &= 78 \\ 8x &= 72 \\ \therefore x &= 9 \end{aligned}$$

Step 5: Substitute the value of x back into equation (1) and solve for y

$$\begin{aligned} y &= x + 2 \\ &= 9 + 2 \\ &= 11 \end{aligned}$$

Step 6: Check that the solution satisfies both original equations

Step 7: Write the final answer

One chocolate milkshake costs $R9,00$ and one fruitshake costs $R11,00$

WORKED EXAMPLE 14: SOLVING WORD PROBLEMS

QUESTION

The product of two consecutive negative integers is 1122. Find the two integers.

SOLUTION

Step 1: Assign variables to the unknown quantities

Let the first integer be n and let the second integer be $n+1$.

Step 2: Set up an equation

$$n(n+1) = 1122$$

Step 3: Expand and solve for n.

$$\begin{aligned} n^2 + n &= 1\ 122 \\ n^2 + n - 1\ 122 &= 0 \\ (n+34)(n-33) &= 0 \\ \therefore n &= -34 \\ \text{or } n &= 33 \end{aligned}$$

Step 4: Find the sign of the integers

It is given that both integers must be negative.

$$\begin{aligned} \therefore n &= -34 \\ n+1 &= -34+1 \\ &= -33 \end{aligned}$$

Step 5: Write the final answer

The two consecutive negative integers are -34 and -33.

6 LITERAL EQUATIONS

A literal equation is one that has several letters or variables. Examples include the area of a circle $(A = \pi r^2)$ and the formula for speed $(v = \frac{D}{t})$. In this section we solve literal equations in terms of one variable. To do this, we use the principles we have learnt about solving equations and apply them to rearranging literal equations. Solving literal equations is also known as changing the subject of the formula.

Keep the following in mind when solving literal equations:

- We isolate the unknown variable by asking "what is it joined to?" and "how is it joined?" We then perform the opposite operation to both sides as a whole.
- If the unknown variable is in two or more terms, then we take it out as a common factor.
- If we have to take the square root of both sides, remember that there will be a positive and a negative answer.
- If the unknown variable is in the denominator, we multiply both sides by the lowest common denominator (LCD) and then continue to solve.

WORKED EXAMPLE 15: SOLVING LITERAL EQUATIONS

QUESTION

The area of a triangle is $A = \frac{1}{2}bh$. What is the height of the triangle in terms of the base and area?

SOLUTION

Step 1: Isolate the required variable

We are asked to isolate the height, so we must rearrange the equation with h on one side of the equals sign and the rest of the variables on the other.

$$\begin{aligned} A &= \frac{1}{2}bh \\ 2A &= bh \\ \frac{2A}{b} &= h \end{aligned}$$

Step 2: Write the final answer

The height of a triangle is given by: $h = \frac{2A}{b}$

WORKED EXAMPLE 16: SOLVING LITERAL EQUATIONS

QUESTION

Given the formula:

$$h = R \times \frac{H}{R + r^2}$$

make R the subject of the formula.

SOLUTION

Step 1: Isolate the required variable

$$\begin{aligned} h\left(R + r^2\right) &= R \times H \\ hR + hr^2 &= HR \\ hr^2 &= HR - hR \\ hr^2 &= R\left(H - h\right) \\ \therefore R &= \frac{hr^2}{H - h} \end{aligned}$$

7 SOLVING LINEAR INEQUALITIES

A linear inequality is similar to a linear equation in that the largest exponent of a variable is 1. The following are examples of linear inequalities.

$$\begin{aligned}&2x+2\leq 1\\&\frac{2-x}{3x+1}\geq 2\\&\frac{4}{3}x-6<7x+2\end{aligned}$$

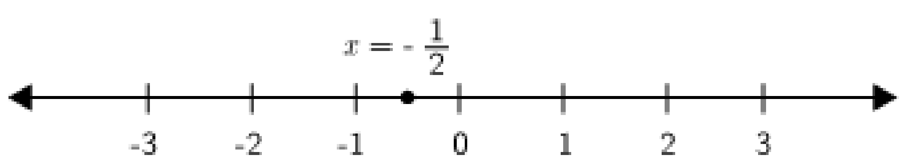
The methods used to solve linear inequalities are similar to those used to solve linear equations. The only difference occurs when there is a multiplication or a division that involves a minus sign. For example, we know that $8>6$. If both sides of the inequality are divided by -2, then we get $-4>-3$, which is not true. Therefore, the inequality sign must be switched around, giving $-4<-3$.

In order to compare an inequality to a normal equation, we shall solve an equation first.

Solve $2x+2=1$:

$$\begin{aligned}2x+2&=1\\2x&=1-2\\2x&=-1\\x&=-\frac{1}{2}\end{aligned}$$

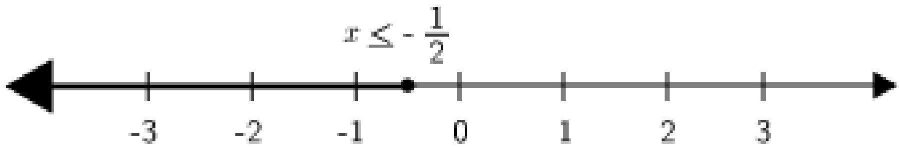
If we represent this answer on a number line, we get:



Now let us solve for x in the inequality $2x+2\leq 1$:

$$\begin{aligned}2x+2&\leq 1\\2x&\leq 1-2\\2x&\leq -1\\x&\leq -\frac{1}{2}\end{aligned}$$

If we represent this answer on a number line, we get:



We see that for the equation there is only a single value of x for which the equation is true. However, for the inequality, there is a range of values for which the inequality is true. This is the main difference between an equation and an inequality.

