CHAPTER 3

Number Patterns

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CONTENTS

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In earlier grades we learned about linear sequences, where the difference between consecutive terms is constant. In this chapter, we will learn about quadratic sequences, where the difference between consecutive terms is not constant, but follows its own pattern.

1 REVISION

Terminology:	
Sequence/pattern	A sequence or pattern is an ordered set of numbers or variables.
Successive/consecutive	Successive or consecutive terms are terms that directly follow one after another in a sequence.
Common difference	The common or constant difference (d) is the difference between any two consecutive terms in a linear sequence.
General term	A mathematical expression that describes the sequence and that generates any term in the pattern by substituting different values for n.
Conjecture	A statement, consistent with known data, that has not been proved true nor shown to be false.

Important: a series is not the same as a sequence or pattern. Different types of series are studied in Grade 12. In Grade 11 we study sequences only.

1.1 Describing patterns

To describe terms in a pattern we use the following notation:

- T_1 is the first term of a sequence.
- T_4 is the fourth term of a sequence.
- T_n is the general term and is often expressed as the n^{th} term of a sequence.

A sequence does not have to follow a pattern but when it does, we can write an equation for the general term. The general term can be used to calculate any term in the sequence. For example, consider the following linear sequence: $1; 4; 7; 10; 13;$ ... The n^{th} term is given by the equation $T_n = 3n - 2$.

You can check this by substituting values for n:

$$T_1 = 3(1) - 2 = 1$$
$$T_2 = 3(2) - 2 = 4$$
$$T_3 = 3(3) - 2 = 7$$
$$T_4 = 3(4) - 2 = 10$$
$$T_5 = 3(5) - 2 = 13$$

If we find the relationship between the position of a term and its value, we can describe the pattern and find any term in the sequence.

1.2 Linear sequences

Definition

Linear sequence

A sequence of numbers in which there is a common difference (d) between any term and the term before it is called a linear sequence.

Important: $d = T_2 - T_1$, not $T_1 - T_2$.

Worked Example 1: Linear Sequence

Question

Determine the common difference (d) and the general term for the following sequence:

$$10; 7; 4; 1; \ldots$$

Solution

Step 1: Determine the common difference

To calculate the common difference, we find the difference between any term and the previous term:

$$\begin{aligned} d &= T_n - T_{n-1} \\ \text{Therefore } d &= T_2 - T_1 \\ &= 7 - 10 \\ &= -3 \\ \text{or } d &= T_3 - T_2 \\ &= 4 - 7 \\ &= -3 \\ \text{or } d &= T_4 - T_3 \\ &= 1 - 4 \\ &= -3 \end{aligned}$$

Step 2: Determine the general term

To find the general term T_n, we must identify the relationship between:

- the **value** of a number in the pattern and
- the **position** of a number in the pattern

postion	1	2	3	4
value	10	7	4	1

We start with the value of the first term in the sequence. We need to write an expression that includes the value of the common difference ($d = -3$) and the position of the term ($n = 1$).

Worked Example 1: Linear Sequence

$$\begin{aligned} T_1 &= 10 \\ &= 10 + (0)(-3) \\ &= 10 + (1 - 1)(-3) \end{aligned}$$

Now we write a similar expression for the second term.

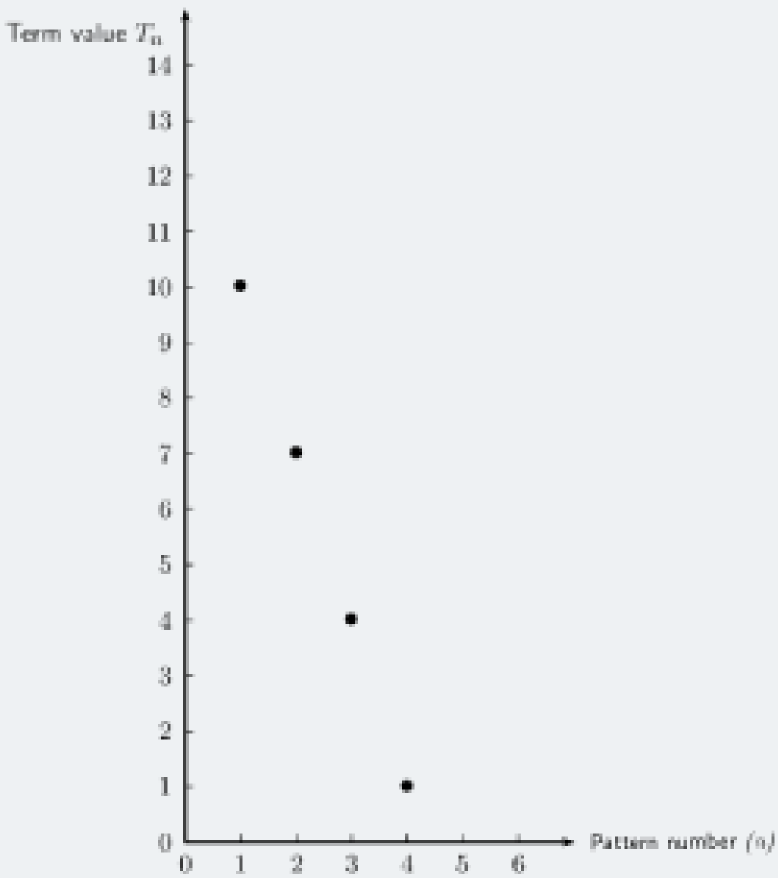
$$\begin{aligned} T_2 &= 7 \\ &= 10 + (1)(-3) \\ &= 10 + (2 - 1)(-3) \end{aligned}$$

