CHAPTER 2

Equations And Inequalities

CONTENTS

MathU Teaching® Emporium

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1 REVISION

1.1 Solving quadratic equations using factorisation

Terminology:	**Definition:**
Expression	An expression is a term or group of terms consisting of numbers, variables and the basic operators ($+, -, \times, \div, x^n$)
Equation	A mathematical statement that asserts that two expressions are equal.
Inequality	An inequality states the relation between two expressions (<,>,$\leq$, $\geq$).
Solution	A value or set of values that satisfy the original problem statement.
Root	A root of an equation is the value of x such that $f(x) = 0$.

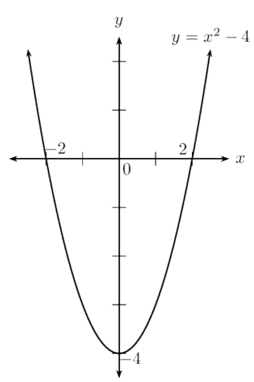
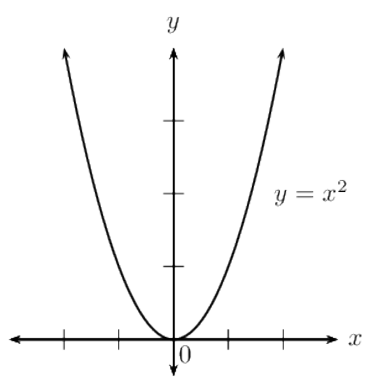
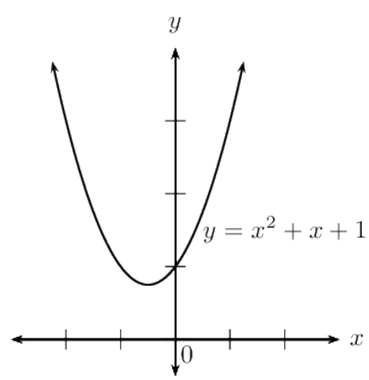
A quadratic equation is an equation of the second degree; the exponent of one variable is 2.
The following are examples of quadratic equations:

$$2x^2 - 5x = 12$$

$$a(a-3) - 10 = 0$$

$$\frac{3b}{b+2} + 1 = \frac{4}{b+1}$$

A quadratic equation has at most two solutions, also referred to as roots. There are some situations, however, in which a quadratic equation has either one solution or no solutions.

		
$y = (x-2)(x+2)$ $= x^2 - 4$ Graph of a quadratic equation with two roots: $x = -2$ and $x = 2$.	$y = x^2$ Graph of a quadratic equation with one root: $x = 0$.	$y = x^2 + x + 1$ Graph of a quadratic equation with no real roots.

One method for solving quadratic equations is factorisation. The standard form of a quadratic equation is $ax^2 + bx + c = 0$ and it is the starting point for solving any equation by factorisation.

It is very important to note that one side of the equation must be equal to zero.

Investigation

Zero product law

Solve the following equations:

1. $6 \times 0 =?$
2. $-25 \times 0 =?$
3. $0 \times 0,69 =?$
4. $7 \times ? = 0$

Now solve for the variable in each of the following:

1. $6 \times m = 0$
2. $32 \times x \times 2 = 0$
3. $11(z - 3) = 0$
4. $(k + 3)(k - 4) = 0$

To obtain the two roots we use the fact that if $a \times b = 0$, then $a = 0$ and/or $b = 0$. This is called the zero product law.

1.2 Method for solving quadratic equations

1. Rewrite the equation in the standard form $ax^2 + bx + c = 0$.
2. Divide the entire equation by any common factor of the coefficients to obtain a simpler equation of the form $ax^2 + bx + c = 0$, where a, b and c have no common factors.
3. Factorise $ax^2 + bx + c = 0$ to be of the form $(rx + s)(ux + v) = 0$.
4. The two solutions are:

   $(rx + s) = 0$ or $(ux + v) = 0$

   So $x = -\frac{s}{r}$ So $x = -\frac{v}{u}$

5. Always check the solution by substituting the answer back into the original equation.

WORKED EXAMPLE 1: Solving quadratic equations using factorisation

QUESTION

Solve for $x : x(x-3) = 10$

SOLUTION **Step 1: Rewrite the equation in the form $ax^2 + bx + c = 0$**

Expand the brackets and subtract 10 from both sides of the equation

$$x^2 - 3x - 10 = 0$$

Step 2: Factorise

$$(x+2)(x-5) = 0$$

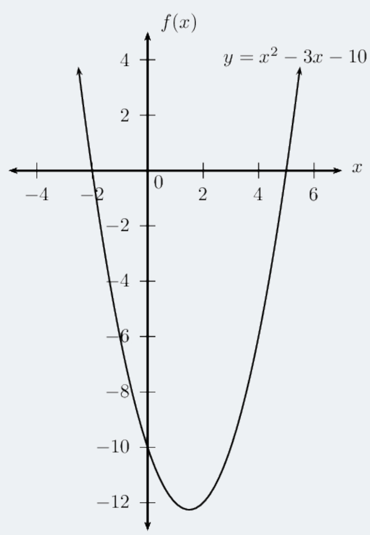
Step 3: Solve for both factors

$$\begin{aligned} x + 2 &= 0 \\ x &= -2 \end{aligned}$$

or

$$\begin{aligned} x - 5 &= 0 \\ x &= 5 \end{aligned}$$

The graph shows the roots of the equation $x = -2$ or $x = 5$. This graph does not form part of the answer as the question did not ask for a sketch. It is shown here for illustration purposes only.



WORKED EXAMPLE 1: Solving quadratic equations using factorisation

WORKED EXAMPLE 1 CONTINUED

Step 4: Check the solution by substituting both answers back into the original equation

Step 5: Write the final answer

Therefore, $x = -2$ or $x = 5$

WORKED EXAMPLE 2: Solving quadratic equations using factorisation

QUESTION

Solve the equation: $2x^2–5x–12 = 0$

SOLUTION

Step 1: There are no common factors

Step 2: The quadratic equation is already in the standard form $ax^2 + bx + c = 0$

Step 3: Factorise

We must determine the combination of factors of 2 and 12 that will give a middle term coefficient of 5.

We find that 2×1 and 3×4 give a middle term coefficient of 5 so we can factorise the equation as

$$(2x + 3)(x - 4) = 0$$

Step 4: Solve for both roots

We have

$$2x + 3 = 0$$
$$x = -\frac{3}{2}$$

or

$$x - 4 = 0$$
$$x = 4$$

Step 5: Check the solution by substituting both answers back into the original equation

Step 6: Write the final answer

Therefore, $x = -\frac{3}{2}$ or $x = 4$

WORKED EXAMPLE 3: Solving quadratic equations using factorisation

QUESTION

Solve for y : $y^2 - 7 = 0$

SOLUTION

Step 1: Factorise as a difference of two squares

We know that

$$\left(\sqrt{7}\right)^2 = 7$$

We can write the equation as

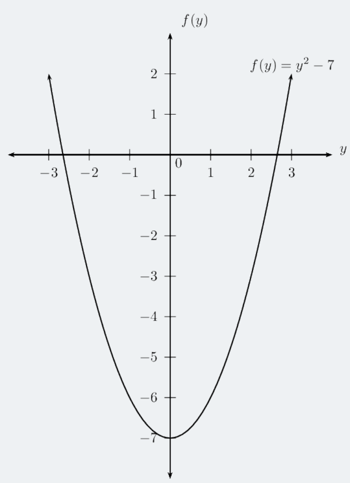
$$y^2 - (\sqrt{7})^2 = 0$$

Step 2: Factorise

$$(y - \sqrt{7})(y + \sqrt{7}) = 0$$

Therefore $y = \sqrt{7}$ or $y = -\sqrt{7}$

Even though the question did not ask for a sketch, it is often very useful to draw the graph. We can let $f(y) = y^2 - 7$ and draw a rough sketch of the graph to see where the two roots of the equation lie.



Step 3: Check the solution by substituting both answers back into the original equation

Step 4: Write the final answer

Therefore, $y = \pm\sqrt{7}$

WORKED EXAMPLE 4: Solving quadratic equations using factorisation

QUESTION

Solve for b :

$$\frac{3b}{b+2}+1=\frac{4}{b+1}$$

SOLUTION

Step 1: Determine the restrictions

The restrictions are the values for b that would result in the denominator being equal to 0, which would make the fraction undefined. Therefore $b \neq -2$ or $b \neq -1$.

Step 2: Determine the lowest common denominator

The lowest common denominator is $(b+2)(b+1)$.

Step 3: Multiply each term in the equation by the lowest common denominator and simplify

$$\begin{aligned}\frac{3b(b+2)(b+1)}{b+2}+(b+2)(b+1) &= \frac{4(b+2)(b+1)}{b+1} \\ 3b(b+1)+(b+2)(b+1) &= 4(b+2) \\ 3b^2+3b+b^2+3b+2 &= 4b+8 \\ 4b^2+2b-6 &= 0 \\ 2b^2+b-3 &= 0\end{aligned}$$

Step 4: Factorise and solve the equation

$$\begin{aligned}(2b+3)(b-1) &= 0 \\ 2b+3=0 &\text{ or } b-1=0 \\ b=-\frac{3}{2} &\text{ or } b=1\end{aligned}$$

Step 5: Check the solution by substituting both answers back into the original equation

Step 6: Write the final answer

Therefore, $b=-1\frac{1}{2}$ or $b=1$

WORKED EXAMPLE 5: Solving quadratic equations using factorisation

QUESTION

Solve for m: $m + 2 = \sqrt{7 + 2m}$

SOLUTION

Step 1: Square both sides of the equation

Before we square both sides of the equation, we must make sure that the radical is the only term on one side of the equation and all other terms are on the other, otherwise squaring both sides will make the equation more complicated to solve.

$$(m + 2)^2 = \left(\sqrt{7 + 2m}\right)^2$$

Step 2: Expand the brackets and simplify

$$\begin{aligned} (m + 2)^2 &= \left(\sqrt{7 + 2m}\right)^2 \\ m^2 + 4m + 4 &= 7 + 2m \\ m^2 + 2m - 3 &= 0 \end{aligned}$$

Step 3: Factorise and solve for m

$$(m - 1)(m + 3) = 0$$

Therefore $m = 1$ or $m = -3$

Step 4: Check the solution by substituting both answers back into the original equation

To find the solution we squared both sides of the equation. Squaring an expression changes negative values to positives and can therefore introduce invalid answers into the solution. Therefore it is very important to check that the answers obtained are valid. To test the answers, always substitute back into the original equation.

WORKED EXAMPLE 5: Solving quadratic equations using factorisation

WORKED EXAMPLE 5 CONTINUED

If $m = 1$:

$$\begin{aligned} \text{RHS} &= \sqrt{7 + 2(1)} \\ &= \sqrt{9} \\ &= 3 \\ \text{LHS} &= 1 + 2 \\ &= 3 \\ \text{LHS} &= \text{RHS} \end{aligned}$$

Therefore $m = 1$ is valid.

If $m = -3$:

$$\begin{aligned} \text{RHS} &= \sqrt{7 + 2(-3)} \\ &= \sqrt{1} \\ &= 1 \\ \text{LHS} &= -3 + 2 \\ &= -1 \\ \text{LHS} &\neq \text{RHS} \end{aligned}$$

Therefore $m = -3$ is not valid.

Step 5: Write the final answer

Therefore, $m = 1$

2 COMPLETING THE SQUARE

Investigation

Completing the square

Can you solve each equation using two different methods?:

1. $x^2 - 4 = 0$
2. $x^2 - 8 = 0$
3. $x^2 - 4x + 4 = 0$
4. $x^2 - 4x - 4 = 0$

Factorising the last equation is quite difficult. Use the previous examples as a hint and try to create a difference of two squares.