Remember: when we divide or multiply both sides of an inequality by a negative number, the direction of the inequality changes. For example, if $x < 1$, then $-x > -1$. Also note that we cannot divide or multiply by a variable.

7.1 Interval notation

$(4; 12)$	Round brackets indicate that the number is not included. This interval includes all real numbers greater than but not equal to 4 and less than but not equal to 12.
$(-\infty; -1)$	Round brackets are always used for positive and negative infinity. This interval includes all real numbers less than, but not equal to -1.
$[1; 13)$	A square bracket indicates that the number is included. This interval includes all real numbers greater than or equal to 1 and less than but not equal to 13.

It is important to note that this notation can only be used to represent an interval of real numbers.

We represent the above answer in interval notation as $\left(-\infty; -\frac{1}{2}\right]$

WORKED EXAMPLE 17: SOLVING LINEAR INEQUALITIES

QUESTION

Solve for r:

$$6 - r > 2$$

Represent the answer on a number line and in interval notation.

SOLUTION

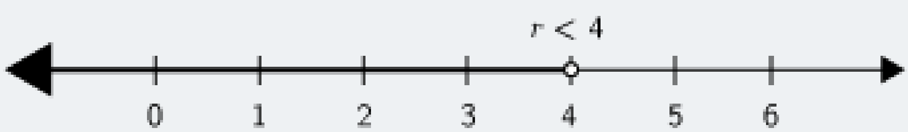
Sep 1: Rearrange and solve for r

$$-r > 2 - 6$$
$$-r > -4$$

Step 2: Multiply by -1 and reverse inequality sign

$$r < 4$$

Step 3: Represent the answer on a number line



Step 4: Represent the answer in interval notation

$$(-\infty; 4)$$

WORKED EXAMPLE 18: SOLVING LINEAR INEQUALITIES

QUESTION

Solve for q:

$$4q + 3 < 2(q + 3)$$

Represent the answer on a number line and in interval notation.

SOLUTION

Sep 1: Expand the bracket

$$4q + 3 < 2(q + 3)$$
$$4q + 3 < 2q + 6$$

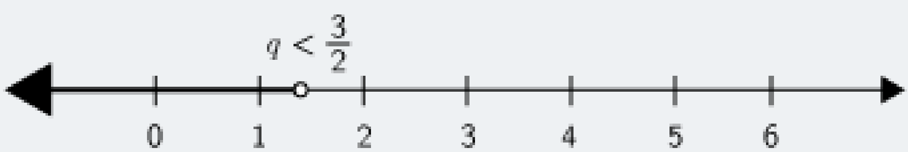
Step 2: Rearrange and solve for q

$$4q + 3 < 2q + 6$$
$$4q - 2q < 6 - 3$$
$$2q < 3$$

Step 3: Divide both sides by 2

$$2q < 3$$
$$q < \frac{3}{2}$$

Step 4: Represent the answer on a number line



Step 5: Represent the answer in interval notation

$$\left(-\infty; \frac{3}{2}\right)$$

WORKED EXAMPLE 19: SOLVING LINEAR INEQUALITIES

QUESTION

Solve for x:

$$5 \leq x + 3 < 8$$

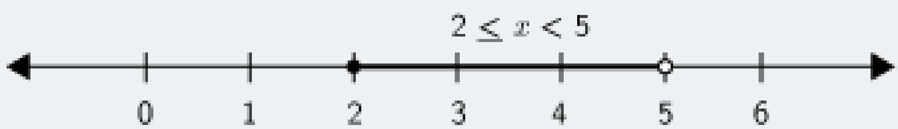
Represent the answer on a number line and in interval notation.

SOLUTION

Sep 1: Subtract 3 from all the parts of the inequality

$$\begin{array}{ccccc} 5-3 & \leq & x+3-3 & < & 8-3 \\ 2 & \leq & x & < & 5 \end{array}$$

Step 2: Represent the answer on a number line



Step 3: Represent the answer in interval notation

$$[2; 5)$$

8 CHAPTER SUMMARY

- A linear equation is an equation where the exponent of the variable is 1. A linear equation has at most one solution.
- A quadratic equation is an equation where the exponent of the variable is at most 2. A quadratic equation has at most two solutions.
- To solve for two unknown variables, two equations are required. These equations are known as a system of simultaneous equations. There are two ways to solve linear simultaneous equations: algebraic solutions and graphical solutions. To solve algebraically we use substitution or elimination methods. To solve graphically we draw the graph of each equation and the solution will be the coordinates of the point of intersection.
- Literal equations are equations that have several letters and variables.
- Word problems require a set of equations that represent the problem mathematically.
- A linear inequality is similar to a linear equation and has the exponent of the variable equal to 1.
- If we divide or multiply both sides of an inequality by any number with a minus sign, the direction of the inequality changes.