We notice a pattern forming that links the position of a number in the sequence to its value.

$$\begin{aligned} T_n &= 10 + (n-1)(-3) \\ &= 10 - 3n + 3 \\ &= -3n + 13 \end{aligned}$$

Step 3: Drawing a graph of the pattern

We can also represent this pattern graphically, as shown below.



Worked Example 1: Linear Sequence

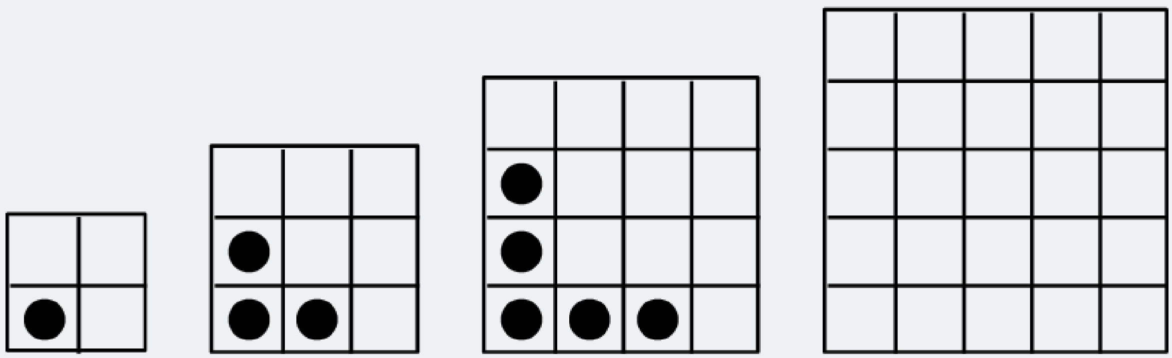
Notice that the position numbers (n) can be positive integers only.

This pattern can also be expressed in words: "each term in the sequence can be calculated by multiplying negative three and the position number, and then adding thirteen."

2 QUADRATIC SEQUENCES

Investigation

Quadratic sequences



1. Study the dotted-tile pattern shown and answer the following questions.

   1.1 Complete the fourth pattern in the diagram.

   1.2 Complete the table below:

pattern number	1	2	3	4	5	20	n
dotted tiles	1	3	5				
difference (d)	-	2					

   1.3 What do you notice about the change in number of dotted tiles?

   1.4 Describe the pattern in words: "The number of dotted tiles...".

   1.5 Write the general term: $T_n =$ ...

   1.6 Give the mathematical name for this kind of pattern.

   1.7 A pattern has 819 dotted tiles. Determine the value of n.

2. Now study the number of blank tiles (tiles without dots) and answer the following questions:

   2.1 Complete the table below:

pattern number	1	2	3	4	5	10
black tiles	3	6	11			
first difference (d)	-	3				
second difference (d)	-	-				

Investigation

2.2 What do you notice about the change in the number of blank tiles?

2.3 Describe the pattern in words: "The number of blank tiles...".

2.4 Write the general term: $T_n = ...$

2.5 Give the mathematical name for this kind of pattern.

2.6 A pattern has 227 blank tiles. Determine the value of n.

2.7 A pattern has 79 dotted tiles. Determine the number of blank tiles.

solution

1

1.1

1.2

pattern number	1	2	3	4	5	20	n
dotted tiles	1	3	5	**7**	**9**	**39**	**2n -1**
difference (d)	-	2	**2**	**2**	**2**	**2**	**2**

1.3 The dotted lines increase by one in each direction as the side of the square grow with the pattern

1.4 The number of dotted lines grow linearly as the sides of the square grows

1.5 $T_n = 2n - 1$

1.6 Linear pattern

1.7 $T_n = 2n - 1$

$819 = 2n - 1$

$820 = 2n$

$n = 410$

$\therefore$ The 410^{th} pattern has 819 dotted tiles

Investigation

Solution

2.