We have seen that expressions of the form $x^2 – b^2$ are known as differences of squares and can be factorised as $(x – b)(x + b)$. This simple factorisation leads to another technique for solving quadratic equations known as completing the square.

Consider the equation $x^2 – 2x – 1 = 0$. We cannot easily factorise this expression. When we expand the perfect square $(x – 1)^2$ and examine the terms we see that $(x – 1)^2 = x^2 – 2x + 1$.

We compare the two equations and notice that only the constant terms are different. We can create a perfect square by adding and subtracting the same amount to the original equation.

$$
\begin{aligned}
x^2 - 2x - 1 &= 0 \\
(x^2 - 2x + 1) - 1 - 1 &= 0 \\
(x^2 - 2x + 1) - 2 &= 0 \\
(x - 1)^2 - 2 &= 0
\end{aligned}
$$

Method 1: Take square roots on both sides of the equation to solve for x.

$$\begin{aligned}(x-1)^2-2&=0\\(x-1)^2&=2\\\sqrt{(x-1)^2}&=\pm\sqrt{2}\\x-1&=\pm\sqrt{2}\\x&=1\pm\sqrt{2}\end{aligned}$$

Therefore $x=1+\sqrt{2}$ or $x=1-\sqrt{2}$

Very important: Always remember to include both a positive and a negative answer when taking the square root, since $2^2=4$ and $(-2)^2=4$.

Method 2: Factorise the expression as a difference of two squares using $2=\left(\sqrt{2}\right)^2$.

We can write

$$\begin{aligned}(x-1)^2-2&=0\\(x-1)^2-\left(\sqrt{2}\right)^2&=0\\\left((x-1)+\sqrt{2}\right)\left((x-1)-\sqrt{2}\right)&=0\end{aligned}$$

The solution is then

$$\begin{aligned}(x-1)+\sqrt{2}&=0\\x&=1-\sqrt{2}\end{aligned}$$

or

$$\begin{aligned}(x-1)-\sqrt{2}&=0\\x&=1+\sqrt{2}\end{aligned}$$

Method for solving quadratic equations by completing the square

1. Write the equation in the standard form $ax^2+bx+c=0$.
2. Make the coefficient of the x^2 term equal to 1 by dividing the entire equation by a.
3. Take half the coefficient of the x term and square it; then add and subtract it from the equation so that the equation remains mathematically correct. In the example above, we added 1 to complete the square and then subtracted 1 so that the equation remained true.
4. Write the left hand side as a difference of two squares.
5. Factorise the equation in terms of a difference of squares and solve for x.

WORKED EXAMPLE 6: SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

QUESTION

Solve by completing the square: $x^2 - 10x - 11 = 0$

SOLUTION

Step 1: The equation is already in the form $ax^2 + bx + c = 0$

Step 2: Make sure the coefficient of the x^2 term is equal to 1

$$x^2 - 10x - 11 = 0$$

Step 3: Take half the coefficient of the x term and square it; then add and subtract it from the equation

The coefficient of the x term is -10. Half of the coefficient of the x term is -5 and the square of it is 25. Therefore $x^2 - 10x + 25 - 25 - 11 = 0$.

Step 4: Write the trinomial as a perfect square

$$(x^2 - 10x + 25) - 25 - 11 = 0$$
$$(x - 5)^2 - 36 = 0$$

Step 5: Method 1: Take square roots on both sides of the equation

$$(x - 5)^2 - 36 = 0$$
$$(x - 5)^2 = 36$$
$$x - 5 = \pm\sqrt{36}$$

Important: When taking a square root always remember that there is a positive and negative answer, since $(6)^2 = 36$ and $(-6)^2 = 36$

Step 6: Solve for x

$$x = -1 \text{ or } x = 11$$

WORKED EXAMPLE 6: SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

WORKED EXAMPLE 6 CONTINUED

Step 7: Method 2: Factorise equation as a difference of two squares

$$(x-5)^2 - (6)^2 = 0$$
$$[(x-5)+6]\,[(x-5)-6] = 0$$

Step 8: Simplify and solve for x

$$(x+1)(x-11) = 0$$
$$\therefore x = -1 \text{ or } x = 11$$

Step 9: Write the final answer

$$x = -1 \text{ or } x = 11$$

Notice that both methods produce the same answer. These roots are rational because 36 is a perfect square.

WORKED EXAMPLE 7: SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

QUESTION

Solve by completing the square: $x^2 - 10x - 11 = 0$

SOLUTION

Step 1: The equation is already in the form $ax^2 + bx + c = 0$

Step 2: Make sure the coefficient of the x^2 term is equal to 1

The coefficient of the x^2 term is 2. Therefore divide the entire equation by 2:

$$x^2 - 3x - 5 = 0$$

Step 3: Take half the coefficient of the x term, square it; then add and subtract it from the equation

The coefficient of the x term is −3, so then $\left(\frac{-3}{2}\right)^2 = \frac{9}{4}$

$$\left(x^2 - 3x + \frac{9}{4}\right) - \frac{9}{4} - 5 = 0$$

Step 4: Write the trinomial as a perfect square

$$\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{20}{4} = 0$$

$$\left(x - \frac{3}{2}\right)^2 - \frac{29}{4} = 0$$

Step 5: Method 1: Take square roots on both sides of the equation

$$\left(x - \frac{3}{2}\right)^2 - \frac{29}{4} = 0$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

$$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}}$$

Remember: When taking a square root there is a positive and a negative answer.

WORKED EXAMPLE 7: SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

WORKED EXAMPLE 7 CONTINUED

Step 6: Solve for x

$$\begin{aligned} x - \frac{3}{2} &= \pm\sqrt{\frac{29}{4}} \\ x &= \frac{3}{2} \pm \frac{\sqrt{29}}{2} \\ &= \frac{3 \pm \sqrt{29}}{2} \end{aligned}$$

Step 7: Method 2: Factorise equation as a difference of two squares

$$\begin{aligned} \left(x - \frac{3}{2}\right)^2 - \frac{29}{4} &= 0 \\ \left(x - \frac{3}{2}\right)^2 - \left(\sqrt{\frac{29}{4}}\right)^2 &= 0 \\ \left(x - \frac{3}{2} - \sqrt{\frac{29}{4}}\right)\left(x - \frac{3}{2} + \sqrt{\frac{29}{4}}\right) &= 0 \end{aligned}$$

Step 8: Solve for x

$$\left(x - \frac{3}{2} - \frac{\sqrt{29}}{2}\right)\left(x - \frac{3}{2} + \frac{\sqrt{29}}{2}\right) = 0$$

Therefore $x = \frac{3}{2} + \frac{\sqrt{29}}{2}$ or $x = \frac{3}{2} - \frac{\sqrt{29}}{2}$

Notice that these roots are irrational since 29 is not a perfect square.

3 QUADRATIC FORMULA

It is not always possible to solve a quadratic equation by factorisation and it can take a long time to complete the square. The method of completing the square provides a way to derive a formula that can be used to solve any quadratic equation. The quadratic formula provides an easy and fast way to solve quadratic equations.

Consider the standard form of the quadratic equation $ax^2 + bx + c = 0$. Divide both sides by $a(a \neq 0)$ to get

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

Now using the method of completing the square, we must halve the coefficient of x and square it. We then add and subtract $\left(\frac{b}{2a}\right)^2$ so that the equation remains true.

$$\begin{aligned} x^2 + \frac{bx}{a} + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a} &= 0 \\ \left(x^2 + \frac{bx}{a} + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a^2} + \frac{c}{a} &= 0 \\ \left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2} &= 0 \end{aligned}$$

We add the constant to both sides and take the square root of both sides of the equation, being careful to include a positive and negative answer.

$$\begin{aligned} \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\ \sqrt{\left(x + \frac{b}{2a}\right)^2} &= \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \\ x + \frac{b}{2a} &= \pm\frac{\sqrt{b^2 - 4ac}}{2a} \\ x &= -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned}$$

Therefore, for any quadratic equation $ax^2 + bx + c = 0$ we can determine two roots

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ or } x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

It is important to notice that the expression $b^2 - 4ac$ must be greater than or equal to zero for the roots of the quadratic to be real. If the expression under the square root sign is less than zero, then the roots are non-real (imaginary).

WORKED EXAMPLE 8: USING THE QUADRATIC FORMULA

QUESTION

Solve for x and leave your answer in simplest surd form: $2x^2 + 3x = 7$

SOLUTION

Step 1: Check whether the expression can be factorised

The expression cannot be factorised, so the general quadratic formula must be used.

Step 2: Write the equation in the standard form $ax^2 + bx + c = 0$

$$2x^2 + 3x - 7 = 0$$

Step 3: Identify the coefficients to substitute into the formula

$$a = 2; \qquad b = 3; \qquad c = -7$$

Step 4: Apply the quadratic formula

Always write down the formula first and then substitute the values of a, b and c.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-7)}}{2(2)} \\ &= \frac{-3 \pm \sqrt{65}}{4} \end{aligned}$$

Step 5: Write the final answer

The two roots are $x = \dfrac{-3 + \sqrt{65}}{4}$ or $x = \dfrac{-3 - \sqrt{65}}{4}$

WORKED EXAMPLE 9: USING THE QUADRATIC FORMULA

QUESTION

Find the roots of the function $f(x) = x^2 - 5x + 8$.

SOLUTION

Step 1: Finding the roots

To determine the roots of $f(x)$, we let $x^2 - 5x + 8 = 0$.

Step 2: Check whether the expression can be factorised

The expression cannot be factorised, so the general quadratic formula must be used.

Step 3: Identify the coefficients to substitute into the formula

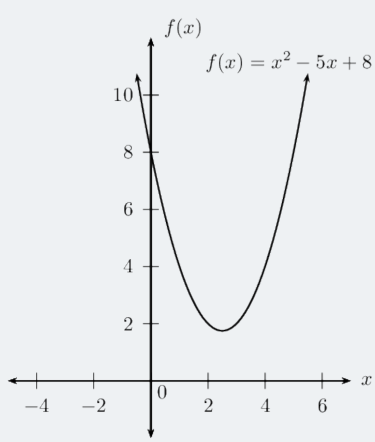
$$a = 1; \qquad b = -5; \qquad c = 8$$

Step 4: Apply the quadratic formula

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(8)}}{2(1)} \\ &= \frac{5 \pm \sqrt{-7}}{2} \end{aligned}$$

Step 5: Write the final answer

There are no real roots for $f(x) = x^2 - 5x + 8$ since the expression under the square root is negative ($\sqrt{-7}$ is not a real number). This means that the graph of the quadratic function has no x-intercepts; the entire graph lies above the x-axis.



4 SUBSTITUTION

It is often useful to make a substitution for a repeated expression in a quadratic equation. This makes the equation simpler and much easier to solve.

WORKED EXAMPLE 10: SOLVING BY SUBSTITUTION

QUESTION

Solve for x: $x^2 - 2x - \dfrac{3}{x^2 - 2x} = 2$

SOLUTION

Step 1: Determine the restrictions for x

The restrictions are the values for x that would result in the denominator being equal to 0, which would make the fraction undefined. Therefore $x \neq 0$ or $x \neq 2$.

Step 2: Substitute a single variable for the repeated expression

We notice that $x^2 - 2x$ is a repeated expression and we therefore let $k = x^2 - 2x$ so that the equation becomes

$$k - \frac{3}{k} = 2$$

Step 3: Determine the restrictions for k

The restrictions are the values for k that would result in the denominator being equal to 0, which would make the fraction undefined. Therefore $k \neq 0$.

