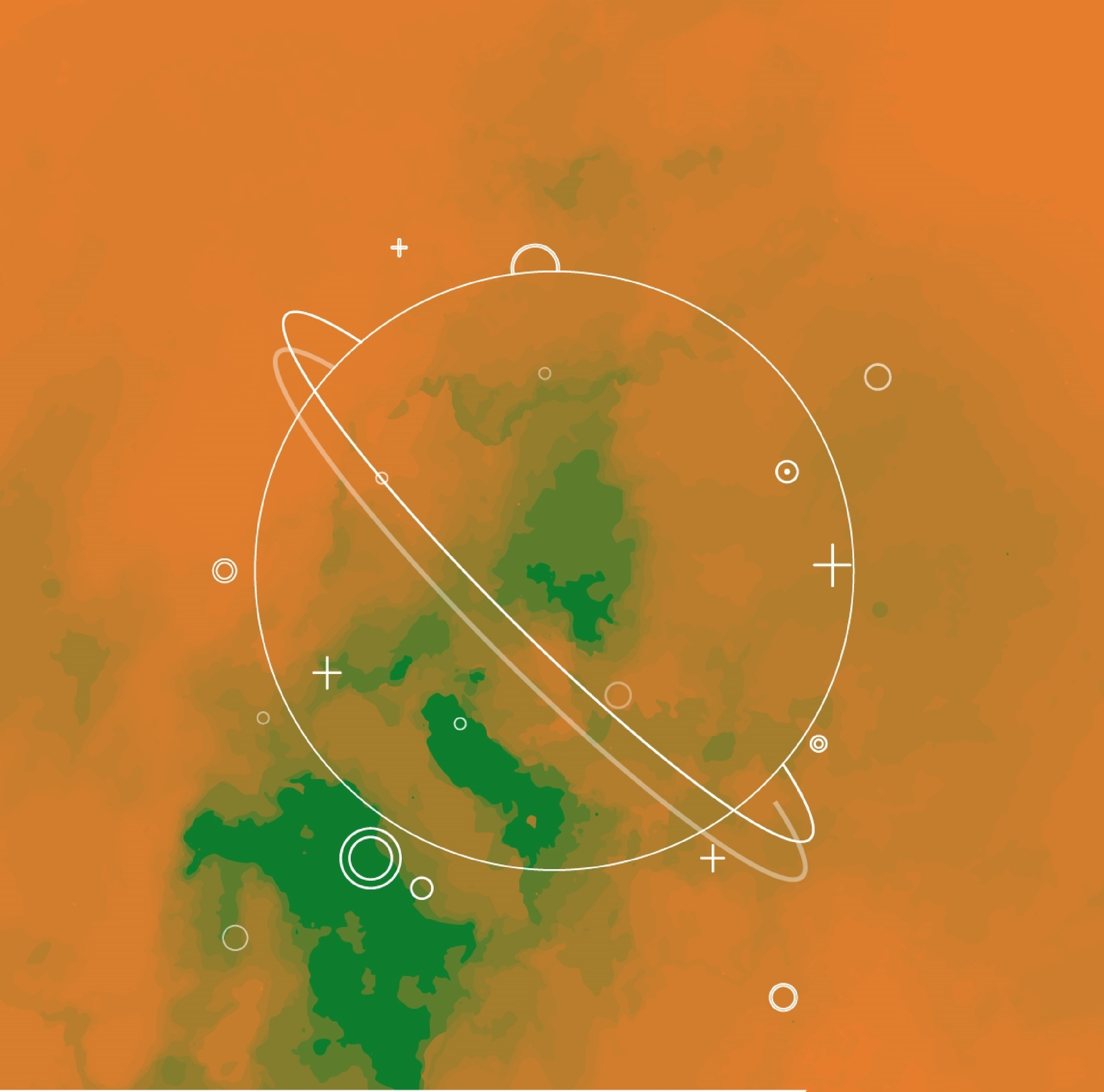


CHAPTER 9

Finance, Growth And Decay

CONTENTS

August 26, 2021

1 REVISION

Simple interest is the interest calculated only on the initial amount invested, the principal amount. Compound interest is the interest earned on the principal amount and on its accumulated interest. This means that interest is being earned on interest. The accumulated amount is the final amount; the sum of the principal amount and the amount of interest earned.