2.1

pattern number	1	2	3	4	5	10
black tiles	3	6	11	**18**	**27**	**102**
first difference (d)	-	3	**5**	**7**	**9**	**19**
second difference (d)	-	-	**2**	**2**	**2**	**2**

2.2 the number of blank tiles increase quadratically to a pattern formed by the second difference

2.3 The number of blank tiles increase quadratically, with the number of tiles being added forming a linear patten with a second difference.

2.4 $\mathsf{T_n} = \mathsf{an}^2 + \mathsf{bn} + \mathsf{c}\ 1:$

$$2a = 2$$
$$\therefore a = 1$$

$2:$

$$3a + b = 3$$
$$3(1) + b = 3$$
$$\therefore b = 0$$

$3:$

$$a + b + c = 3$$
$$1 + 0 + c = 3$$
$$\therefore c = 2$$
$$\mathsf{T_n} = \mathsf{n}^2 + 2$$

2.5 Quadratic pattern

2.6 $\mathsf{T_n} = \mathsf{n}^2 + 2$

$227 = \mathsf{n}^2 + 2$

$225 = \mathsf{n}^2$

$n = 15 \therefore$ The 15^{th} patten has 227 blank tiles

2.7 $\mathsf{T_n} = \mathsf{n}^2 + 2$

$\mathsf{T_n} = (79)^2 + 2$

$T_n = 6243$

$\therefore$ The 79^{th} has 6243 blank tiles

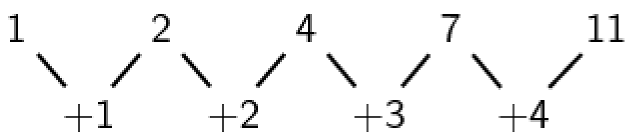
Definition

Quadratic sequence

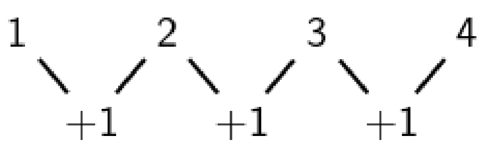
A quadratic sequence is a sequence of numbers in which the second difference between any two consecutive terms is constant.

Consider the following example: $1; 2; 4; 7; 11; ...$

The first difference is calculated by finding the difference between consecutive terms:



The second difference is obtained by taking the difference between consecutive first differences:

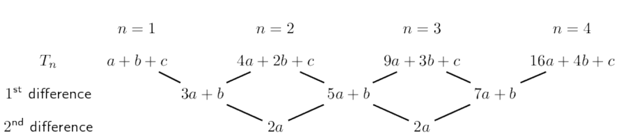


We notice that the second differences are all equal to 1. Any sequence that has a common second difference is a **quadratic sequence**.

It is important to note that the first differences of a quadratic sequence form a sequence. This sequence has a constant difference between consecutive terms. In other words, a linear sequence results from taking the first differences of a quadratic sequence.

General case

If the sequence is quadratic, the n^{th} term is of the form $T_n = an^2 + bn + c$.



In each case, the common second difference is a $2a$.

Worked Example 2: Quadratic Sequences

Question

Write down the next two terms and determine an equation for the n^{th} term of the sequence $5; 12; 23; 38; ...$

Solution

Step 1: Find the first differences between the terms

$5 \quad 12 \quad 23 \quad 38$

$+7 \quad +11 \quad +15$

Step 2: Find the second differences between the terms

$7 \quad 11 \quad 15$

$+4 \quad +4$

So there is a common second difference of 4. We can therefore conclude that this is a quadratic sequence of the form $T_n = an^2 + bn + c$.

Continuing the sequence, the next first differences will be:

$...15 \quad 19 \quad 23...$

$+4 \quad +4$

Step 3: Finding the next two terms in the sequence

The next two terms will be:

$...38 \quad 57 \quad 80...$

$+19 \quad +23$

So the sequence will be: $5; 12; 23; 38; 57; 80; ...$

Worked Example 2: Quadratic Sequences

Step 4: Determine the general term for the sequence

To find the values of a, b and c for $T_n = 2n^2 + n + 2$ we look at the first 3 terms in the sequence:

$$\begin{aligned} n = 1: &\quad T_1 = a + b + c \\ n = 2: &\quad T_2 = 4a + 2b + c \\ n = 3: &\quad T_3 = 9a + 3b + c \end{aligned}$$