9 EXERCISES

9.1 Exercise 1

1. Solve the following equations (assume all denominators are non-zero):

   1.1 $2y - 3 = 7$

   1.2 $5x = 2x + 45$

   1.3 $23x - 12 = 6 + 3x$

   1.4 $12 - 6x + 34x = 2x - 24 - 64$

   1.5 $6x + 3x = 4 - 5(2x - 3)$

   1.6 $18 - 2p = p + 9$

   1.7 $\frac{4}{p} = \frac{16}{24}$

   1.8 $-(-16 - p) = 13p - 1$

   1.9 $3f - 10 = 10$

   1.10 $3f + 16 = 4f - 10$

   1.11 $10f + 5 = -2f - 3f + 80$

   1.12 $2c = c - 8$

   1.13 $8(f - 4) = 5(f - 4)$

   1.14 $6 = 6(f + 7) + 5f$

   1.15 $-7x = 8(1 - x)$

   1.16 $5 - \frac{7}{b} = \frac{2b+4}{b}$

   1.17 $\frac{x+2}{4} - \frac{x-6}{3} = \frac{1}{2}$

   1.18 $1 = \frac{3a-4}{2a+6}$

   1.19 $\frac{2-5a}{3} - 6 = \frac{4a}{3} + 2 - a$

   1.20 $2 - \frac{4}{b+5} = \frac{3b}{b+5}$

   1.21 $3 - \frac{y-2}{4} = 4$

   1.22 $1,5x + 3,125 = 1,25x$

   1.23 $3 = 1 - 2c$

   1.24 $1,3(2,7x + 1) = 4,1 - x$

   1.25 $6,5x - 4,15 = 7 + 4,25x$

   1.26 $\frac{1}{3}P + \frac{1}{2}P - 10 = 0$

1.27 $\frac{5}{4}(x-1)-\frac{3}{2}(3x+2)=0$

1.28 $\frac{1}{5}(x-1)=\frac{1}{3}(x-2)+3$

1.29 $\frac{5}{2a}+\frac{1}{6a}-\frac{3}{a}=2$

1.30 $4b+5=-7$

1.31 $-3y=0$

1.32 $16y+4=-10$

1.33 $12y+0=144$

1.34 $7+5y=62$

1.35 $55=5x+\frac{3}{4}$

9.2 Exercise 2

1. Write the following in standard form.

   1.1 $(r+4)(5r-4)=-16$

   1.2 $(3r-8)(2r-3)=-15$

   1.3 $(d+5)(2d+4)=8$

2. Solve the following equations:

   2.1 $x^2+2x-15=0$