Step 4: Solve for k

$$\begin{aligned} k - \frac{3}{k} &= 2 \\ k^2 - 3 &= 2k \\ k^2 - 2k - 3 &= 0 \\ (k+1)(k-3) &= 0 \end{aligned}$$

Therefore $k = -1$ or $k = 3$

We check these two roots against the restrictions for k and confirm that both are valid.

WORKED EXAMPLE 10: SOLVING BY SUBSTITUTION

WORKED EXAMPLE 10 CONTINUED **Step 5: Use values obtained for k to solve for the original variable** x

For $k = -1$

$$x^2 - 2x = -1$$
$$x^2 - 2x + 1 = 0$$
$$(x - 1)(x - 1) = 0$$
$$\text{Therefore } x = 1$$

For $k = 3$

$$x^2 - 2x = 3$$
$$x^2 - 2x - 3 = 0$$
$$(x + 1)(x - 3) = 0$$
$$\text{Therefore } x = -1 \text{ or } x = 3$$

We check these roots against the restrictions for x and confirm that all three values are valid.

Step 6: Write the final answer

The roots of the equation are $x = -1$, $x = 1$ or $x = 3$.

5 FINDING THE EQUATION

We have seen that the roots are the solutions obtained from solving a quadratic equation. Given the roots, we are also able to work backwards to determine the original quadratic equation.

WORKED EXAMPLE 11: FINDING AN EQUATION WHEN THE ROOTS ARE GIVEN

QUESTION

Find an equation with roots 13 and -5.

SOLUTION

Step 1: Assign a variable and write roots as two equations

$$x = 13 \text{ or } x = -5$$

Use additive inverses to get zero on the right-hand sides

$$x - 13 = 0 \text{ or } x + 5 = 0$$

Step 2: Write down as the product of two factors

$$(x - 13)(x + 5) = 0$$

Notice that the signs in the brackets are opposite of the given roots.

Step 3: Expand the brackets

$$x^2 - 8x - 65 = 0$$

Note that if each term in the equation is multiplied by a constant then there could be other possible equations which would have the same roots. For example,

Multiply by 2:

$$2x^2 - 16x - 130 = 0$$

Multiply by -3:

$$-3x^2 + 24x + 195 = 0$$

WORKED EXAMPLE 11: FINDING AN EQUATION WHEN THE ROOTS ARE FRACTIONS

QUESTION

Find an equation with roots $-\frac{3}{2}$ and 4.

SOLUTION

Step 1: Assign a variable and write roots as two equations

$$x = 4 \text{ or } x = -\frac{3}{2}$$

Use additive inverses to get zero on the right-hand sides

$$x - 4 = 0 \text{ or } x + \frac{3}{2} = 0$$

Multiply the second equation through by 2 to remove the fraction.

$$x - 4 = 0 \text{ or } 2x + 3 = 0$$

Step 2: Write down as the product of two factors

$$(2x + 3)(x - 4) = 0$$

Step 3: Expand the brackets

The quadratic equation is $2x^2 - 5x - 12 = 0$.

6 NATURE OF ROOTS

Investigation

Investigating the nature of roots

1. Use the quadratic formula to determine the roots of the quadratic equations given below and take special note of:
   - the expression under the square root sign and
   - the type of number for the final answer (rational/irrational/real/imaginary)

   (a) $x^2 - 6x + 9 = 0$

   (b) $x^2 - 4x + 3 = 0$

   (c) $x^2 - 4x - 3 = 0$

   (d) $x^2 - 4x + 7 = 0$

2. Choose the appropriate words from the table to describe the roots obtained for the equations above.

rational	unequal	real
imaginary	not perfect square	equal
perfect square	irrational	undefined

3. The expression under the square root, $b^2 - 4ac$, is called the discriminant. Can you make a conjecture about the relationship between the discriminant and the roots of quadratic equations?

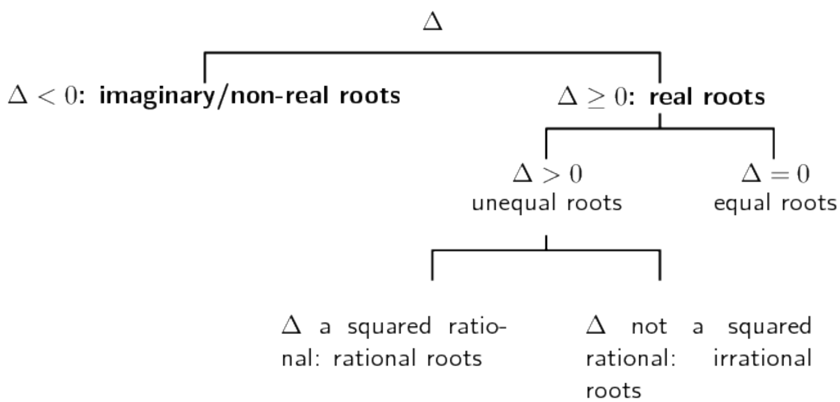
6.1 The discriminant

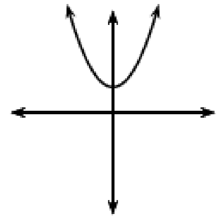
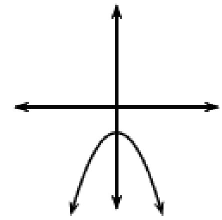
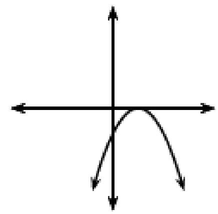
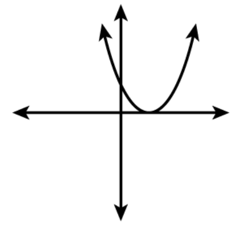
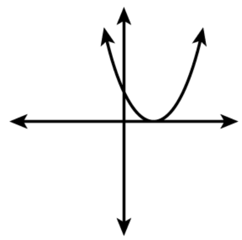
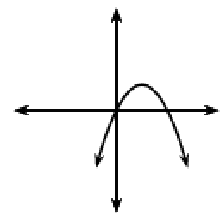
The discriminant is defined as $\Delta = b^2 - 4ac$

This is the expression under the square root in the quadratic formula. The discriminant determines the nature of the roots of a quadratic equation. The word 'nature' refers to the types of numbers the roots can be — namely real, rational, irrational or imaginary. Δ is the Greek symbol for the letter D.

For a quadratic function $f(x) = ax^2 + bx + c$, the solutions to the equation $f(x) = 0$ are given by the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{\Delta}}{2a}$

- If $\Delta < 0$, then roots are imaginary (non-real) and beyond the scope of this book.
- If $\Delta \geq 0$, the expression under the square root is non-negative and therefore roots are real. For real roots, we have the following further possibilities.
- If $\Delta = 0$, the roots are equal and we can say that there is only one root.
- If $\Delta > 0$, the roots are unequal and there are two further possibilities.
- Δ is the square of a rational number: the roots are rational.
- Δ is not the square of a rational number: the roots are irrational and can be expressed in decimal or surd form.



Nature of roots	Discriminant	$a > 0$	$a \geq 0$
Roots are non-real	$\Delta < 0$		
Roots are real and equal	$\Delta = 0$		
Roots are real and unequal: • rational roots • irrational roots	$\Delta > 0$ • $\Delta =$ squared rational • $\Delta =$ not squared rational		

WORKED EXAMPLE 13: NATURE OF ROOTS

QUESTION

Show that the roots of $x^2–2x–7 = 0$ are irrational.

SOLUTION

Step 1: Interpret the question

For roots to be real and irrational, we need to calculate and show that it is greater than zero and not a perfect square.

Step 2: Check that the equation is in standard form $ax^2 + bx + c = 0$

$$x^2 - 2x - 7 = 0$$

Step 3: Identify the coefficients to substitute into the formula for the discriminant

$$a = 1; \qquad b = -2; \qquad c = -7$$

Step 4: Write down the formula and substitute values

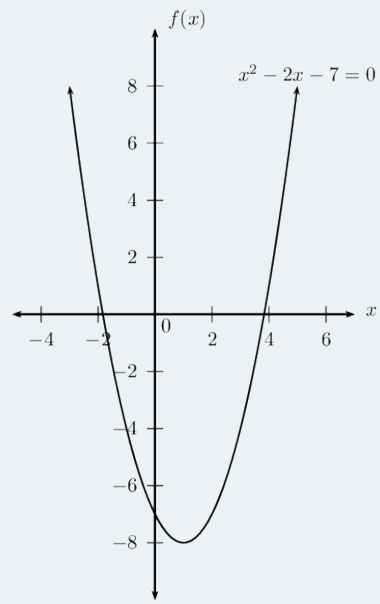
$$\begin{aligned} \Delta &= b^2 - 4ac \\ &= (-2)^2 - 4(1)(-7) \\ &= 4 + 28 \\ &= 32 \end{aligned}$$

We know that $32 > 0$ and is not a perfect square.

WORKED EXAMPLE 13: NATURE OF ROOTS

WORKED EXAMPLE 13 CONTINUED

The graph below shows the roots of the equation $x^2 – 2x – 7 = 0$. Note that the graph does not form part of the answer and is included for illustration purposes only.



Step 5: Write the final answer

We have calculated that $\Delta > 0$ and is not a perfect square, therefore we can conclude that the roots are **real**, **unequal** and **irrational**.

WORKED EXAMPLE 14: NATURE OF ROOTS

QUESTION

For which value(s) of k will the roots of $6x^2 + 6 = 4kx$ be real and equal?

SOLUTION

Step 1: Interpret the question

For roots to be real and equal, we need to solve for the value(s) of k such that $= 0$.

Step 2: Check that the equation is in standard form $ax^2 + bx + c = 0$

$$6x^2 - 4kx + 6 = 0$$

Step 3: Identify the coefficients to substitute into the formula for the discriminant

$$a = 6; \qquad b = -4k; \qquad c = 6$$

Step 4: Write down the formula and substitute values

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= (-4k)^2 - 4(6)(6) \\ &= 16k^2 - 144\end{aligned}$$

For roots to be real and equal, $= 0$.

$$\begin{aligned}\Delta &= 0 \\ 16k^2 - 144 &= 0 \\ 16(k^2 - 9) &= 0 \\ (k-3)(k+3) &= 0\end{aligned}$$

Therefore $k = 3$ or $k = -3$.

Step 5: Check both answers by substituting back into the original equation

For $k = 3$

$$\begin{aligned}6x^2 - 4(3)x + 6 &= 0 \\ 6x^2 - 12x + 6 &= 0 \\ x^2 - 2x + 1 &= 0 \\ (x-1)(x-1) &= 0 \\ (x-1)^2 &= 0 \\ \text{Therefore } x &= 1\end{aligned}$$

WORKED EXAMPLE 14: NATURE OF ROOTS

WORKED EXAMPLE 14 CONTINUED

We see that for $k = 3$ the quadratic equation has real, equal roots $x = 1$.

For $k = -3$

$$\begin{aligned} 6x^2 - 4(-3)x + 6 &= 0 \\ 6x^2 + 12x + 6 &= 0 \\ x^2 + 2x + 1 &= 0 \\ (x+1)(x+1) &= 0 \\ (x+1)^2 &= 0 \\ \text{Therefore } x &= -1 \end{aligned}$$

We see that for $k = -3$ the quadratic equation has real, equal roots $x = -1$.

Step 6: Write the final answer For the roots of the quadratic equation to be **real** and **equal**, $k = 3$ or $k = -3$.

WORKED EXAMPLE 15: NATURE OF ROOTS

QUESTION

Show that the roots of $(x+h)(x+k)=4d^2$ are real for all real values of h, k and d.

SOLUTION

Step 1: Interpret the question

For roots to be real, we need to calculate Δ and show that $\Delta \geq 0$ for all real values of h, k and d.