Formula for simple interest :

$$A = P(1 + in)$$

Formula for compound interest :

$$A = P(1 + i)^n$$

where

$$\begin{aligned} A &= accumulated\, amount \\ P &= principal\, amount \\ i &= interest\, rate\, written\, as\, a\, decimal \\ n &= time\, period\, in\, years \end{aligned}$$

WORKED EXAMPLE 1: SIMPLE AND COMPOUND INTEREST

QUESTION

Sam wants to invest **R3 450** for 5 years. Wise Bank offers a savings account which pays simple interest at a rate of **12**, **5**% per annum, and Grand Bank offers a savings account paying compound interest at a rate of **10**, **4**% per annum. Which bank account would give Sam the greatest accumulated balance at the end of the 5 year period?

SOLUTION

Step 1 : **Calculation using the simple interest formula**

Write down the known variables and the simple interest formula

$$\begin{aligned} P &= 3\,450 \\ i &= 0,125 \\ n &= 5 \\ A &= P(1 + in) \end{aligned}$$

Substitute the values to determine the accumulated amount for the Wise Bank savings account.

$$\begin{aligned} A &= 3\,450(1 + 0.125 \times 5) \\ &= R5\,606,25 \end{aligned}$$

Step 2 : **Calculation using the compound interest formula**

Write down the known variables and the compound interest formula.

$$\begin{aligned} P &= 3\,450 \\ i &= 0,104 \\ n &= 5 \\ A &= P(1 + i)^n \end{aligned}$$

Substitute the values to determine the accumulated amount for the Grand Bank savings account.

$$\begin{aligned} A &= 3\,450(1 + 0.104)^5 \\ &= R5\,658,02 \end{aligned}$$

Step 3 : **Write the final answer**

The Grand Bank savings account would give Sam the highest accumulated balance at the end of the 5 year period.

WORKED EXAMPLE 2: FINDING i

QUESTION

Bongani decides to put $R30\,000$ in an investment account. What compound interest rate must the investment account achieve for Bongani to double his money in **6** years? Give your answer correct to one decimal place.

SOLUTION

Step 1 : Write down the known variables and the compound interest formula

$$A = 60\,000$$
$$P = 30\,000$$
$$n = 6$$
$$A = P(1+i)^n$$

Step 2 : Substitute the values and solve for i

$$60\,000 = 30\,000(1+i)^6$$
$$\frac{60\,000}{30\,000} = (1+i)^6$$
$$2 = (1+i)^6$$
$$\sqrt[6]{2} = 1+i$$
$$\sqrt[6]{2} - 1 = i$$
$$\therefore i = 0{,}122...$$

Step 3 : Write the final answer and comment

We round up to a rate of **12**, **3**% p.a. to make sure that Bongani doubles his investment.

2 SIMPLE AND COMPOUND DEPRECIATION

As soon as a new car leaves the dealership, its value decreases and it is considered “second-hand”. Vehicles, equipment, machinery and other similar assets, all lose value over time as a result of usage and age. This loss in value is called depreciation. Assets that have a relatively long useful lifetime, such as machines, trucks, farming equipment etc., depreciate slower than assets like office equipment, computers, furniture etc. which need to be replaced more often and therefore depreciate more quickly.

Depreciation is used to calculate the value of a company’s assets, which determines how much tax a company must pay. Companies can take depreciation into account as an expense, and thereby reduce their taxable income. A lower taxable income means that the company will pay less income tax to SARS (South African Revenue Service).