   2.2 $4x^2-12x=-9$

   2.3 $20m+25m^2=0$

   2.4 $2x^2-5x-12=0$

   2.5 $-75x^2-290x=240$

   2.6 $2x=\frac{1}{3}x^2-3x+14\frac{2}{3}$

   2.7 $x^2-4x=-4$

   2.8 $-x^2+4x-6=4x^2-14x+3$

   2.9 $t^2=3t$

   2.10 $x^2-10x=-25$

   2.11 $x^2=18$

   2.12 $p^2-7p-18=0$

   2.13 $p^2-6p=7$

   2.14 $4x^2-17x-77=0$

   2.15 $14x^2+5x=6$

2.16 $2x^2 - 2x = 12$

2.17 $(2a-3)^2 = -16$

2.18 $(x-6)^2 - 24 = 1$

2.19 $9x^2 - 6x - 8 = 0$

2.20 $5x^2 + 21x - 54 = 0$

2.21 $4z^2 + 12z + 8 = 0$

2.22 $-b^2 + 7b - 12 = 0$

2.23 $-3a^2 + 27a - 54 = 0$

2.24 $4y^2 - 9 = 0$

2.25 $4x^2 + 16x - 9 = 0$

3. Solve the following equations. (note the restrictions that apply):

3.1 $3y = \frac{54}{2y}$

3.2 $\frac{a+1}{3a-4} + \frac{9}{2a+5} + \frac{2a+3}{2a+5} = 0$

3.3 $\frac{x^2-2x-3}{x+1} = 0$

3.4 $x + 2 = \frac{6x-12}{x-2}$

3.5 $\frac{3(a^2+1)+10a}{3a+1} = 1$

3.6 $\frac{3}{9a^2-3a+1} - \frac{3a+4}{27a^3+1} = \frac{1}{9a^2-1}$

3.7 $\frac{10z}{3} = 1 - \frac{1}{3z}$

3.8 $x + 2 = \frac{18}{x} - 1$

3.9 $y - 3 = \frac{5}{4} - \frac{1}{y}$

3.10 $\frac{1}{2}(b-1) = \frac{1}{3}(\frac{2}{b} + 4)$

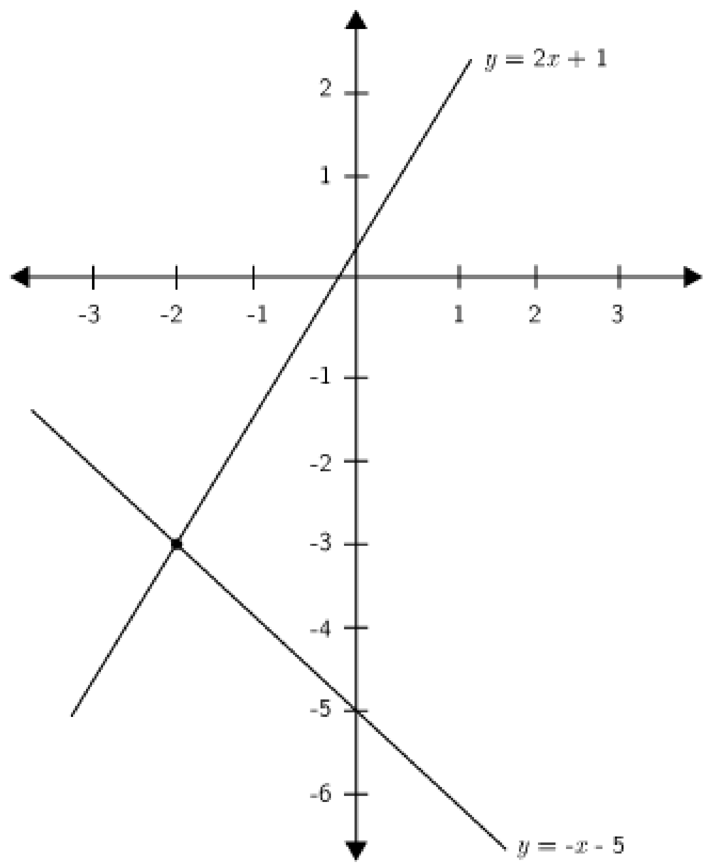
3.11 $3(y+1) = \frac{4}{y} + 2$

3.12 $z^4 - 1 = 0$

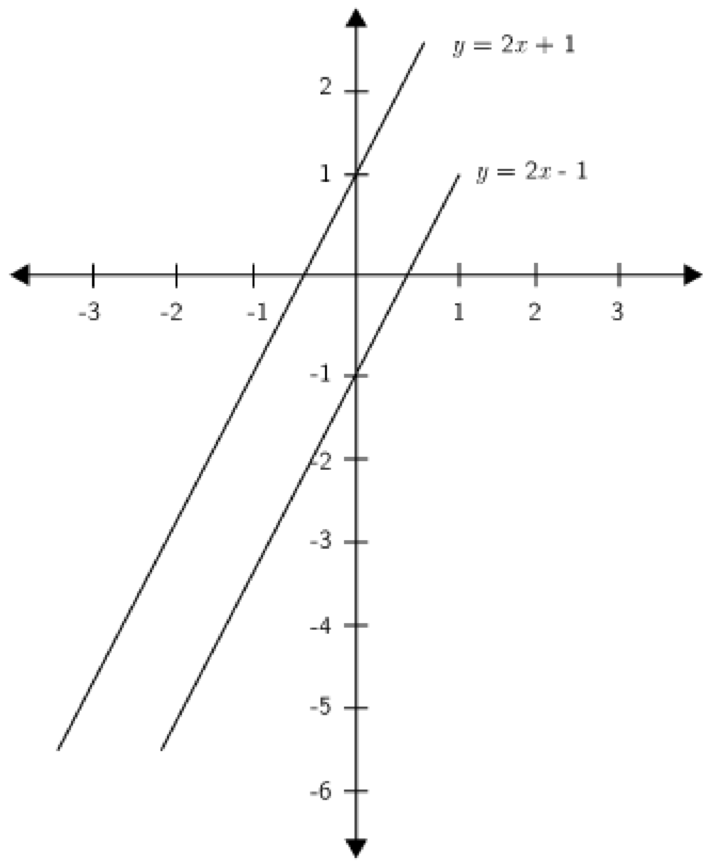
3.13 $b^4 - 13b^2 + 36 = 0$

9.3 Exercise 3

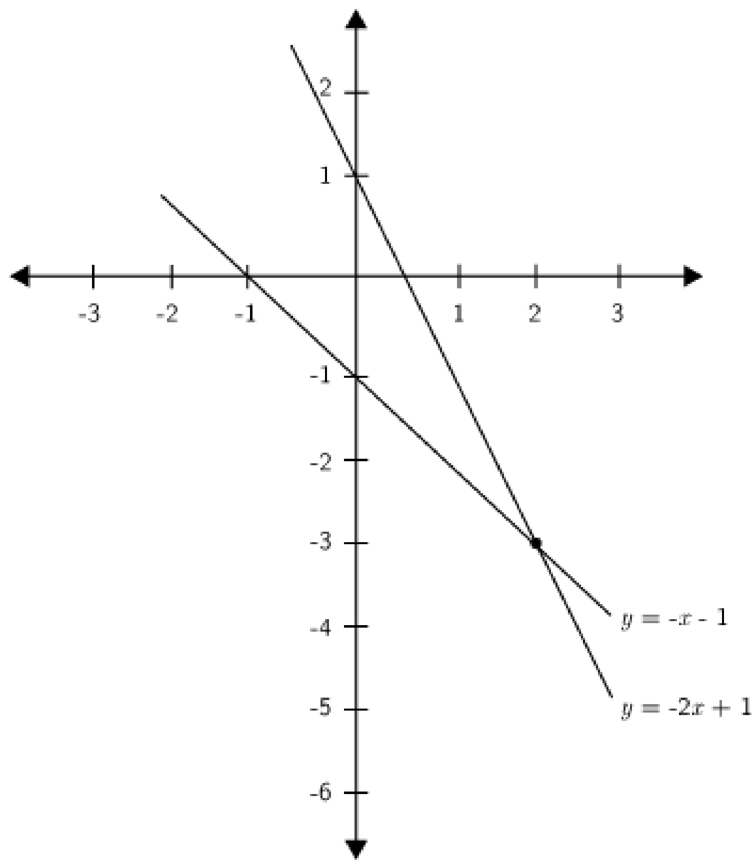
1. Look at the graph below. Solve the equations $y = 2x + 1$ and $y = -x - 5$ simultaneously.