We solve a set of simultaneous equations to determine the values of a, b and c

We know that $T_1 = 5$, $T_2 = 12$ and $T_3 = 23$

$$\begin{aligned} a + b + c &= 5 \\ 4a + 2b + c &= 12 \\ 9a + 3b + c &= 23 \end{aligned}$$

$$\begin{aligned} T_2 - T_1 &= 4a + 2b + c - (a + b + c) \\ 12 - 5 &= 4a + 2b + c - a - b - c \\ 7 &= 3a + b \ldots(1) \end{aligned}$$

$$\begin{aligned} T_3 - T_2 &= 9a + 3b + c - (4a + 2b + c) \\ 23 - 12 &= 9a + 3b + c - 4a - 2b - c \\ 11 &= 5a + b \ldots(2) \end{aligned}$$

$$\begin{aligned} (2) - (1) &= 5a + b - (3a + b) \\ 11 - 7 &= 5a + b - 3a - b \\ 4 &= 2a \\ \therefore a &= 2 \end{aligned}$$

$$\begin{aligned} \text{Using equation } (1): \quad 3(2) + b &= 7 \\ \therefore b &= 1 \\ \text{And using} \quad a + b + c &= 5 \\ 2 + 1 + c &= 5 \\ \therefore c &= 1 \end{aligned}$$

Step 5: Write the general term for the sequence

$$T_n = 2n^2 + n + 2$$

Worked Example 3: Plotting a graph of terms in a sequence

<u>Question</u>

Consider the following sequence:

$$3; 6; 10; 15; 21; \ldots$$

1. Determine the general term (T_n) for the sequence.
2. Is this a linear or a quadratic sequence?
3. Plot a graph of T_n vs n.

<u>Solution</u>

Step 1: Determine the first and second differences

	$n = 1$		$n = 2$		$n = 3$		$n = 4$
T_n	3		6		10		15
1^{st} difference		3		4		5	
2^{nd} difference			1		1		

We see that the first differences are not constant and form the sequence $3; 4; 5; \ldots$ and that there is a common second difference of 1. Therefore the sequence is quadratic and has a general term of the form $T_n = an^2 + bn + c$.

Step 2: Determine the general term T_n

To find the values of a, b and c for $T_n = an^2 + bn + c$ we look at the first 3 terms in the sequence:

$$\begin{aligned} n = 1: &\quad T_1 = a + b + c \\ n = 2: &\quad T_2 = 4a + 2b + c \\ n = 3: &\quad T_3 = 9a + 3b + c \end{aligned}$$

We solve this set of simultaneous equations to determine the values of a, b and c. We know that $T_1 = 3$, $T_2 = 6$ and $T_3 = 10$.

$$\begin{aligned} a + b + c &= 3 \\ 4a + 2b + c &= 6 \\ 9a + 3b + c &= 10 \end{aligned}$$

$$\begin{aligned} T_2 - T_1 &= 4a + 2b + c - (a + b + c) \\ 6 - 3 &= 4a + 2b + c - a - b - c \\ 3 &= 3a + b \ldots(1) \end{aligned}$$

Worked Example 3: Plotting a graph of terms in a sequence

$$\begin{aligned} T_3 - T_2 &= 9a + 3b + c - (4a + 2b + c) \\ 10 - 6 &= 9a + 3b + c - 4a - 2b - c \\ 4 &= 5a + b \ldots(2) \end{aligned}$$

$$\begin{aligned} (2) - (1) &= 5a + b - (3a + b) \\ 4 - 3 &= 5a + b - 3a - b \\ 1 &= 2a \\ \therefore a &= \tfrac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{Using equation } (1): \quad 3(\tfrac{1}{2}) + b &= 3 \\ \therefore b &= \tfrac{3}{2} \\ \text{And using} \quad a + b + c &= 3 \\ \tfrac{1}{2} + \tfrac{3}{2} + c &= 3 \\ \therefore c &= 1 \end{aligned}$$

Therefore the general term for the sequence is $T_n = \frac{1}{2}n^2 + \frac{3}{2}n + 1$.