Step 2: Check that the equation is in standard form $ax^2+bx+c=0$

Expand the brackets and gather like terms

$$\begin{aligned}(x+h)(x+k) &= 4d^2\\ x^2+hx+kx+hk-4d^2 &= 0\\ x^2+(h+k)x+(hk-4d^2) &= 0\end{aligned}$$

Step 3: Identify the coefficients to substitute into the formula for the discriminant

$$a=1; \qquad b=h+k; \qquad c=hk-4d^2$$

Step 4: Write down the formula and substitute values

$$\begin{aligned}\Delta &= b^2-4ac\\ &= (h+k)^2-4(1)(hk-4d^2)\\ &= h^2+2hk+k^2-4hk+16d^2\\ &= h^2-2hk+k^2+16d^2\\ &= (h-k)^2+(4d)^2\end{aligned}$$

For roots to be real, $\Delta \geq 0$.

$$\begin{aligned}\text{We know that } (4d)^2 &\geq 0\\ \text{and } (h-k)^2 &\geq 0\\ \text{so then } (h-k)^2+(4d)^2 &\geq 0\\ \text{therefore } \Delta &\geq 0\end{aligned}$$

Step 5: Write the final answer

We have shown that $\Delta \geq 0$, therefore the roots are real for all **real** values of h,k and d.

7 QUADRATIC INEQUALITIES

Quadratic inequalities can be of the following forms:

$$ax^2 + bx + c > 0$$
$$ax^2 + bx + c \geq 0$$
$$ax^2 + bx + c < 0$$
$$ax^2 + bx + c \leq 0$$

To solve a quadratic inequality we must determine which part of the graph of a quadratic function lies above or below the x-axis. An inequality can therefore be solved graphically using a graph or algebraically using a table of signs to determine where the function is positive and negative.

WORKED EXAMPLE 16: SOLVING QUADRATIC INEQUALITIES

QUESTION

Solve for x: $x^2 - 5x + 6 \geq 0$

SOLUTION

Step 1: Factorise the quadratic

$$(x-3)(x-2) \geq 0$$

Step 2: Determine the critical values of x

From the factorised quadratic we see that the values for which the inequality is equal to zero are $x = 3$ or $x = 2$. These are called the critical values of the inequality and they are used to complete a table of signs.

Step 3: Complete a table of signs

We must determine where each factor of the inequality is positive and negative on the number line:

- to the left (in the negative direction) of the critical value
- equal to the critical value
- to the right (in the positive direction) of the critical value

In the final row of the table we determine where the inequality is positive and negative by finding the product of the factors and their respective signs.

Critical values		$x = 2$		$x = 3$	
$x - 3$	$-$	$-$	$-$	0	$+$
$x - 2$	$-$	0	$+$	$+$	$+$
$f(x) = (x-3)(x-2)$	$+$	0	$-$	0	$+$

From the table we see that $f(x)$ is greater than or equal to zero for $x \leq 2$ or $x \geq 3$.

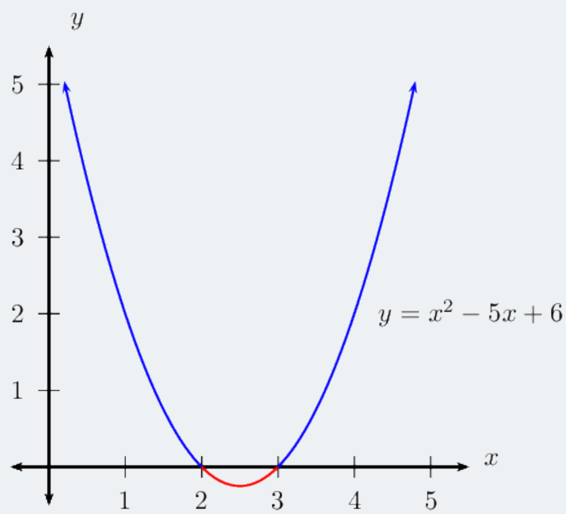
Step 4: A rough sketch of the graph

The graph below does not form part of the answer and is included for illustration purposes only. A graph of the quadratic helps us determine the answer to the inequality. We can find the answer graphically by seeing where the graph lies above or below the x-axis.

WORKED EXAMPLE 16: SOLVING QUADRATIC INEQUALITIES

WORKED EXAMPLE 16 CONTINUED

- From the standard form, $x^2 - 5x + 6$, $a > 0$ and therefore the graph is a "smile" and has a minimum turning point.
- From the factorised form, $(x-3)(x-2)$, we know the x-intercepts are $(2;0)$ and $(3;0)$.



The graph is above or on the x-axis for $x \leq 2$ or $x \geq 3$

Step 5: Write the final answer and represent on a number line

$$x^2 - 5x + 6 \geq 0 \text{ for } x \leq 2 \text{ or } x \geq 3$$

WORKED EXAMPLE 17: SOLVING QUADRATIC INEQUALITIES

QUESTION

Solve for x: $4x^2 - 4x + 1 \leq 0$

SOLUTION

Step 1: Factorise the quadratic

$$\begin{aligned}(2x-1)(2x-1) &\leq 0 \\ (2x-1)^2 &\leq 0\end{aligned}$$

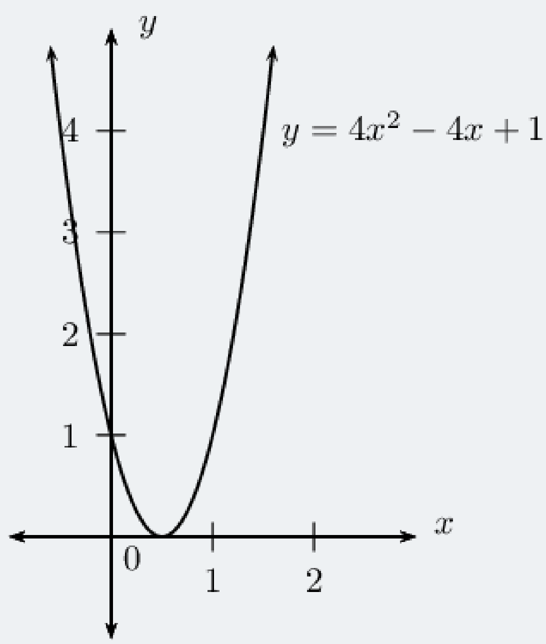
Step 2: Determine the critical values of x

From the factorised quadratic we see that the values for which the inequality is equal to zero is $x = \frac{1}{2}$. We know that $a^2 > 0$ for any real number $a, a \neq 0$, so then $(2x-1)^2$ will never be negative.

Step 3: A rough sketch of the graph

The graph below does not form part of the answer and is included for illustration purposes only.

- From the standard form, $4x^2 - 4x + 1$, $a > 0$ and therefore the graph is a "smile" and has a minimum turning point.
- From the factorised form, $(2x-1)(2x-1)$,we know there is only one x-intercept at $\left(\frac{1}{2}; 0\right)$.



Notice that no part of the graph lies below the x-axis.

WORKED EXAMPLE 17: SOLVING QUADRATIC INEQUALITIES

WORKED EXAMPLE 17 CONTINUED

Step 4: Write the final answer and represent on a number line

$$4x^2 - 4x + 1 \leq 0 \text{ for } x = \frac{1}{2}$$

WORKED EXAMPLE 18: SOLVING QUADRATIC INEQUALITIES

QUESTION

Solve for x: $-x^2 - 3x + 5 > 0$

SOLUTION

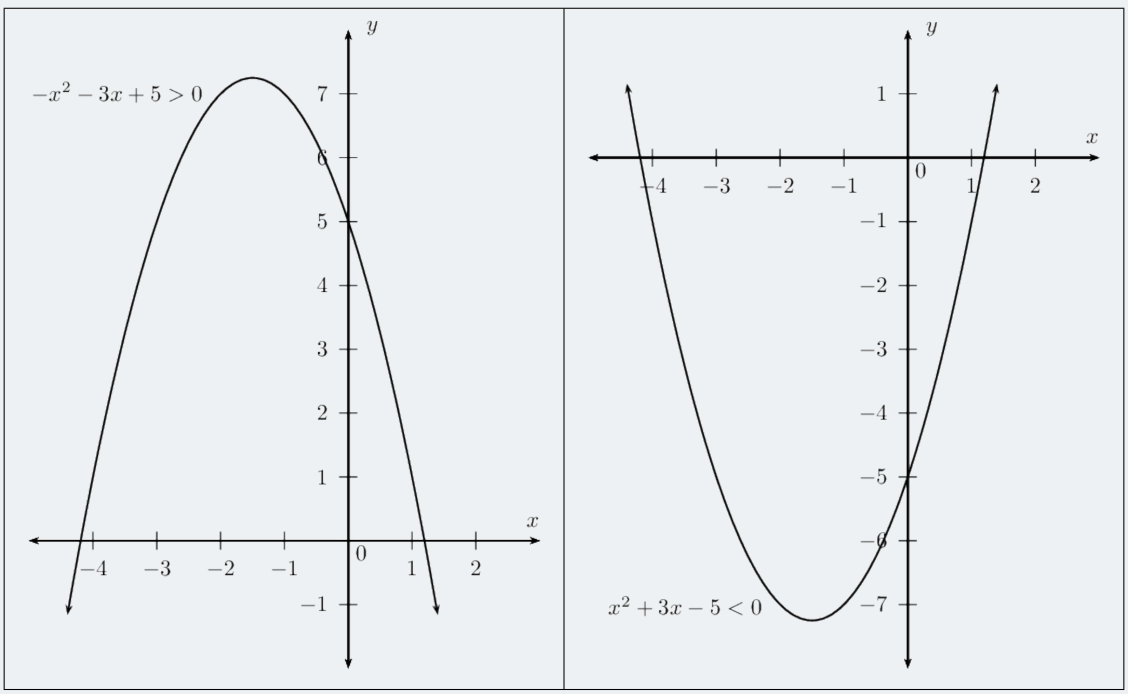
Step 1: Examine the form of the inequality

Notice that the coefficient of the x^2 term is -1. **Remember** that if we multiply or divide an inequality by a negative number, then the inequality sign changes direction. So we can write the same inequality in different ways and still get the same answer, as shown below.

$$-x^2 - 3x + 5 > 0$$

Multiply by -1 and change direction of the inequality sign

$$x^2 + 3x - 5 < 0$$



From this rough sketch, we can see that both inequalities give the same solution; the values of x that lie between the two x-intercepts.

WORKED EXAMPLE 18: SOLVING QUADRATIC INEQUALITIES

WORKED EXAMPLE 18 CONTINUED

Step 2: Factorise the quadratic

We notice that $-x^2 - 3x + 5 > 0$ cannot be easily factorised. So we let $-x^2 - 3x + 5 = 0$ and use the quadratic formula to determine the roots of the equation.

$$-x^2 - 3x + 5 = 0$$
$$x^2 + 3x - 5 = 0$$

$$\therefore x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-5)}}{2(1)}$$
$$= \frac{-3 \pm \sqrt{29}}{2}$$
$$x_1 = \frac{-3 - \sqrt{29}}{2} \approx -4,19$$
$$x_2 = \frac{-3 + \sqrt{29}}{2} \approx 1,19$$

Therefore we can write, correct to one decimal place,

$$x^2 + 3x - 5 < 0$$
$$\text{as } (x - 1,19)(x + 4,19) < 0$$

Step 3: Determine the critical values of x

From the factorised quadratic we see that the critical values are $x = 1,19$ and $x = -4,19$.