2. Look at the graph below. Solve the equations $y = 2x - 1$ and $y = 2x + 1$ simultaneously.



3. Look at the graph below and solve the equations $y = -2x + 1$ and $y = -x - 1$ simultaneously.



4. Solve the equations simultaneously:

 4.1 $-10x = -1$ and $-4x + 10y = -9$

 4.2 $-10x - 10y = -2$ and $2x + 3y = 2$

 4.3 $\frac{1}{x} + \frac{1}{y} = 3$ and $\frac{1}{x} - \frac{1}{y=11}$

 4.4 $y = \frac{2(x^2+2)-3}{x^2+2}$ and $y = 2 - \frac{3}{x^2+2}$

 4.5 $3a + b = \frac{6}{2a}$ and $3a^2 = 3 - ab$

 4.6 $3x - 14y = 0$ and $x - 4y + 1 = 0 - ab$

 4.7 $x + y = 8$ and $3x + 2y = 21$

 4.8 $y = 2x + 1$ and $x + 2y + 3 = 0$

 4.9 $5x - 4y = 69$ and $2x + 3y = 23$

4.10 $x + 3y = 26$ and $5x + 4y = 75$

4.11 $3x - 4y = 19$ and $2x - 8y = 2$

4.12 $\frac{a}{2} + b = 4$ and $\frac{a}{4} - \frac{b}{4} = 1$

4.13 $-10x + y = -1$ and $-10x - 2y = 5$

5. Solve graphically and check your answer algebraically:

5.1 $y + 2x = 0$ and $y - 2x - 4 = 0$

5.2 $x + 2y = 1$ and $\frac{x}{3} + \frac{y}{2} = 1$

5.3 $y - 2 = 6x$ and $y - x = -3$

5.4 $2x + y = 5$ and $3x - 2y = 4$

5.5 $5 = x + y$ and $x = y - 2$

9.4 Exercise 4

1. Two jets are flying towards each other from airports that are 1 200 km apart.

   One jet is flying at $250 km{\cdot}h^{-1}$ and the other jet at $350 km{\cdot}h^{-1}$.

   If they took off at the same time, how long will it take for the jets to pass each other?

2. Two boats are moving towards each other from harbours that are $144km$ apart. One boat is moving at 63 km.h $^{-1}$ and the other boat at 81 km.h^{-1} . If both boats started their journey at the same time, how long will they take to pass each other?

3. Zwelibanzi and Jessica are friends. Zwelibanzi takes Jessica's civil technology test paper and will not tell her what her mark is. He knows that Jessica dislikes word problems so he decides to tease her. Zwelibanzi says: "I have 12 marks more than you do and the sum of both our marks is equal to 148 . What are our marks?"

4. Kadesh bought 20 shirts at a total cost of R980. If the large shirts cost R 50 and the small shirts cost R40, how many of each size did he buy?

5. The diagonal of a rectangle is 25 cm more than its width. The length of the rectangle is 17 cm more than its width. What are the dimensions of the rectangle?

6. The sum of 27 and 12 is equal to 73 more than an unknown number. Find the unknown number.

7. A group of friends is buying lunch. Here are some facts about their lunch:

1. A milkshake costs R7 more than a wrap.
2. The group buys 8 milkshakes and 2 wraps
3. The total cost for the lunch is R 326.

Determine the individual prices for the lunch items.

8. The two smaller angles in a right-angled triangle are in the ratio of $1 : 2$. What are the sizes of the two angles?

9. The length of a rectangle is twice the breadth. If the area is 128 cm^2 , determine the length and the breadth.

10. If 4 times a number is increased by 6, the result is 15 less than the square of the number. Find the number.

11. The length of a rectangle is 2 cm more than the width of the rectangle. The perimeter of the rectangle is 20 cm. Find the length and the width of the rectangle.

12. Stephen has 1 litre of a mixture containing 69% salt. How much water must Stephen add to make the mixture 50% salt? Write your answer as a fraction of a litre.

13. The sum of two consecutive odd numbers is 20 and their difference is 2 . Find the two numbers.

14. The denominator of a fraction is 1 more than the numerator. The sum of the fraction and its reciprocal is $\frac{5}{2}$. Find the fraction.

15. Masindi is 21 years older than her daughter, Mulivhu. The sum of their ages is 37 . How old is Mulivhu?

16. Tshamano is now five times as old as his son Murunwa. Seven years from now, Tshamano will be three times as old as his son. Find their ages now.

17. If adding one to three times a number is the same as the number, what is the number equal to?

18. If a third of the sum of a number and one is equivalent to a fraction whose denominator is the number and numerator is two, what is the number?

19. A shop owner buys 40 sacks of rice and mealie meal worth R $5\,250$ in total. If the rice costs R 150 per sack and mealie meal costs R 100 per sack, how many sacks of mealie meal did he buy?

20. There are 100 bars of blue and green soap in a box. The blue bars weigh 50 g per bar and the green bars 40 g per bar. The total mass of the soap in the box is $4,66$ kg. How many bars of green soap are in the box?

21. Lisa has 170 beads. She has blue, red and purple beads each weighing 13 g, 4 g and 8 g respectively. If there are twice as many red beads as there are blue beads and all the beads weigh $1,216$ kg, how many beads of each type does Lisa have?