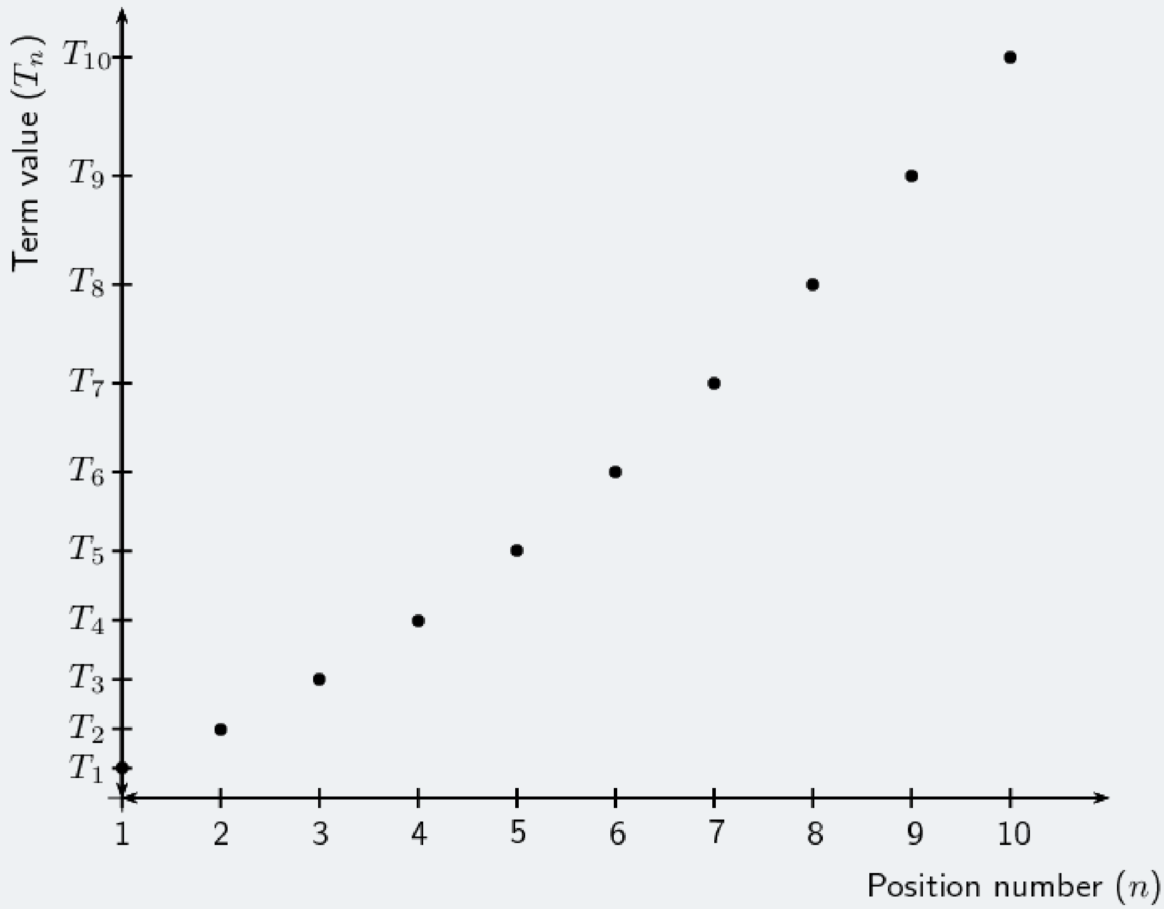
Step 3: Plot a graph of T_n vs n

Use the general term for the sequence,$T_n = \frac{1}{2}n^2 + \frac{3}{2}n + 1$, to complete the table.

(n):	1	2	3	4	5	6	7	8	9	10
T_n	3	6	10	15	21	28	36	45	55	66

Worked Example 3: Plotting a graph of terms in a sequence

Use the table to plot the graph:



In this case it would not be accurate to join these points, since n indicates the position of a term in a sequence and can therefore only be a positive integer. We can, however, see that the plot of the points lies in the shape of a parabola.

Worked Example 4: Olympic games soccer event

Question

In the first stage of the soccer event at the Olympic Games, there are teams from four different countries in each group. Each country in a group must play every other country in the group once.

1. How many matches will be played in each group in the first stage of the event?
2. How many matches would be played if there are 5 teams in each group?
3. How many matches would be played if there are 6 teams in each group?
4. Determine the general formula of the sequence.

Solution

Step 1: Determine the number of matches played if there are 4 teams in a group

Let the teams from four different countries be A, B, C and D.

teams in a group	**matches played**
A	AB, AC, AD
B	BC, BD
C	CD
D	
4	$3+2+1=6$

AB means that team A plays team B and BA would be the same match as AB. So if there are four different teams in a group, each group plays 6 matches.

Step 2: Determine the number of matches played if there are 5 teams in a group

Let the teams from five different countries be A, B, C, D and E.

teams in a group	**matches played**
A	AB, AC, AD, AE
B	BC, BD, BE
C	CD, CE
D	DE
E	
5	$4+3+2+1=10$

So if there are five different teams in a group, each group plays 10 matches.