Step 4: Complete a table of signs

Critical values		$x = -4,19$		$x = 1,19$	
19$x + 4,2$	$-$	0	$+$	$+$	$+$
$x - 1,19$	$-$	$-$	$-$	0	$+$
$f(x) = (x + 4,19)(x - 1,19)$	$+$	0	$-$	0	$+$

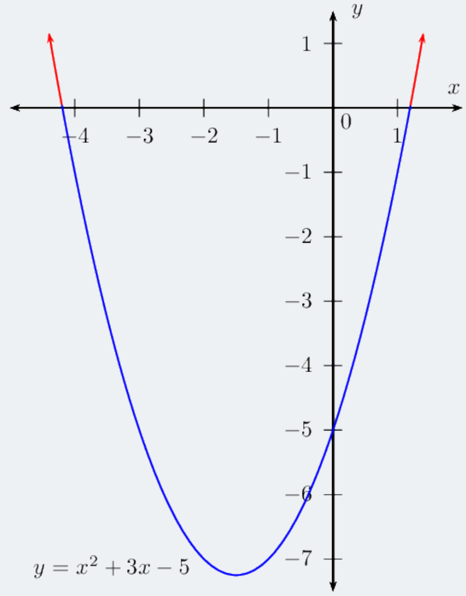
From the table we see that the function is negative for $-4,19 < x < 1,19$

WORKED EXAMPLE 18: SOLVING QUADRATIC INEQUALITIES

WORKED EXAMPLE 18 CONTINUED

Step 5: A sketch of the graph

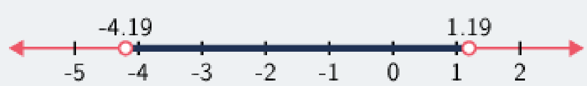
- From the standard form, x^2+3x-5, $a>0$ and therefore the graph is a "smile" and has a minimum turning point.
- From the factorised form, $(x-1,19)(x+4,19)$, we know the x-intercepts are $(-4,19;0)$ and $(1,19;0)$.



From the graph we see that the function lies below the x-axis between $-4,19$ and $1,19$.

Step 6: Write the final answer and represent on a number line

$$x^2+3x-5<0 \text{ for } -4{,}19<x<1{,}19$$



Important: When working with an inequality in which the variable is in the denominator, a different approach is needed. Always remember to check for restrictions.

WORKED EXAMPLE 19: SOLVING QUADRATIC INEQUALITIES

QUESTION

Solve for x:

1. $\frac{2}{x+3} = \frac{1}{x-3}, x \neq \pm 3$
2. $\frac{2}{x+3} \leq \frac{1}{x-3}, x \neq \pm 3$

SOLUTION

Step 1: Solving the equation

To solve this equation we multiply both sides of the equation by $(x+3)(x–3)$ and simplfy:

$$\frac{2}{x+3} \times (x+3)(x-3) = \frac{1}{x-3} \times (x+3)(x-3)$$
$$2(x-3) = x+3$$
$$2x-6 = x+3$$
$$x = 9$$

Step 2: Solving the inequality

It is very important to recognise that we cannot use the same method as above to solve the inequality. If we multiply or divide an inequality by a negative number, then the inequality sign changes direction. We must rather simplify the inequality to have a lowest common denominator and use a table of signs to determine the values that satisfy the inequality.

Step 3: Subtract $\frac{1}{x-3}$ from both sides of the inequality

$$\frac{2}{x+3} - \frac{1}{x-3} \leq 0$$

Step 4: Determine the lowest common denominator and simplify the fraction

$$\frac{2(x-3)-(x+3)}{(x+3)(x-3)} \leq 0$$
$$\frac{x-9}{(x+3)(x-3)} \leq 0$$

Keep the denominator because it affects the final answer.

Step 5: Determine the critical values of x

From the factorised inequality we see that the critical values are $x = –3$, $x = 3$ and $x = 9$.

WORKED EXAMPLE 18: SOLVING QUADRATIC INEQUALITIES

WORKED EXAMPLE 18 CONTINUED

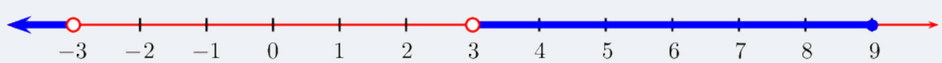
Step 6: Complete a table of signs

Critical values		$x = -3$		$x = 3$		$x = 9$	
$x + 3$	$-$	undef	$+$	$+$	$+$	$+$	$+$
$x-3$	$-$	$-$	$-$	undef	$+$	$+$	$+$
$x-9$	$-$	$-$	$-$	$-$	$-$	0	$+$
$f(x) = \dfrac{x-9}{(x+3)(x-3)}$	$-$	undef	$+$	undef	$-$	0	$+$

From the table we see that the function is less than or equal to zero for $x < -3$ or $3 < x \leq 9$. We do not include $x = -3$ or $x = 3$ in the solution because of the restrictions on the denominator.

Step 7: Write the final answer and represent on a number line

$$x < -3 \quad \text{or} \quad 3 < x \leq 9$$



8 SIMULTANEOUS EQUATIONS

Simultaneous linear equations can be solved using three different methods: substitution, elimination or using a graph to determine where the two lines intersect. For solving systems of simultaneous equations with linear and non-linear equations, we mostly use the substitution method. Graphical solution is useful for showing where the two equations intersect.

In general, to solve for the values of n unknown variables requires a system of n independent equations.

An example of a system of simultaneous equations with one linear equation and one quadratic equation is

$$y - 2x = -4$$
$$x^2 + y = 4$$

8.1 Solving by substitution

- Use the simplest of the two given equations to express one of the variables in terms of the other.

- Substitute into the second equation. By doing this we reduce the number of equations and the number of variables by one.
- We now have one equation with one unknown variable which can be solved.
- Use the solution to substitute back into the first equation to find the value of the other unknown variable.

WORKED EXAMPLE 20: SIMULTANEOUS EQUATIONS

QUESTION

Solve for x and y:

$$y - 2x = -4 \qquad \ldots (1)$$
$$x^2 + y = 4 \qquad \ldots (2)$$

SOLUTION

Step 1: Make y the subject of the first equation

$$y = 2x - 4$$

Step 2: Substitute into the second equation and simplify

$$x^2 + (2x - 4) = 4$$
$$x^2 + 2x - 8 = 0$$

Step 3: Factorise the equation

$$(x + 4)(x - 2) = 0$$
$$\therefore x = -4 \text{ or } x = 2$$

Step 4: Substitute the values of x back into the first equation to determine the corresponding y-values

If $x = -4$:

$$y = 2(-4) - 4$$
$$= -12$$

If $x = 2$:

$$y = 2(2) - 4$$
$$= 0$$

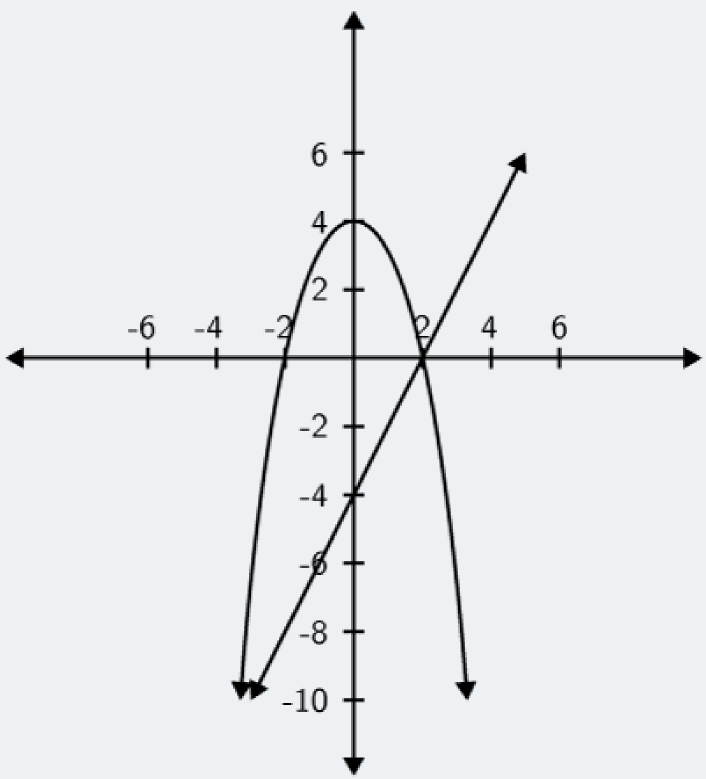
Step 5: Check that the two points satisfy both original equations

WORKED EXAMPLE 20: SIMULTANEOUS EQUATIONS

WORKED EXAMPLE 20 CONTINUED

Step 6: Write the final answer

The solution is $x = -4$ and $y = -12$ or $x = 2$ and $y = 0$. These are the coordinate pairs for the points of intersection as shown below.



8.2 Solving by elimination

- Make one of the variables the subject of both equations.
- Equate the two equations; by doing this we reduce the number of equations and the number of variables by one.
- We now have one equation with one unknown variable which can be solved.
- Use the solution to substitute back into either original equation, to find the corresponding value of the other unknown variable.

WORKED EXAMPLE 21: SIMULTANEOUS EQUATIONS

QUESTION

Solve for x and y:

$$y = x^2 - 6x \qquad \ldots(1)$$
$$y + \frac{1}{2}x - 3 = 0 \qquad \ldots(2)$$

SOLUTION

Step 1: Make y the subject of the second equation

$$y + \frac{1}{2}x - 3 = 0$$
$$y = -\frac{1}{2}x + 3$$

Step 2: Equate the two equations and solve for x

$$x^2 - 6x = -\frac{1}{2}x + 3$$
$$x^2 - 6x + \frac{1}{2}x - 3 = 0$$
$$2x^2 - 12x + x - 6 = 0$$
$$2x^2 - 11x - 6 = 0$$
$$(2x+1)(x-6) = 0$$
$$\text{Therefore } x = -\frac{1}{2} \text{ or } x = 6$$

Step 3: Substitute the values for x back into the second equation to calculate the corresponding y-values

If $x = -\frac{1}{2}$:

$$y = -\frac{1}{2}\left(-\frac{1}{2}\right) + 3$$
$$\therefore y = 3\frac{1}{4}$$

This gives the point $\left(-\frac{1}{2}; 3\frac{1}{4}\right)$

WORKED EXAMPLE 21: SIMULTANEOUS EQUATIONS

WORKED EXAMPLE 21 CONTINUED

If $x = 6$:

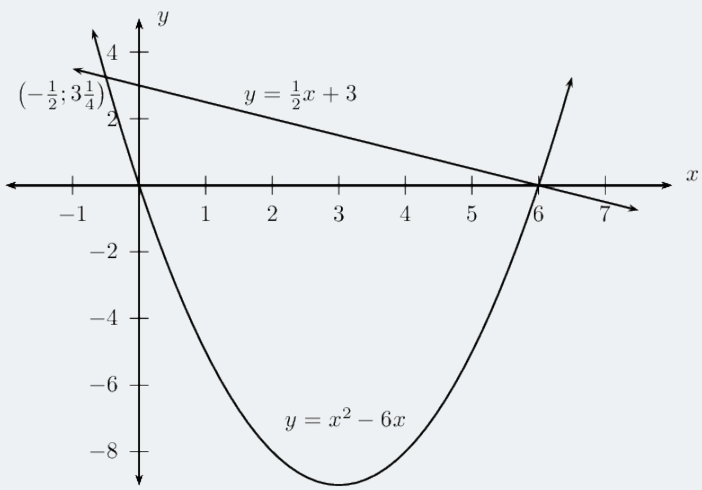
$$\begin{aligned} y &= -\frac{1}{2}(6) + 3 \\ &= -3 + 3 \\ \therefore y &= 0 \end{aligned}$$

This gives the point $(6;0)$

Step 4: Check that the two points satisfy both original equations

Step 5: Write the final answer

The solution is $x = -\frac{1}{2}$ and $y = 3\frac{1}{4}$ or $x = 6$ and $y = 0$. These are the coordinate pairs for the points of intersection as shown below.