9.5 Exercise 5

1. Solve for x in the following formula: $2x + 4y = 2$
2. Make a the subject of the formula: $s = ut\frac{1}{2}at^2$.
3. Solve for $n : pV = nRT$
4. Make x the subject of the formula: $\frac{1}{b} + \frac{2b}{x} = 2$.
5. Solve for r: $V = \pi r^2 h$
6. Solve for $h : E = \frac{hc}{\lambda}$
7. Solve for h :$A = 2\pi rh + 2\pi r$
8. Make λ the subject of the formula: $t = \frac{D}{f\lambda}$
9. Solve for m: $E = mgh + \frac{1}{2}mv^2$.
10. Solve for $x : x^2 + x(a + b) + ab = 0$
11. Solve for b: $c = \sqrt{a^2 + b^2}$
12. Make U the subject of the formula: $\frac{1}{V} = \frac{1}{U} + \frac{1}{W}$
13. Solve for r: $A = \pi R^2 + \pi r^2$
14. $F = \frac{9}{5}C + 32°$ is the formula for converting temperature in degrees Celsius to degrees Fahrenheit. Derive a formula for converting degrees Fahrenheit to degrees Celcius.
15. $V = \frac{4}{3}\pi r^3$ is the formula for determining the volume of a soccer ball. Express the radius in terms of the volume.
16. Solve for x in : $x^2 - ax - 3x = 4 + a$
17. Solve for x in : $x^2 - 4a + bx^2 - 4b = 0$
18. Solve for x in : $v^2 = u^2 + 2ax$ if $v = 2, u = 0,3, a = 0,5$
19. Solve for u in $f' = f\frac{v}{v-u}$, if$v = 13, f = 40, f' = 50$
20. Solve for h in $I = \frac{bh^2}{12}$ if $b = 18, I = 384$
21. Solve for r_2 in $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$ if $R = \frac{3}{2}, r_1 = 2$

9.6 Exercise 6

1. Look at the number line and write down the inequality it represents.

(a)

(b)

(c)

(d)