We can calculate two different kinds of depreciation: simple decay and compound decay. Decay is also a term used to describe a reduction or decline in value. Simple decay is also called straight-line depreciation and compound decay can also be referred to as reducing-balance depreciation. In the straight-line method the value of the asset is reduced by a constant amount each year, which is calculated on the principal amount. In reducing-balance depreciation we calculate the depreciation on the reduced value of the asset. This means that the value of an asset decreases by a different amount each year.

2.1 Simple depreciation

WORKED EXAMPLE 3: STRAIGHT-LINE DEPRECIATION

QUESTION

A new smartphone costs **R6 000** and depreciates at **22**% p.a. on a straight-line basis. Determine the value of the smartphone at the end of each year over a 4 year period.

SOLUTION

Step 1 : Calculate depreciation amount

$$\begin{aligned} Depreciation &= 6\,000 \times \frac{22}{100} \\ &= 1\,320 \end{aligned}$$

Therefore the smartphone depreciates by **R1 320** every year.

Step 2 : Complete a table of values

Year	Value at beginning of year	Depreciation amount	Value at end of year
1	R6 000	R1 320	R4 680
2	R4 680	R1 320	R3 360
3	R3 360	R1 320	R2 040
4	R2 040	R1 320	R720

We notice that

$$Total\,depreciation = P \times i \times n$$

where

$$\begin{aligned} P &= principal\,amount \\ i &= interest\,rate\,written\,as\,a\,decimal \\ n &= time\,period\,in\,years \end{aligned}$$

Therefore the depreciated value of the asset (also called the book value) can be calculated as:

$$A = P(1 - in)$$

Note the similarity to the simple interest formula $A = P(1 + in)$. Interest increases the value of the principal amount, whereas with simple decay, depreciation reduces the value of the principal amount.

Important: to get an accurate answer do all calculations in one step on your calculator. Do not round off answers in your calculations until the final answer. In the worked examples in this chapter, we use dots to show that the answer has not been rounded off. We always round the final answer to two decimal places (cents).

WORKED EXAMPLE 4: STRAIGHT-LINE DEPRECIATION METHOD

QUESTION

A car is valued at **R240 000**. If it depreciates at **15**% p.a. using straight-line depreciation, calculate the value of the car after **5** years.

SOLUTION

Step 1 : Write down the known variables and the simple decay formula

$$P = 240\,000$$
$$i = 0,15$$
$$n = 5$$
$$A = P(1 - in)$$

Step 2 : Substitute the values and solve for A

$$\begin{aligned} A &= 240\,000(1 - 0,15 \times 5) \\ &= 240\,000(0,25) \\ &= 60\,000 \end{aligned}$$

Step 3 : Write the final answer

At the end of **5** years, the car is worth **R60 000**.

WORKED EXAMPLE 5: SIMPLE DECAY

<u>QUESTION</u>

A small business buys a photocopier for **R12 000**. For the tax return the owner depreciates this asset over **3** years using a straight-line depreciation method. What amount will he fill in on his tax form at the end of each year?

<u>SOLUTION</u>

Step 1 : Write down the known variables

The owner of the business wants the photocopier to have a book value of **R0** after **3** years.

$$\begin{aligned} A &= 0 \\ P &= 12\,000 \\ n &= 3 \end{aligned}$$

Therefore we can calculate the annual depreciation as

$$\begin{aligned} Depreciation &= \frac{P}{n} \\ &= \frac{12\,000}{3} \\ &= R4\,000 \end{aligned}$$

Step 2 : Determine the book value at the end of each year

$$\begin{aligned} Book\,value\,end\,of\,first\,year &= 12\,000 - 4\,000 \\ &= R8\,000 \end{aligned}$$

$$\begin{aligned} Book\,value\,end\,of\,second\,year &= 8\,000 - 4\,000 \\ &= R4\,000 \end{aligned}$$

$$\begin{aligned} Book\,value\,end\,of\,first\,year &= 4\,000 - 4\,000 \\ &= R0 \end{aligned}$$

2.2 Compound depreciation

WORKED EXAMPLE 6: REDUCING-BALANCE DEPRECIATION

QUESTION

A second-hand farm tractor worth **R60 000** has a limited useful life of **5** years and depreciates at **20**% p.a. on a reducing-balance basis. Determine the value of the tractor at the end of each year over the **5** year period.