Worked Example 4: Olympic games soccer event

Step 3: Determine the number of matches played if there are 6 teams in a group

Let the teams from four different countries be A, B, C and D,E and F.

teams in a group	**matches played**
A	AB, AC, AD, AE, AF
B	BC, BD, BE, BF
C	CD, CE, CF
D	DE, DF
E	EF
F	
5	$5 + 4 + 3 + 2 + 1 = 15$

So if there are six different teams in a group, each group plays 15 matches.

We continue to increase the number of teams in a group and find that a group of 7 teams plays 21 matches and a group of 8 teams plays 28 matches.

Step 4: Consider the sequence

We examine the sequence to determine if it is linear or quadratic:

	$n = 1$		$n = 2$		$n = 3$		$n = 4$		$n = 5$
T_n	6		10		15		21		$28 \ldots$
first difference		4		5		6		7	
second difference			1		1		1		

We see that the first differences are not constant and that there is a common second difference of 1. Therefore the sequence is quadratic and has a general term of the form $T_n = an^2 + bn + c$.

Step 5: Determine the general term T_n

To find the values of a, b and c for $T_n = an^2 + bn + c$ we look at the first 3 terms in the sequence:

$$\begin{aligned} n = 1: &\quad T_1 = a + b + c \\ n = 2: &\quad T_2 = 4a + 2b + c \\ n = 3: &\quad T_3 = 9a + 3b + c \end{aligned}$$

We solve this set of simultaneous equations to determine the values of a, b and c. We know that $T_1 = 6$, $T_2 = 10$ and $T_3 = 15$.

$$\begin{aligned} a + b + c &= 6 \\ 4a + 2b + c &= 10 \\ 9a + 3b + c &= 15 \end{aligned}$$

Worked Example 4: Olympic games soccer event

$$\begin{aligned} T_2 - T_1 &= 4a + 2b + c - (a + b + c) \\ 10 - 6 &= 4a + 2b + c - a - b - c \\ 4 &= 3a + b \ldots(1) \end{aligned}$$

$$\begin{aligned} T_3 - T_2 &= 9a + 3b + c - (4a + 2b + c) \\ 15 - 10 &= 9a + 3b + c - 4a - 2b - c \\ 5 &= 5a + b \ldots(2) \end{aligned}$$

$$\begin{aligned} (2) - (1) &= 5a + b - (3a + b) \\ 5 - 4 &= 5a + b - 3a - b \\ 1 &= 2a \\ \therefore a &= \tfrac{1}{2} \end{aligned}$$

Using equation (1): $3(\frac{1}{2}) + b = 4$

$\therefore b = \frac{5}{2}$

And using $a + b + c = 6$

$\frac{1}{2} + \frac{5}{2} + c = 6$

$\therefore c = 3$

Therefore the general term for the sequence is $T_n = \frac{1}{2}n^2 + \frac{5}{2}n + 3$.

3 SUMMARY

- T_n is the general term of a sequence.
- **Successive** or **consecutive** terms are terms that follow one after another in a sequence.
- A **linear** sequence has a common difference (d) between any two successive terms.

$$d = T_n - T_{n\text{-}1}$$

- A **quadratic** sequence has a common second difference between any two successive terms.
- The general term for a quadratic sequence is

$$T_n = an^2 + bn + c$$

- A general quadratic sequence:

	$n=1$		$n=2$		$n=3$		$n=4$
T_n	$a+b+c$		$4a+2b+c$		$9a+3b+c$		$16a+4b+c$
1st difference		$3a+b$		$5a+b$		$7a+b$	
2nd difference			$2a$		$2a$		

4 EXERCISES

4.1 Exercise 1

1. Write down the next three terms in each of the following sequences:
   $45;\ 29;\ 13;\ -3;\ \ldots$

2. The general term is given for each sequence below. Calculate the missing terms.

   2.1 $-4;\ -9;\ -14;\ \ldots;\ -24$
   $T_n = 1 - 5n$

   2.2 $6;\ \ldots;\ 24;\ \ldots;\ 42$
   $T_n = 9n - 3$

3. Find the general formula for the following sequences and then find T_{15} and T_{30} :

   3.1 $13;\ 16;\ 19;\ 22;\ \ldots$

   3.2 $18;\ 24;\ 30;\ 36;\ldots$

   3.3 $-10;\ -15;\ -20;\ -25;\ldots$

4. The seating in a classroom is arranged so that the first row has 20 desks, the second row has 22 desks, the third row has 24 desks and so on. Calculate how many desks are in the ninth row.