2. Solve for x and show your answer in interval notation:

2.1 $3x + 4 > 5x + 8$

2.2 $3 \leq 4 - x \leq 16$

2.3 $\frac{-7y}{3} - 5 > -7$

2.4 $1 \leq 1 - 2y < 9$

2.5 $-2 < \frac{x-1}{-3} < 7$

2.6 $3(x-1) - 2 \leq 6x + 4$

2.7 $\frac{x-7}{3} > \frac{2x-3}{2}$

2.8 $-4(x-1) < x + 2$

2.9 $\frac{1}{2}x + \frac{1}{3}(x-1) \geq \frac{5}{6}x - \frac{1}{3}$

2.10 $-2 \leq x - 1 < 3$

2.11 $-5 < 2x - 3 \leq 7$

2.12 $7(3x+2) - 5(2x-3) > 7$

2.13 $\frac{5x-1}{-6} \geq \frac{1-2x}{3}$

3. Solve x for and show your answer in interval notation:

3.1 $2x - 1 < 3(x + 11)$

3.2 $x - 1 < -4(x - 6)$

3.3 $\frac{x-1}{8} \leq \frac{2(x-2)}{3}$

3.4 $\frac{x+2}{4} \leq \frac{-2(x-4)}{7}$

3.5 $\frac{1}{5}x - \frac{5}{4}(x+2) > \frac{1}{4}x + 3$

3.6 $\frac{1}{5}x - \frac{2}{5}(x+2) > \frac{4}{2}x + 3$

3.7 $4x + 3 < -3$ or $4x + 3 > 5$

3.8 $4 \geq -6x - 6 \geq -3$

4. Solve for the unknown variable and show your answer on a number line.

4.1 $6b - 3 > b + 2, b \in \mathbb{Z}$

4.2 $3a - 1 < 4a + 6, a \in \mathbb{N}$

4.3 $\frac{b-3}{2} + 1 < \frac{b}{4} - 4, b \in \mathbb{R}$

4.4 $\frac{4a+7}{3} - 5 > a - \frac{2}{3}, a \in \mathbb{R}$

10 ANSWERS FOR EXERCISES

10.1 Exercise 1

1.1 $y = 5$

1.2 $x = 15$

1.3 $x = \frac{9}{10}$

1.4 $x = -\frac{50}{13}$

1.5 $x = 1$

1.6 $p = 3$

1.7 $p = 6$

1.8 $p = \frac{17}{12}$

1.9 $f = \frac{20}{3}$

1.10 $f = 26$

1.11 $f = 5$

1.12 $c = -8$

1.13 $f = 4$

1.14 $f = -\frac{36}{11}$

1.15 $x = 8$

1.16 $b = 5$

1.17 $x = 24$

1.18 $a = 10$

1.19 $a = -\frac{11}{3}$

1.20 $b = 6$

1.21 $y = -2$

1.22 $x = -12{,}5$

1.23 $c = -1$

1.24 $= \frac{280}{451}$

1.25 $= \frac{223}{45}$

1.26 $p = 12$

1.27 $x = -\frac{17}{13}$

1.28 $x = -19$

1.29 $a = -\frac{1}{6}$

1.30 $b = -3$

1.31 $y = 0$

1.32 $= -\frac{7}{8}$

1.33 $y = 12$

1.34 $y = 11$

1.35 $x = \frac{217}{20}$

10.2 Exercise 2

1.1 $5r^2 + 16r = 0$

1.2 $6r^2 - 25r + 39 = 0$

1.3 $2d^2 + 15d + 17 = 0$

2.1 $x = -5$ or $x = 3$

2.2 $x = \frac{3}{2}$

2.3 $m = 0$ or $m = -\frac{4}{5}$

2.4 $x = -\frac{3}{2}$ or $x = 4$

2.5 $x = \frac{6}{5}$ or $x = \frac{8}{3}$

2.6 $x = 4$ or $x = 11$

2.7 $x = 2$

2.8 $x = \frac{3}{5}$ or $x = 3$

2.9 $t = 0$ or $t = 3$

2.10 $x = 5$

2.11 $x = \sqrt{18}$ or $x = -\sqrt{18}$

2.12 $p = -2$ or $p = 9$

2.13 $p = 7$ or $p = -1$

2.14 $x = -\frac{11}{4}$ or $x = 7$

2.15 $x = -\frac{6}{7}$ or $x = \frac{1}{2}$

2.16 $x = 3$ or $x = -2$

2.17 $a = -\frac{1}{2}$ or $a = 3,5$

2.18 $x = 11$ or $x = 1$

2.19 $x = -\frac{2}{3}$ or $x = \frac{4}{3}$

2.20 $x = \frac{9}{5}$ or $x = -6$

2.21 $z = -2$ or $z = -1$

2.22 $b = 3$ or $b = 4$

2.23 $a = 3$ or $a = 6$

2.24 $y = \frac{3}{2}$ or $y = -\frac{3}{2}$

2.25 $x = \frac{1}{2}$ or $x = -\frac{9}{2}$

3.1 $y = 3$ or $y = -3$

3.2 $a = -\frac{43}{8}$ or $a = 1$

3.3 $x = 3$

3.4 $x = 4$

3.5 $a = -2$

3.6 $a = 0$ or $a = \frac{2}{3}$

3.7 The function does not intersect the z-axis

3.8 $x = 3$ or $x = -6$

3.9 $y = \frac{1}{4}$ or $y = 4$

3.10 $b = -\frac{1}{3}$ or $b = 4$

3.11 $y = -\frac{4}{3}$ or $y = 1$

3.12 $x = 4$ or $x = -4$

3.13 $z = 1$ or $z = -1$

3.14 $b = \pm 2$ or $b = \pm 3$

10.3 Exercise 3

1. $x = -2$ and $y = -3$
2. No Solution.
3. $x = 2$ and $y = -3$.

4.1 $x = \frac{1}{10}$ and $y = -\frac{43}{50}$

4.2 $x = -\frac{7}{5}$ and $y = \frac{8}{5}$

4.3 $x = \frac{1}{7}$ and $y = -\frac{1}{4}$

4.4 x can be any real number, $\frac{1}{2} \leq y \leq 2$.

4.5 a and b can be any real number except for 0.

4.6 $x = -7$ and $y = -\frac{3}{2}$

4.7 $x = 5$ and $y = 3$

4.8 $x = -1$ and $y = -1$

4.9 $x = 13$ and $y = -1$

4.10 $x = 11$ and $y = 5$

4.11 $x = 9$ and $y = 2$

4.12 $a = \frac{16}{3}$ and $b = \frac{4}{3}$

4.13 $x = -\frac{1}{10}$ and $y = -2$

5.1 $x = -1$
$y = 2$

5.2 $y = -4$
$x = 9$

5.3 $x = -1$
$y = -4$

5.4 $x = 2$
$y = 1$

5.5 $x = \frac{3}{2}$
$y = \frac{7}{2}$

10.4 Exercise 4

1. It will take take the jets 2 hours to pass each other.
2. The boats will meet after 1 hour.
3. Zwelibanzi achieved 80 marks and Jessica achieved 68 marks.
4. Kadesh buys 18 large shirts and 2 small shirts.
5. width $w = 28$ cm
   length $l = (w + 17) = 45$ cm and
   diagonal $d = (w + 25) = 53$ cm.
6. The unknown number is -34
7. A milkshake costs R 34 and a wrap costs R27.
8. The sizes of the angles are $30°$ and $60°$.
9. $b = 8$ cm and $l = 2b = 16$ cm
10. We are not told if the number is positive or negative.
    Therefore the number is 7 or -3 .
11. length: 6 cm, width: 4 cm
12. $\frac{19}{50}$ litres must be added.
13. The two numbers are 9 and 11.
14. The fraction is $frac12$
15. Mulivhu is 8 years old.
16. Murunwa is 7 years old and Tshamano is 35 years old.
17. Let the number be x. $x = -\frac{1}{2}$
18. Let the number be x. $x = 2$ or $x = -3$
19. 15 sacks of melie meal were bought
20. 34 bars of green soap are in the box
21. Lisa has 48 blue beads, 96 red beads and 36 purple beads.

10.5 Exercise 5

1. $x = 1 - 2y$
2. $\frac{2(s-ut)}{t^2} = a$
   Note restriction: $t \neq 0$
3. $\frac{pV}{RT} = n$
   Note restrictions: $R \neq 0; T \neq 0$
4. $x = \frac{-2b^2}{1-2b}$
   Note restriction: $b \neq \frac{1}{2}$
5. $\pm\sqrt{\frac{V}{\pi h}} = r$
   Note restriction: $h \neq 0$
6. $\frac{E\lambda}{c} = h$
   Note restriction: $c \neq 0$
7. $\frac{A-2\pi r}{2\pi r} = h$
   Note restriction: $r \neq 0$
8. $\lambda = \frac{D}{tf}$
   Note restrictions: $t \neq 0; f \neq 0$
9. $\frac{E}{gh+\frac{1}{2}v^2} = m$
   Note restriction: $gh + \frac{1}{2}v^2 \neq 0$
10. $x = -a$ or $x = -b$
11. $b = \pm\sqrt{c^2 - a^2}$
12. $U = \frac{VW}{W-V}$
    Note restriction: $W \neq V$
13. $r = \pm\sqrt{\frac{\pi R^2 - A}{\pi}}$
14. $\frac{5(F-3^\circ)}{9} = C$
15. $r = \sqrt[3]{\frac{3}{4}V\pi}$
16. $x = a + 4$ or $x = -1$
17. $x = 2$ or $x = -2$
18. $x = 3,91$
19. $u = 2,6$