SOLUTION

Step 1 : **Write down the known variables**

$$P = 60\,000$$
$$i = 0,2$$
$$n = 5$$

When we calculate depreciation using the reducing-balance method:

1. the depreciation amount changes for each year.
2. the depreciation amount gets smaller each year.
3. the book value at the end of a year becomes the principal amount for the next year.
4. the asset will always have some value (the book value will never equal zero).

Step 2 : **Completeatableofvalues**

Year	Book value	Depreciation	Value at end of year
1	R60 000	60 000$\times 0,2 = 12000$	R48 000
2	R48 000	48 000$\times 0,2 = 9600$	R38 400
3	R38 400	38 400$\times 0,2 = 7680$	R30 720
4	R30 720	30 720$\times 0,2 = 6144$	R24 576
5	R24 576	24 576$\times 0,2 = 4915,20$	R19 660,80

Notice in the example above that we could also write the book value at the end of each year as:

$$\begin{aligned}
Book\,value\,end\,of\,first\,year &= 60\,000(1-0,2) \\
Book\,value\,end\,of\,second\,year &= 48\,000(1-0,2) = 60\,000(1-0,2)^2 \\
Book\,value\,end\,of\,third\,year &= 38\,400(1-0,2) = 60\,000(1-0,2)^3 \\
Book\,value\,end\,of\,fourth\,year &= 30\,720(1-0,2) = 60\,000(1-0,2)^4 \\
Book\,value\,end\,of\,fifth\,year &= 24\,576(1-0,2) = 60\,000(1-0,2)^5
\end{aligned}$$

Using the formula for simple decay and the observed pattern in the calculation above, we obtain the following formula for compound decay:

$$A = P(1-i)^n$$

where

$$\begin{aligned}
A &= book\,value\,or\,depreciated\,value \\
P &= principal\,amount \\
i &= interest\,rate\,written\,as\,a\,decimal \\
n &= time\,period\,in\,years
\end{aligned}$$

n Again, notice the similarity to the compound interest formula $A = P(1+i)^n$.

WORKED EXAMPLE 7: REDUCING-BALANCE DEPRECIATION

QUESTION

The number of pelicans at the Berg river mouth is decreasing at a compound rate of **12**% p.a. If there are currently **3 200** pelicans in the wetlands of the Berg river mouth, what will the population be in **5** years?

SOLUTION

Step 1 : Write down the known variables and the compound decay formula

$$P = 3\,200$$
$$i = 0,12$$
$$n = 5$$

$$A + P(1-i)^n$$

Step 2 : Substitute the values and solve for A

$$\begin{aligned} A &= 3\,200(1-0,12)^5 \\ &= 3\,200(0,88)^5 \\ &= 1\,688,7421... \end{aligned}$$

Step 3 : Write the final answer

In **5** years, the pelican population will be approximately **1 689**.

WORKED EXAMPLE 8: COMPOUND DECAY

QUESTION

1. A school buys a minibus for **R950 000**, which depreciates at **13, 5**% per annum. Determine the value of the minibus after **3** years if the depreciation is calculated:

   (a) on a straight-line basis.

   (b) on a reducing-balance basis.

   (c) Which is the better option?