WORKED EXAMPLE 22: SIMULTANEOUS EQUATIONS

QUESTION

Solve for x and y:

$$y = \frac{5}{x-2} \qquad \ldots (1)$$
$$y + 1 = 2x \qquad \ldots (2)$$

SOLUTION

Step 1: Make y the subject of the second equation

$$\begin{aligned} y + 1 &= 2x \\ y &= 2x - 1 \end{aligned}$$

Step 2: Equate the two equations and solve for x

$$\begin{aligned} 2x - 1 &= \frac{5}{x-2} \\ (2x-1)(x-2) &= 5 \\ 2x^2 - 5x + 2 &= 5 \\ 2x^2 - 5x - 3 &= 0 \\ (2x+1)(x-3) &= 0 \\ \text{Therefore } x &= -\frac{1}{2} \text{ or } x = 3 \end{aligned}$$

Step 3: Substitute the values for x back into the second equation to calculate the corresponding y-values

If $x = -\frac{1}{2}$:

$$\begin{aligned} y &= 2(-\frac{1}{2}) - 1 \\ \therefore y &= -2 \end{aligned}$$

This gives the point $(-\frac{1}{2}; -2)$.

If $x = 3$:

$$\begin{aligned} y &= 2(3) - 1 \\ &= 5 \end{aligned}$$

This gives the point $(3; 5)$.

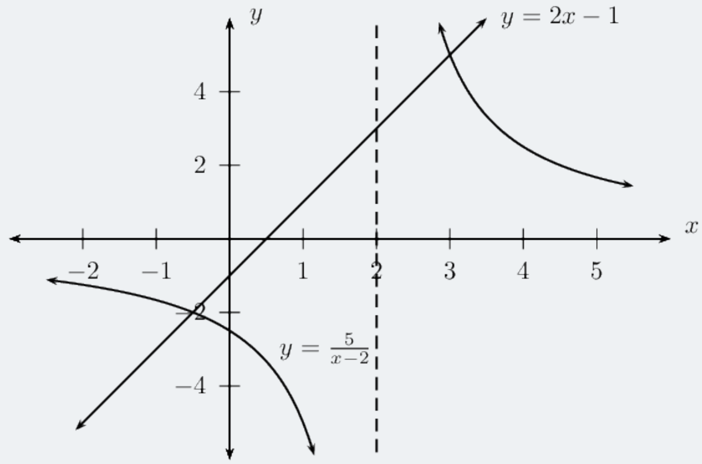
WORKED EXAMPLE 22: SIMULTANEOUS EQUATIONS

WORKED EXAMPLE 22 CONTINUED

Step 4: Check that the two points satisfy both original equations

Step 5: Write the final answer

The solution is $x = -\frac{1}{2}$ and $y = -2$ or $x = 3$ and $y = 5$. These are the coordinate pairs for the points of intersection as shown below.



8.3 Solving graphically

- Make y the subject of each equation.
- Draw the graph of each equation on the same system of axes.
- The final solutions to the system of equations are the coordinates of the points where the two graphs intersect.

WORKED EXAMPLE 23: SIMULTANEOUS EQUATIONS

QUESTION

Solve graphically for x and y:

$$y + x^2 = 1 \qquad \ldots (1)$$
$$y - x + 5 = 0 \qquad \ldots (2)$$

SOLUTION

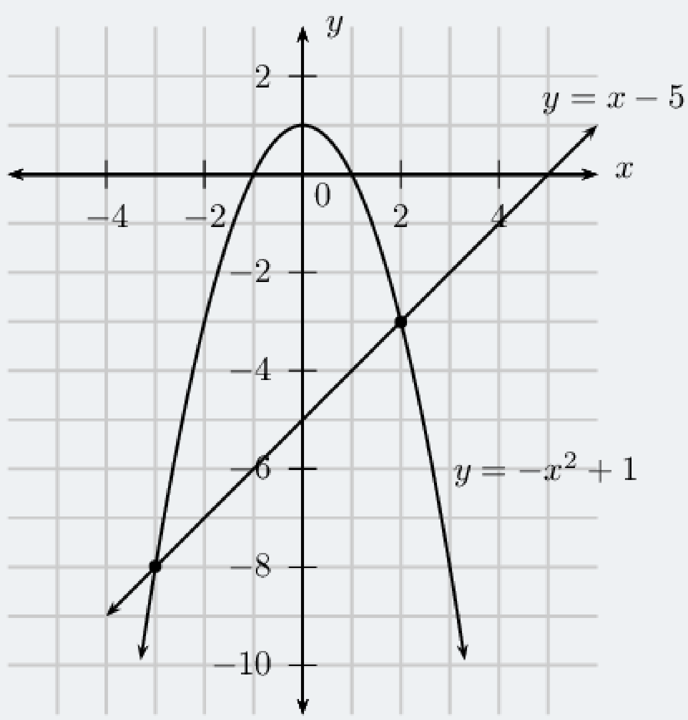
Step 1: Make y the subject of both equations

$$\begin{aligned} y + x^2 &= 1 \\ y &= -x^2 + 1 \end{aligned}$$

and for the second equation

$$\begin{aligned} y - x + 5 &= 0 \\ y &= x - 5 \end{aligned}$$

Step 2: Draw the straight line graph and parabola on the same system of axes



WORKED EXAMPLE 23: SIMULTANEOUS EQUATIONS

WORKED EXAMPLE 23 CONTINUED

Step 3: Determine where the two graphs intersect

From the diagram we see that the graphs intersect at $(-3; -8)$ and $(2; -3)$.

Step 4: Check that the two points satisfy both original equations

Step 5: Write the final answer

The solutions to the system of simultaneous equations are $(-3; -8)$ and $(2; -3)$.

9 WORD PROBLEMS

Solving word problems requires using mathematical language to describe real-life contexts. Problem-solving strategies are often used in the natural sciences and engineering disciplines (such as physics, biology, and electrical engineering) but also in the social sciences (such as economics, sociology and political science). To solve word problems we need to write a set of equations that describes the problem mathematically.

Examples of real-world problem solving applications are:

- modelling population growth;
- modelling effects of air pollution;
- modelling effects of global warming;
- computer games;
- in the sciences, to understand how the natural world works;
- simulators that are used to train people in certain jobs, such as pilots, doctors and soldiers;
- in medicine, to track the progress of a disease.

9.1 Problem solving strategy

1. Read the problem carefully.
2. What is the question and what do we need to solve for?
3. Assign variables to the unknown quantities, for example, x and y.
4. Translate the words into algebraic expressions by rewriting the given information in terms of the variables.

5. Set up a system of equations.
6. Solve for the variables using substitution.
7. Check the solution.
8. Write the final answer.

Investigation

Simple word problems

Write an equation that describes the following real-world situations mathematically:

1. Mohato and Lindiwe both have colds. Mohato sneezes twice for each sneeze of Lindiwe's. If Lindiwe sneezes x times, write an equation describing how many times they both sneezed.
2. The difference of two numbers is 10 and the sum of their squares is 50. Find the two numbers.
3. Liboko builds a rectangular storeroom. If the diagonal of the room is $\sqrt{1\ 312}$ m and the perimeter is 80 m, determine the dimensions of the room.
4. It rains half as much in July as it does in December. If it rains y mm in July, write an expression relating the rainfall in July and December.
5. Zane can paint a room in 4 hours. Tlali can paint a room in 2 hours. How long will it take both of them to paint a room together?
6. 25 years ago, Arthur was 5 years more than a third of Bongani's age. Today, Bongani is 26 years less than twice Arthur's age. How old is Bongani?
7. The product of two integers is 95. Find the integers if their total is 24.

WORKED EXAMPLE 24: GYM MEMBERSHIP

QUESTION

The annual gym subscription for a single member is $R1\ 000$, while an annual family membership is $R1\ 500$. The gym is considering increasing all membership fees by the same amount. If this is done then a single membership would cost $\frac{5}{7}$ of a family membership. Determine the amount of the proposed increase.

SOLUTION

Step 1: Identify the unknown quantity and assign a variable

Let the amount of the proposed increase be x.

Step 2: Use the given information to complete a table

	now	after increase
single	$1\ 000$	$1\ 000 + x$
family	$1\ 500$	$1\ 500 + x$

Step 3: Set up an equation

$$1\ 000 + x = \frac{5}{7}(1\ 500 + x)$$

Step 4: Solve for x

$$\begin{aligned} 7\ 000 + 7x &= 7\ 500 + 5x \\ 2x &= 500 \\ x &= 250 \end{aligned}$$

Step 5: Write the final answer

The proposed increase is $R250$.

WORKED EXAMPLE 25: CORNER COFFEE HOUSE

QUESTION

Erica has decided to treat her friends to coffee at the Corner Coffee House. Erica paid $R54,00$ for four cups of cappuccino and three cups of filter coffee. If a cup of cappuccino costs $R3,00$ more than a cup of filter coffee, calculate how much a cup of each type of coffee costs?

SOLUTION

Step 1: Method 1: identify the unknown quantities and assign two variables

Let the cost of a cappuccino be x and the cost of a filter coffee be y.

Step 2: Use the given information to set up a system of equations

$$\begin{aligned} 4x + 3y &= 54 \qquad \ldots(1) \\ x &= y + 3 \qquad \ldots(2) \end{aligned}$$

Step 3: Solve the equations by substituting the second equation into the first equation

$$\begin{aligned} 4(y+3) + 3y &= 54 \\ 4y + 12 + 3y &= 54 \\ 7y &= 42 \\ y &= 6 \end{aligned}$$

If $y = 6$, then using the second equation we have

$$\begin{aligned} x &= y + 3 \\ &= 6 + 3 \\ &= 9 \end{aligned}$$

Step 4: Check that the solution satisfies both original equations

Step 5: Write the final answer

A cup of cappuccino costs $R9$ and a cup of filter coffee costs $R6$.

Step 6: Method 2: identify the unknown quantities and assign one variable

Let the cost of a cappuccino be x and the cost of a filter coffee be $x – 3$.

WORKED EXAMPLE 25: CORNER COFFEE HOUSE

WORKED EXAMPLE 25 CONTINUED

Step 7: Use the given information to set up an equation

$$4x + 3(x - 3) = 54$$

Step 8: Solve for x

$$\begin{aligned} 4x + 3(x - 3) &= 54 \\ 4x + 3x - 9 &= 54 \\ 7x &= 63 \\ x &= 9 \end{aligned}$$

Step 9: Write the final answer

A cup of cappuccino costs $R9$ and a cup of filter coffee costs $R6$.

WORKED EXAMPLE 26: TAPS FILLING A CONTAINER

QUESTION

Two taps, one more powerful than the other, are used to fill a container. Working on its own, the less powerful tap takes 2 hours longer than the other tap to fill the container. If both taps are opened, it takes 1 hour, 52 minutes and 30 seconds to fill the container. Determine how long it takes the less powerful tap to fill the container on its own.

SOLUTION

Step 1: Identify the unknown quantities and assign two variables

Let the time taken for the less powerful tap to fill the container be x and let the time taken for the more powerful tap be x–2.