20. $h = \pm 16$

21. $r_2 = 6$

10.6 Exercise 6

1.1 $x < -1$ and $x \geq 6; x \in \mathbb{R}$

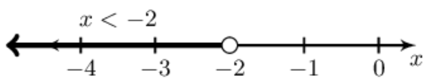
1.2 $3 < x < 6; x \in \mathbb{R}$

1.3 $x \in \mathbb{R}; x \neq 3; x \neq 6$

1.4 $x > -10; x \in \mathbb{R}$

2.1 $x < -2$

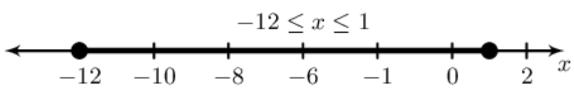
Represented on a number line:



In interval notation: $(-\infty; -2)$

2.2 $1 \geq x \geq -12$

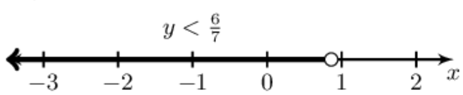
Represented on a number line:



In interval notation: $[1; 12]$

2.3 $y < \frac{6}{7}$

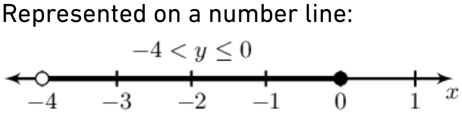
Represented on a number line:



In interval notation: $\left(-\infty; \frac{6}{7}\right)$

2.4 $-4 < y \leq 0$

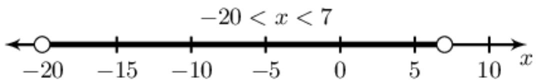
Represented on a number line:



In interval notation: $(-4; 0]$

2.5 $-20 < x < 7$

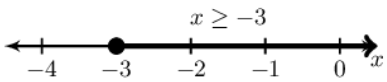
Represented on a number line:



In interval notation: $(-20; 7)$

2.6 $x \geq -3$

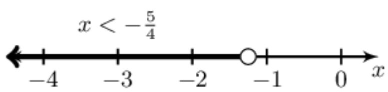
Represented on a number line:



In interval notation: $[-3; \infty)$

2.7 $x < -\frac{5}{4}$

Represented on a number line:

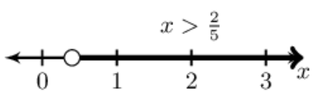


In interval notation: (-linfty;

$-$ lfrac$\{5\}\{4\}\}$

2.8 $x > \frac{2}{5}$

Represented on a number line:

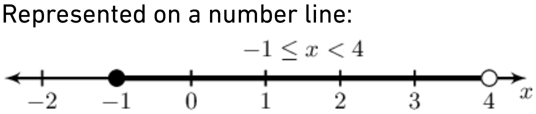


In interval notation: $\left(\frac{2}{5}; \infty\right)$

2.9 The inequality is true for all real values of x.

2.10 $-1 < x < 4$

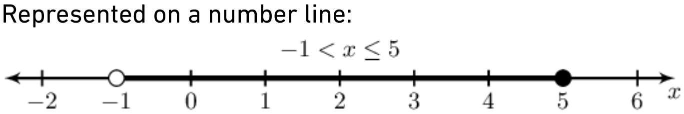
Represented on a number line:



In interval notation: $[-1; 4)$

2.11 $-1 < x \leq 5$

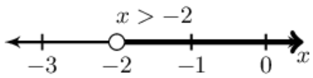
Represented on a number line:



In interval notation: $(-1; 5]$

2.12 $x > -2$

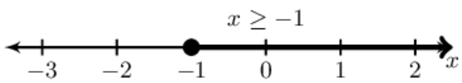
Represented on a number line:



In interval notation: $(-2; \infty)$

2.13 $x \geq -1$

Represented on a number line:



In interval notation: $[-1; \infty)$

3.1 $(-34; \infty)$

3.2 $(-\infty; 5)$

3.3 $x \in \left[\frac{29}{13}; \infty\right)$

3.4 $x \in \left(-\infty; \frac{6}{5}\right]$

3.5 $\left(-\infty; -\frac{55}{13}\right)$

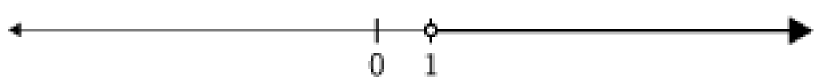
3.6 $\left(-\infty; -\frac{21}{11}\right]$

3.7 $\left(-\infty; -\frac{3}{2}\right) \cup \left(\frac{1}{2}; \infty\right)$

3.8 $-\frac{5}{3} \geq -\frac{1}{2}$

4.1 $6b - 3 > b + 2, b \in \mathbb{Z}$

$b > 1$

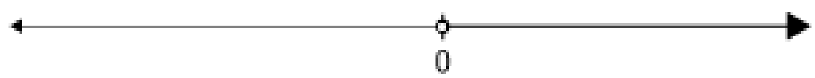


4.2 $3a - 1 < 4a + 6, a \in \mathbb{N}$

$a > -7$

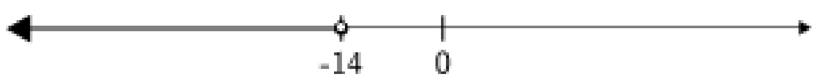
However we are told that $a \in \mathbb{N}$

and so $a > 0$.



4.3 $\frac{b-3}{2} + 1 < \frac{b}{4} - 4, b \in \mathbb{R}$

$b < -14$



4.4 $\frac{4a+7}{3} - 5 > a - \frac{2}{3}, a \in \mathbb{N}$