SOLUTION

Step 1 : Write down known variables

$$P = 950\,000$$
$$i = 0,135$$
$$n = 3$$

Step 2 : Use the simple decay formula and solve for A

$$\begin{aligned} A &= 950\,000(1 - 0,135)^3 \\ &= 950\,000(0,865)^3 \\ &= 614\,853,89 \\ \therefore &= R614\,853,89 \end{aligned}$$

Step 4 : Interpret the answers

After a period of **3** years, the value of the minibus calculated on the straight-line method is less than the value of the minibus calculated on the reducing-balance method. The value of the minibus depreciated less on the reducing-balance basis because the amount of depreciation is calculated on a smaller amount every year, whereas the straight-line method is based on the full value of the minibus every year.

WORKED EXAMPLE 9: COMPOUND DEPRECIATION

QUESTION

Farmer Jack bought a tractor and it has depreciated by **20**% p.a. on a reducing-balance basis. If the current value of the tractor is **R52 429**, calculate how much Farmer Jack paid for his tractor if he bought it **7** years ago.

SOLUTION

Step 1 : Write down known variables and compound decay formula

$$A = 52\,429$$
$$i = 0,2$$
$$n = 7$$

$$A = P(1-i)^n$$

Step 2 : Substitute the values and solve for P

$$52\,429 = P(1-0,2)^7$$
$$= P(0,8)^7 \therefore P = \frac{52\,429}{(0,8)^7}$$
$$= 250\,000,95...$$

Step 3 : Write the final answer

7 years ago, Farmer Jack paid **R250 000** for his tractor.

2.3 Finding i

WORKED EXAMPLE 10: FINDING i FOR SIMPLE DECAY

QUESTION

After **4** years, the value of a computer is halved. Assuming simple decay, at what annual rate did it depreciate? Give your answer correct to two decimal places.

SOLUTION

Step 1 : Write down known variables and simple decay formula

Let the value of the computer be x, therefore:

$$A = \frac{x}{2}$$
$$P = x$$
$$n = 4$$

$$A = P(1 - in)$$

Step 2 : Substitute the values and solve for i

$$\frac{x}{2} = x(1 - 3i)$$
$$\frac{1}{2} = 1 - 3i$$
$$\therefore 3i = 1 - \frac{1}{2}$$
$$\therefore i = 0,1667$$

Step 3 : Write the final answer

The computer depreciated at a rate of **16**, **67**% p.a.

WORKED EXAMPLE 11: FINDING i FOR COMPOUND DECAY

QUESTION

Cristina bought a fridge at the beginning of **2 009** for **R8 999** and sold it at the end of **2 011** for **R4 500**. At what rate did the value of her fridge depreciate assuming a reducing-balance method? Give your answer correct to two decimal places.

SOLUTION **Step 1 : Write down known variables and compound decay formula**

$$A = 4\,500$$
$$P = 8\,999$$
$$n = 3$$
$$A = P(1-i)^n$$

Step 2 : Substitute the values and solve for i

$$4\,500 = 8\,999(1-i)^3$$
$$\frac{4\,500}{8\,999} = (1-i)^3$$
$$\sqrt[3]{\frac{4\,500}{8\,999}} = 1-i$$
$$\therefore i = 1 - \sqrt[3]{\frac{4\,500}{8\,999}}$$
$$= 0,206$$

Step 3 : Write the final answer

Cristina's fridge depreciated at a rate of **20**, **6**% p.a.

3 TIMELINES

Interest can be compounded more than once a year. For example, an investment can be compounded monthly or quarterly. Below is a table of compounding terms and their corresponding numeric value (**p**). When amounts are compounded more than once per annum, we multiply the number of years by **p** and we also divide the interest rate by **p**.

Term	**p**
yearly / annually	1
half-yearly / bi-annually	2
quarterly	4
monthly	12
weekly	52
daily	365

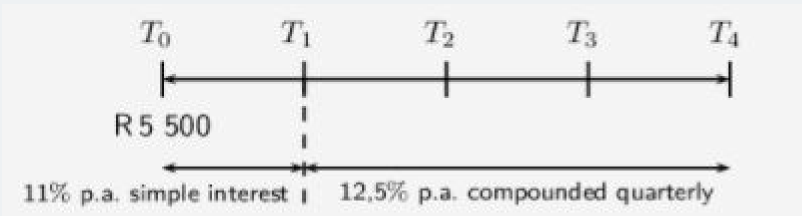
WORKED EXAMPLE 12: TIMELINES

QUESTION

R5 500 is invested for a period of **4** years in a savings account. For the first year, the investment grows at a simple interest rate of **11**% p.a. and then at a rate of **12**, **5**% p.a. compounded quarterly for the rest of the period. Determine the value of the investment at the end of the **4** years.