Step 2: Convert all units of time to be the same First we must convert 1 hour, 52 minutes and 30 seconds to hours:

$$1+\frac{52}{60}+\frac{30}{(60)^2}=1,875 \text{ hours}$$

Step 3: Use the given information to set up a system of equations

Write an equation describing the two taps working together to fill the container:

$$\frac{1}{x}+\frac{1}{x-2}=\frac{1}{1,875}$$

Step 4: Multiply the equation through by the lowest common denominator and simplify

$$\begin{aligned}1,875(x-2)+1,875x &= x(x-2)\\ 1,875x-3,75+1,875x &= x^2-2x\\ 0 &= x^2-5,75x+3,75\end{aligned}$$

Multiply the equation through by 4 to make it easier to factorise (or use the quadratic formula)

$$\begin{aligned}0 &= 4x^2-23x+15\\ 0 &= (4x-3)(x-5)\end{aligned}$$

Therefore $x=\frac{3}{4}$ or $x=5$

We have calculated that the less powerful tap takes $\frac{3}{4}$ hours or 5 hours to fill the container, but we know that when both taps are opened it takes $1,875$ hours. We can therefore discard the first solution $x=\frac{3}{4}$ hours.

So the less powerful tap fills the container in 5 hours and the more powerful tap takes 3 hours.

WORKED EXAMPLE 26: TAPS FILLING A CONTAINER

WORKED EXAMPLE 26 CONTINUED

Step 5: Check that the solution satisfies the original equation

Step 6: Write the final answer

The less powerful tap fills the container in 5 hours and the more powerful tap takes 3 hours.

10 SUMMARY

- Zero product law: if $a \times b = 0$, then $a = 0$ and/or $b = 0$.
- Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Discriminant: $\Delta = b^2 - 4ac$

Nature of roots	Discriminant
Roots are non-real	$\Delta < 0$
Roots are real and equal	$\Delta = 0$
Roots are real and unequal: • rational roots • irrational roots	$\Delta > 0$ • $\Delta =$ squared rational number • $\Delta =$ not squared rational number

11 EXERCISES

11.1 Exercise 1

1. Solve the following quadratic equations by factorisation. Answers may be left in surd form, where applicable.

   1.1 $7t^2 + 14t = 0$

   1.2 $12y^2 + 24y + 12 = 0$

   1.3 $16s^2 = 400$

   1.4 $y^2 - 5y + 6 = 0$

   1.5 $y^2 + 5y - 36 = 0$

   1.6 $-y^2 - 11y - 24 = 0$

   1.7 $y^2 - 5ky + 4k^2 = 0$

   1.8 $2y^2 - 61 = 101$

   1.9 $2y^2 - 10 = 0$

   1.10 $-8 + h^2 = 28$

   1.11 $y^2 - 4 = 10$

   1.12 $y^2 + 28 = 100$

   1.13 $f(2f + 1) = 15$

   1.14 $\frac{5y}{y-2} + \frac{3}{y} + 2 = \frac{-6}{y^2-2y}$

   1.15 $\frac{x+9}{x^2-9} + \frac{1}{x+3} = \frac{2}{x-3}$

   1.16 $\frac{y-2}{y+1} = \frac{2y+1}{y-7}$

   1.17 $1 + \frac{t-2}{t-1} = \frac{5}{t^2-4t+3} + \frac{10}{3-t}$

   1.18 $\frac{4}{m+3} + \frac{4}{4-m^2} = \frac{5m-5}{m^2+m-6}$

11.2 Exercise 2

1. Solve the following equations by completing the square:

   1.1 $x^2 + 10x–2 = 0$

   1.2 $x^2 + 4x + 3 = 0$

   1.3 $p^2 - 5 = -8p$

   1.4 $2(6x + x^2) = -4$

   1.5 $x^2 + 5x + 9 = 0$

   1.6 $t^2 + 30 = 2(10 - 8t)$

   1.7 $3x^2 + 6x - 2 = 0$

   1.8 $z^2 + 8z - 6 = 0$

   1.9 $2z^2 = 11z$

   1.10 $5 + 4z - z^2 = 0$

2. Solve for k in terms of $a : k^2 + 6k + a = 0$

3. Solve for y in terms of p, q and $r : py^2 + qy + r = 0$

4. Write down the maximum or minimum value for each of the following and the x -value where this minimum or maximum occurs

   4.1 $2(x + 3)^2 - 5$

   4.2 $x^2 - 5x - 7$

   4.3 $-3x^2 + 2x + 6$

   4.4 $2x^2 - 5x + 4$

5. A landowner wants to erect a fence on a rectangular piece of land. He has 200 m of fencing available. Calculate the dimensions of the rectangle so that he can enclose the maximum area. What is this maximum area?

11.3 Exercise 3

1. Solve the following using the quadratic formula:

   1.1 $3t^2 + t - 4 = 0$

   1.2 $x^2 - 5x - 3 = 0$

   1.3 $2t^2 + 6t + 5 = 0$

   1.4 $2p\,(2p + 1) = 2$

1.5 $-3t^2 + 5t - 8 = 0$

1.6 $5t^2 + 3t - 3 = 0$

1.7 $t^2 - 4t + 2 = 0$

1.8 $9(k^2 - 1) = 7k$

1.9 $3f - 2 = -2f^2$

1.10 $t^2 + t + 1 = 0$

11.4 Exercise 4

1. Solve the following equations by substitution:

   1.1 $-24 = 10(x^2 + 5x) + (x^2 + 5x)^2$

   1.2 $(x^2 - 2x)^2 - 8 = 7(x^2 - 2x)$

   1.3 $x^2 + 3x - \frac{56}{x(x+3)} = 26$

   1.4 $x^2 - 18 + x + \frac{72}{x^2 + x} = 0$

   1.5 $x^2 - 4x + 10 - 7(4x - x^2) = -2$

   1.6 $\frac{9}{x^2 + 2x - 12} = x^2 + 2x - 12$

2. Determine a quadratic equation for a graph that has roots 3 and –2.

3. Find a quadratic equation for a graph that has x-intercepts of $(-4; 0)$ and $(4; 0)$

4. Determine a quadratic equation of the form $ax^2 + bx + c = 0$, where a, b and c are integers, that has roots –$\frac{1}{2}$ and 3

5. Determine the value of k and the other root of the quadratic equation kx^2–$7x + 4$=0 given that one of the roots is $x = 1$.

6. One root of the equation $2x^2$–$3x = p$ is $2,5$. Find p and the other root.

7 Solve the following quadratic equations by either factorisation, using the quadratic formula or completing the square:

   7.1 $24y^2 + 61y - 8 = 0$

   7.2 $8x^2 + 16x = 42$

   7.3 $9t^2 = 24t - 12$

   7.4 $-5y^2 + 0y + 5 = 0$

7.5 $3m^2 + 12 = 15m$

7.6 $49y^2 + 0y - 25 = 0$

7.7 $72 = 66w - 12w^2$

7.8 $-40y^2 + 58y - 12 = 0$

7.9 $37n + 72 - 24n^2 = 0$

7.10 $6y^2 + 7y - 24 = 0$

7.11 $3 = x(2x - 5)$

7.12 $-18y^2 - 55y - 25 = 0$

7.13 $-25y^2 + 25y - 4 = 0$

7.14 $8(1 - 4g^2) + 24g = 0$

7.15 $9y^2 - 13y - 10 = 0$

7.16 $(7p - 3)(5p + 1) = 0$

7.17 $-81y^2 - 99y - 18 = 0$

7.18 $14y^2 - 81y + 81 = 0$

11.5 Exercise 5

1. Determine the nature of the roots for each of the following equations:

   1.1 $x^2 + 3x = -2$

   1.2 $x^2 + 9 = 6x$

   1.3 $6y^2 - 6y - 1 = 0$

   1.4 $4t^2 - 19t - 5 = 0$

   1.5 $z^2 = 3$

   1.6 $0 = p^2 + 5p + 8$

   1.7 $x^2 = 36$

   1.8 $4m + m^2 = 1$

   1.9 $11 - 3x + x^2 = 0$

   1.10 $y^2 + \frac{1}{4} = y$

2. Given, $x^2 + bx - 2 + k\left(x^2 + 3x + 2\right) = 0\ (k \neq -1)$

   2.1 Show that the discriminant is given by: $\Delta = k^2 + 6bk + b^2 + 8$

   2.2 If $b = 0$, discuss the nature of the roots of the equation.

   2.3 If $b = 2$, find the value(s) of k for which the roots are equal.

3 What is the nature of roots for $k^2x^2 + 2 = kx - x^2$ for all real values for k.

4. Consider equation $x^2 + 12x = 3kx^2 + 2$ with real roots .

   4.1 Find the greatest value of value k such that $k \in \mathbb{Z}$

   4.2 Find one rational value of k for which the above equation has rational roots

5. Consider the equation: $k = \frac{x^2-4}{2x-5}$ Where , $x \neq \frac{5}{2}$

   5.1 Find a value of k for which the roots are equal.

   5.2 Find an integer k for which the roots of the equation will be rational and unequal

6. If b and c can take on only the values 1, 2 or 3, determine all pairs $(b; c)$ such that $x^2 + bx + c = 0$ has real roots

7. Given, $x^2 - (a + b)\,x + ab - p^2 = 0$

   7.1 Prove that the roots of the equation are real for all real values of a, b and p.

   7.2 When will the roots of the equation be equal?

11.6 Exercise 6

1. Solve the following inequalities and show each answer on a number line:

   1.1 $x^2 - x < 12$

   1.2 $3x^2 > -x + 4$

   1.3 $y^2 < -y - 2$

   1.4 $(3 - t)(1 + t) > 0$

   1.5 $x \geq -4x^2$

   1.6 $2x^2 + x + 6 \leq 0$

   1.7 $\frac{x}{x-3} < 2, x \neq 3$

   1.8 $\frac{x^2-4}{x-7} \geq 0, x \neq 7$

   1.9 $\frac{x+2}{x} - 1 \geq 0, x \neq 0$

   1.10 $s^2 - 4s > -6$

   1.11 $0 \geq 7x^2 - x + 8$

2. Draw a sketch of the following inequalities and solve for x .

   2.1 $2x^2 - 18 > 0$

   2.2 $5 - x^2 \leq 0$

   2.3 $x^2 < 0$

   2.4 $0 \leq 6x^2$

11.7 Exercise 7

1. Solve the following systems of equations algebraically. Leave your answer in surd form, where appropriate.

   1.1 $y + x = 5$ and $y - x^2 + 3x - 5 = 0$

   1.2 $y = 6 - 5x + x^2$ and $y - x + 1 = 0$

   1.3 $y = \frac{2x+2}{4}$ and $y - 2x^2 + 3x + 5 = 0$

   1.4 $a - 2b - 3 = 0$ and $a - 3b^2 + 4 = 0$

   1.5 $x^2 - y + 2 = 3x$ and $4x = 8 + y$

   1.6 $2y + x^2 + 25 = 7x$ and $3x = 6y + 96$

2. Solve the following systems of equations graphically. Check your solutions by also solving algebraically.

   2.1 $x^2 - 1 - y = 0$ and $y + x - 5 = 0$

   2.2 $x + y - 10 = 0$ and $x^2 - 2 - y = 0$

   2.3 $xy = 12$ and $7 = x + y$

   2.4 $6 - 4x - y = 0$ and $12 - 2x^2 - y = 0$

3. Solve the following systems of equations algebraically:

   3.1 $\frac{1}{x} - \frac{1}{y} = \frac{1}{3}$ and $\frac{1}{x^2} + \frac{1}{y^2} = \frac{5}{9}$

   3.2 $2x + 3y = 5$ and $(2x + 3y)(4x - 3y) = 25$

   3.3 $4^{x+y} = 2^{y+4}$ and $y^2 + xy = 70$

11.8 Exercise 8

1. Mr. Tsilatsila builds a fence around his rectangular vegetable garden of 8 m ^{2} . If the length is twice the breadth. Determine the dimensions of Mr. Tsilatsila's vegetable garden.

2. Kevin has played a few games of ten-pin bowling. In the third game, Kevin scored 80 more than in the second game. In the first game Kevin scored 110 less than the third game. His total score for the first two games was 208 . If he wants an average score of 146 , what must he score on the fourth game?