5. Determine and complete the following questions:

   5.1 Complete the following:
   $13 + 31 = \ldots$
   $24 + 42 = \ldots$
   $38 + 83 = \ldots$

   5.2 Look at the numbers on the left-hand side, what do you notice about the unit digit and the tens-digit?

   5.3 Investigate the pattern by trying other examples of $2-$ digit numbers.

   5.4 Make a conjecture about the pattern that you notice.

   5.5 Prove this conjecture.

6. Consider the following sequence:
   $7;\ 12;\ 17;\ 22;\ \ldots$

   6.1 What type of sequence is this? Prove this statement.

   6.2 Write down the next three terms of the sequence.

   6.3 Find the general formula for this sequence.

   6.4 Find the value of n if $T_n = 97$

7. Consider the following linear sequence:
   $6;\ 2;\ -2;\ \ldots;\ -390$

   7.1 Write down the common difference.

   7.2 Find the general formula of the sequence.

   7.3 Write down the value of T_6 .

   7.4 Determine the number of terms in the sequence.

8. A linear pattern has a constant difference of 4 and the 17^{th} term is 67 . Determine the value of the 14^{th} term.

9. Cubes of volume 1 cm^3 are stacked on top of each other to form a tower:

   9.1 What is the general term for the height of the tower?

   9.2 What type of sequence is this?

   9.3 Now consider the number of cubes in each tower:
   How many cubes will there be in the forth tower?

   9.4 Determine the general term for the sequence that describes the pattern of the cubes.

   9.5 Determine the general term for this sequence.

   9.6 How many cubes are needed for tower number 21 ?

   9.7 How high will a tower of 496 cubes be?

4.2 Exercise 2

1. Determine the second difference between the terms for the following sequences:

   1.1 $5;\ 20;\ 45;\ 80;\ \ldots$

   1.2 $6;\ 11;\ 18;\ 27;\ \ldots$

   1.3 $1;\ 4;\ 9;\ 16;\ \ldots$

   1.4 $3;\ 0;\ -5;\ -12;\ \ldots$

   1.5 $1;\ 3;\ 7;\ 13;\ \ldots$

   1.6 $0;\ -6;\ -16;\ -30;\ \ldots$

   1.7 $-1;\ 2;\ 9;\ 20;\ \ldots$

   1.8 $1;\ -3;\ -9;\ -17;\ \ldots$

   1.9 $3a+1;\ 12a+1;\ 27a+1;\ 48a+1;\ \ldots$

   1.10 $2;\ 10;\ 24;\ 44;\ \ldots$

   1.11 $t-2;\ 4t-1;\ 9t;\ 16t+1;\ \ldots$

2. Complete the sequence by filling in the missing term:

   2.1 $11;\ 21;\ 35;\ \ldots;\ 75$

   2.2 $20;\ \ldots;\ 42;\ 56;\ 72$

   2.3 $\ldots;\ 37;\ 65;\ 101$

   2.4 $3;\ \ldots;\ -13;\ -27;\ -45$

   2.5 $24;\ 35;\ 48;\ \ldots;\ 80$

   2.6 $\ldots;\ 11;\ 26;\ 47$

3. Use the general term to generate the first four terms in each sequence:

   3.1 $T_n = n^2 + 3n - 1$

   3.2 $T_n = -n^2 - 5$

   3.3 $T_n = 3n^2 - 2n$

   3.4 $T_n = -2n^2 + n + 1$

4. Calculate the second difference for each of the following quadratic sequences:

   4.1 $3;\ 4;\ 10;\ 21;\ \ldots$

   4.2 $4;\ 9;\ 16;\ 25;\ \ldots$

   4.3 $7;\ 17;\ 31;\ 49;\ \ldots$

   4.4 $2;\ 10;\ 26;\ 50;\ 82\ \ldots$

   4.5 $31;\ 30;\ 27;\ 22;\ 15;\ \ldots$

5. Find the first five terms of the quadratic sequence defined by: $T_n = 5n^2 + 3n + 4$

6. $T_n = 4n^2 + 5n + 10$, find T_9 .

7. Given $T_n = 2n^2$, for which value of n does $T_n = 32$.

8. Consider the quadratic sequence $16;\ 27;\ 42;\ 61;\ \ldots$

   8.1 Write down the next two terms.

   8.2 Find the general formula.

9. Show that the following sequence is a quadratic sequence: $7;\ 9;\ 12;\ 16;\ \ldots$

10. Consider the following quadratic number pattern: $40;\ 30;\ 23;\ 19$

   10.1 Determine the next 3 terms in the sequence.

   10.2 Determine the general term of the sequence.

   10.3 Determine the value of term 25 .

   10.4 Which term in the sequence is equal to 173 ?

4.3 Exercise 3

1. Consider the following quadratic pattern:
   $7;\ 18;\ 31;\ 46$

   1.1 Determine the next 3 terms in the sequence.

   1.2 Determine the general term of the sequence.

   1.3 Determine the value of the 20^{th} term.

   1.4 Which term in the sequence is equal to 178 ?

   1.5 Determine the general term for the first differences of this number pattern.

   1.6 Between which TWO consecutive terms of the quadratic number pattern will be the first difference be equal to 35 ?

2. Determine the values of x and y in the quadratic pattern:
   $1,\ x,\ 12,\ 22,\ y$

3. A quadratic number pattern has a second term of 2 . The first differences of the quadratic pattern are given by: $5;\ 3;\ 2;\ldots$