3. When an object is dropped or thrown downward, the distance, d , that it falls in time, t , is described by the following equation:$s = 5t^2 + v_0 t$ In this equation, v_0 is the initial velocity, in m $\cdot$ s^{-1} . Distance is measured in meters and time is measured in seconds. Use the equation to find how long it takes a tennis ball to reach the ground if it is thrown downward from a hot-air balloon that is 500 m high. The tennis ball is thrown at an initial velocity of 5 m $\cdot$ s^{-1} .

4. The table below lists the times that Sheila takes to walk the given distances.

time (minutes)	5	10	15	20	25	30
distance (km)	1	2	3	4	5	6

4.1 Find the equation that describes the relationship between time and distance.

4.2 How long will it take Sheila to walk 21 km?

4.3 How far will Sheila walk in 7 minutes?

4.4 If Sheila were to walk half as fast as she is currently walking, how would the graph of her distances and times look like?

5. The power P (in watts) supplied to a circuit by a 12 volt battery is given by the formula $P = 12I–0,5I^2$ where I is the current in amperes.

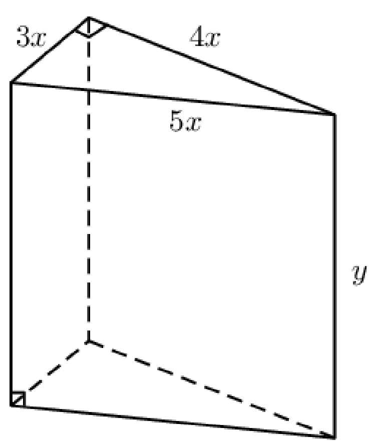
5.1 Since both power and current must be greater than 0 , find the limits of the current that can be drawn by the circuit.

5.2 What is the maximum current that can be drawn?

5.3 How much power is supplied to the circuit when the current is 10A.

5.4 At what value of current will the power supplied be a maximum?

6. A wooden block is made as shown in the diagram. The ends are right-angled triangles having sides $3x$, $4x$ and $5x$ .The length of the block is y . The total surface area of the block is 3600 cm^2



Prove that $y = \frac{300-x^2}{x}$

7. The difference between two numbers is 8 and the product of the same two numbers is 20 . Determine the two numbers.

8. A brother is currently two times the age of his sister. Two years ago he was four times as old as his sister was then. How old is the sister now?

9. The sum of three consecutive integers is 219 . Determine the value of the integers.

10. If the length of a bathroom is increased by 3 m and the breadth of the bathroom is increased by 1 m, the area of the bathroom is tripled. Determine the dimensions of the new bathroom if the area of the old bathroom is 10 m ^{2} .

12 ANSWERS FOR EXERCISES

12.1 Exercise 1

1. 1.1 $t = 0$ or $t = -2$
   1.2 $y = -1$
   1.3 $s = \pm 5$
   1.4 $y = 3$ or $y = 2$
   1.5 $y = 4$ or $y = -9$
   1.6 $y = -3$ or $y = -8$
   1.7 $y = 6$ or $y = 7$
   1.8 $x = -7$ or $x = -2$
   1.9 $y = 4k$ or $y = k$
   1.10 $y = 9$ or $y = -9$
   1.11 $y = \sqrt{5}$ or $y = -\sqrt{5}$
   1.12 $h = \pm 6$
   1.13 $y = \sqrt{14}$ or $y = -\sqrt{14}$
   1.14 $y = \pm 6\sqrt{2}$
   1.15 $f = \frac{5}{2}$ or $f = -3$
   1.16 $y = \frac{1}{7}$
   1.17 $x \in \mathbb{R}, x \neq \pm 3$
   1.18 $y = -13$ or $y = 1$
   1.19 $t = \frac{3}{2}$ or $t = -2$
   1.20 $m = -6$

12.2 Exercise 2

1.1 $x = -5 - 3\sqrt{3}$ or $x = -5 + 3\sqrt{3}$

1.2 $x = -1$ or $x = -3$

1.3 $p = -4 \pm \sqrt{21}$

1.4 $x = -3 \pm \sqrt{7}$

1.5 No real solution

1.6 $t = -8 \pm 3\sqrt{6}$

1.7 $x = -1 \pm \sqrt{\frac{5}{3}}$

1.8 $z = -4 \pm \sqrt{22}$

1.9 $z = \frac{11}{2}$ or $z = 0$

1.10 $z = 5$ or $z = -1$

2. $k = -3 \pm \sqrt{9 - a}$

3. $y = \frac{-q \pm \sqrt{q^2 - 4pr}}{2p}$

4.1 Minimum value of -5 at $x = -3$

4.2 Minimum value of $-\frac{53}{4}$ at $x = \frac{5}{2}$

4.3 Maximum value of $\frac{19}{3}$ at $x = \frac{1}{3}$

4.4 Minimum value of $\frac{7}{8}$ at $x = \frac{5}{4}$

5. 2500 m^2

12.3 Exercise 3

1. 1.1 $t = 1$ or $t = \frac{-4}{3}$

   1.2 $x = \frac{5+\sqrt{37}}{2}$ or $x = \frac{5-\sqrt{37}}{2}$

   1.3 No real solution

   1.4 $p = \frac{1}{2}$ or $p = -1$

   1.5 No real solution

   1.6 $t = \frac{-3+\sqrt{69}}{10}$ or $t = \frac{-3-\sqrt{69}}{10}$

   1.7 $t = 2 \pm \sqrt{2}$

   1.8 $k = \frac{7+\sqrt{373}}{18}$ or $k = \frac{7-\sqrt{373}}{18}$

   1.9 $f = \frac{1}{2}$ or $f = -2$

   1.10 No real solution

12.4 Exercise 4

1. 1.1 $x = -1$,
   $x = -4$,
   $x = -2$ or
   $x = -3$

1.2 $x = 1$,
$x = 4$ or
$x = -2$

1.3 $x = -7$,
$x = 4$,
$x = -2$ or
$x = -1$

1.4 $x = -4$,
$x = 3$,
$x = -3$ or
$x = 2$

1.5 $x = \frac{8+\sqrt{40}}{4}$ or $x = \frac{8-\sqrt{40}}{4}$

1.6 $x = -5$,
$x = 3$,
$x = -1 + \sqrt{10}$ or
$x = -1 - \sqrt{10}$

2. $x^2 - x - 6 = 0$

3. $x^2 - 16 = 0$

4. $2x^2 - 5x - 3 = 0$

5. $x = 1$ or $x = \frac{4}{3}$

6. $x = 2\frac{1}{2}$ or $x = -1$

7. 7.1 $y = \frac{1}{8}$ or $y = -\frac{8}{3}$

7.2 $x = \frac{3}{2}$ or $x = -\frac{7}{2}$

7.3 $t = \frac{2}{3}$ or $t = 2$

7.4 $y = 1$ or $y = -1$

7.5 $m = 1$ or $m = 4$

7.6 $y = \frac{5}{7}$ or $y = -\frac{5}{7}$

7.7 $w = \frac{3}{2}$ or $w = 4$

7.8 $y = \frac{6}{5}$ or $y = \frac{1}{4}$

7.9 $n = \frac{8}{3}$ or $n = -\frac{9}{8}$

7.10 $y = -\frac{8}{3}$ or $y = \frac{3}{2}$

7.11 $x = -\frac{1}{2}$ or $x = 3$

7.12 $y = -\frac{5}{2}$ or $y = -\frac{5}{9}$

7.13 $y = \frac{4}{5}$ or $y = \frac{1}{5}$

7.14 $g = -\frac{1}{4}$ or $g = 1$

7.15 $y = 2$ or $y = -\frac{5}{9}$

7.16 $p = \frac{3}{7}$ or $p = -\frac{1}{5}$

7.17 $y = -\frac{2}{9}$ or $y = -1$

7.18 $y = \frac{9}{2}$ or $y = \frac{9}{7}$

12.5 Exercise 5

1. 1.1 Real, unequal and rational.
   1.2 Real and equal.
   1.3 Real, Unequal and Irrational.
   1.4 Real, Unequal and Rational.
   1.5 Real, Unequal and Irrational
   1.6 Non-real
   1.7 Real, Unequal and Rational
   1.8 Real, Unequal and Irrational.
   1.9 Non-real
   1.10 Real and Equal.

2. 2.1 $\Delta = k^2 + 6bk + b^2 + 8$
   2.2 The roots are real and unequal. We cannot say if the roots are rational or irrational
   2.3 $k = -6 \pm 2\sqrt{6}$

3. Non-real.

4. 4.1 $k = 6$
   4.2 $k = \frac{1}{3}$

5. 5.1 $k = 4$ or $k = 1$
   5.2 $k = 0$ or $k = 5$

6. $(b; c) = (2; 1), (3; 1), (3; 2)$

7. 7.1 Real
   7.2 $a = b$ and $p = 0$

12.6 Exercise 6

1. 1.1 $-3 < x < 4$

   1.2 $x < -\frac{4}{3}$ or $x > 1$

   1.3 No solution

   1.4 $-1 < t < 3$

   1.5 $x \leq -\frac{1}{4}$ or $x \geq 0$

   1.6 Inequality is never true.

   1.7 $x < 3$ or $x > 6$ with $x \neq 3$

   1.8 $-2 \leq x \leq 2$ and $x >$ 7 with $x \neq 7$

   1.9 $x > 0$ with x not equal to 0

   1.10 True for all real values of s

   1.11 True for all real values of x

2. 2.1 $x < -3$ or $x > 3$

   2.2 $x \leq -\sqrt{5}$ or $x \geq \sqrt{5}$

   2.3 No solution

   2.4 True for all real values of x

12.7 Exercise 7

1. 1.1 $x = 0$ and $y = 5$ or $x = 2$ and $y = 3$

   1.2 $x = 3 \pm \sqrt{2}$ and $y = 2 \pm \sqrt{2}$

   1.3 $x = -1$ and $y = 0$ or $x = \frac{11}{4}$ and $y = \frac{15}{8}$

   1.4 $a = \frac{11 \pm \sqrt{88}}{3}$ and $b = \frac{2 \pm \sqrt{88}}{6}$

   1.5 $x = 5$ and $y = 12$ or $x = 2$ and $y = 0$

   1.6 $x =$ 7 and $y = -\frac{25}{2}$ or $x = -1$ and $y = -\frac{33}{2}$

2. 2.1 $(-3; 8)$ and $(2; 3)$

   2.2 $(-4; 14)$ and $(3; 7)$

   2.3 $(3; 4)$ and $(4; 3)$

   2.4 $(3; -6)$ and $(-1; 10)$

3. 3.1 $x = \frac{3}{2}$ and $y = 3$ or $x = -3$ and $y = -\frac{3}{2}$

   3.2 $x = \frac{5}{3}$ and $y = \frac{5}{9}$

   3.3 $x = 9$ and $y = -14$ or $x = -3$ or $y = 10$

12.8 Exercise 8

1. Breadth is 2 m and the length is 4 m
2. $d = 187$
3. $t \approx 10,5$ s
4. 4.1 $t = 5d$
   4.2 105 minutes
   4.3 $1,4$ km
   4.4 The graph will be steeper and lie closer to the y -axis.
5. 5.1 $I = 0$ or $I = 24$
   5.2 $24A$
   5.3 $70W$
   5.4 $12A$
6. $y = \frac{300-x^2}{x}$
7. $x = 10$ and $y = 2$ or $x = -2$ and $y = -10$
8. The brother is currently 6 years old and the sister is currently 3 years old.
9. $x = 72, y = 73, z = 74$
10. $x = 15$ and $y = \frac{2}{3}$ or $x = 2$ and $y = 5$