   3.1 Determine the first four terms of the quadratic sequence.

   3.2 Determine the general formula for the number pattern.

4. Given that the general form for a quadratic sequence is $T_n = an^2 + bn + c$, let d be the first difference and D be the second common difference.

   4.1 Show that $a = \frac{D}{2}$.

   4.2 Show that $b = d - \frac{3}{2}D$.

   4.3 Show that $c = T_1 - d + D$.

   4.4 Hence, show that $T_n = \frac{D}{2}n^2 + \left(d - \frac{3}{2}D\right)n + (T_1 - d + D)$.

5 ANSWERS FOR EXERCISES

5.1 Exercise 1

1. $-19; -35; -51$
2. 2.1 $T_4 = -19$

   2.2 $T_2 = 15$ $T_4 = 33$
3. 3.1 $T_n = 10 + 3n$ $T_{10} = 40$ $T_{15} = 55$ $T_{30} = 100$

   3.2 $T_n = 12 + 6n$ $T_{10} = 72$ $T_{15} = 102$ $T_{30} = 192$

   3.3 $T_n = -5 - 5n$ $T_{10} = -55$ $T_{15} = -80$ $T_{30} = -155$
4. 36
5. 5.1 $44; 66; 121$

   5.2 The unit digit and tens-digit have swapped position.

   5.3 $45 + 54 = 99$ $71 + 17 = 88$

   5.4 The sum of the two numbers will always be 11 times the sum of the two digits.
6. 6.1 Arithmetic. Constant first difference of 5 .

   6.2 $27; 32; 37$

   6.3 $T_n = 5n + 2$

   6.4 $n = 19$
7. 7.1 -4

   7.2 $T_n = -4n + 10$

   7.3 $T_6 = -14$

   7.4 $n = 100$
8. $T_{14} = 55$
9. 9.1 $T_n = n + 1$

   9.2 Linear

   9.3 15

   9.4 $T_n = \frac{1}{2}n^2 + \frac{3}{2}n + 1$

   9.5 Quadratic

   9.6 253

   9.7 31cm

5.2 Exercise 2

1. 1.1 10

   1.2 2

   1.3 2

   1.4 2

   1.5 2

   1.6 -4

   1.7 4

   1.8 -2

   1.9 $6a$

   1.10 6

   1.11 $2t$

2. 2.1 53

   2.2 30

   2.3 17

   2.4 -3

   2.5 63

   2.6 2

3. 3.1 $T_1 = 3;\ T_2 = 9;\ T_3 = 17;\ T_4 = 27$

   3.2 $T_1 = -6;\ T_2 = -9;\ T_3 = -14;\ T_4 = -21$

   3.3 $T_1 = 1;\ T_2 = 8;\ T_3 = 21;\ T_4 = 40$

   3.4 $T_1 = 0;\ T_2 = -5;\ T_3 = -14;\ T_4 = -27$

4. 4.1 1

   4.2 2

   4.3 4

   4.4 8

   4.5 -2

5. $T_1 = 12$
   $T_2 = 30$
   $T_3 = 58$
   $T_4 = 96$
   $T_5 = 144$

6. $T_n = 379$

7. $n = 4$

8. 8.1 $T_5 = 84$
   $T_6 = 111$

   8.2 $T_n = 2n^2 + 5n + 9$

9. Constant second difference of 1

10. 10.1 $18;\ 20;\ 25$

    10.2 $T_n = \frac{3}{2}n^2 - \frac{29}{2}n + 53$

    10.3 628

    10.4 $n = 15$

5.3 Exercise 3

1. 1.1 $63;\ 82;\ 103$

   1.2 $T_n = n^2 + 8n - 2$

   1.3 $T_{20} = 558$

   1.4 $n = 10$

   1.5 $T_n = 2n + 9$

   1.6 $n = 13$

2. $x = 5$
   $y = 35$

3. 3.1 $-3;\ 2;\ 5;\ 7$

   3.2 $T_n = -n^2 + 8n - 10$