SOLUTION

Step 1 : **Draw a timeline and write down known variables**



Remember to show when the additional deposits of **R8 000** and **R2 000** where made into the account. It is very important to note that the interest rate changes at $\mathbf{T_4}$.

We break this question down into parts and consider each amount separately.

Step 2 : The initial deposit at T_0

Between T_0 and T_4:

We notice that interest for the first **4** years is compounded half-yearly, therefore:

$$\begin{aligned} n_1 &= 4 \times 2 \\ &= 8 \\ and\, i_1 &= \frac{0,12}{2} \end{aligned}$$

Between T_4 and T_6

$$\begin{aligned} n_2 &= 2 \\ and\, i_2 &= 0,085 \end{aligned}$$

Therefore the total growth of the initial deposit over the **6** years is:

$$\begin{aligned} A &= P(1+i_1)^{n_1}(1+i_2)^{n^2} \\ &= 150\,000(1+\frac{0,12}{2})^8(1+0,085)^2 \end{aligned}$$

Step 3 : The deposit at T_1

Interest on this deposit is compounded half-yearly for **3** years, therefore:

$$\begin{aligned} n_3 &= 3 \times 2 \\ &= 6 \\ and\, i_3 &= \frac{0,012}{2} \end{aligned}$$

Between T_4 and T_6:

$$\begin{aligned} n_4 &= 2 \\ and\, i_4 &= 0,085 \end{aligned}$$

Therefore the total growth of the deposit over the **5** years is:

$$\begin{aligned} A &= P(1+i_3)^{n_3}(1+i_4)^{n_4} \\ &= 8\,000(1+\frac{0,12}{2})^6(1+0,085)^2 \end{aligned}$$

Step 4 : **The deposit at T_5**

Accumulate interest for only **1** year:

$$\begin{aligned} A &= P(1+i)^n \\ &= 2\,000(1+0,085)^1 \end{aligned}$$

Step 5 : **Determine the total calculation**

To get as accurate an answer as possible, we do the the calculation on the calculator in one step. Using the memory and answer recall function on the calculator, we avoid rounding off until we get the final answer.

$$\begin{aligned} A &= 150\,000(1+\frac{0,12}{2})^8(1+0,085)^2 + 8\,000(1+\frac{0,12}{2})^6(1+0,085)^2 + 2\,000(1+0,085)^1 \\ &= R\,296\,977,00 \end{aligned}$$

Step 6 : **Write the final answer**

The value of the investment at the end of the **6** years is **R296 977**, **00**.

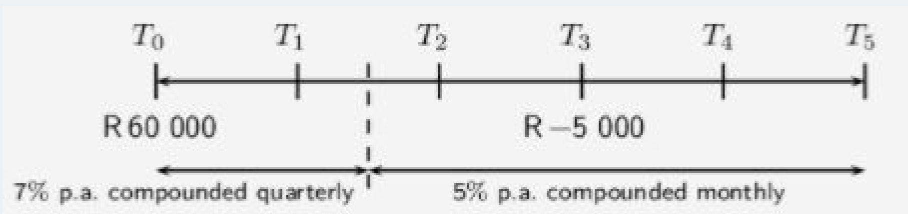
WORKED EXAMPLE 14: TIMELINES

QUESTION

R60 000 is invested in an account which offers interest at **7**% p.a. compounded quarterly for the first **18** months. Thereafter the interest rate changes to **5**% p.a. compounded monthly. Three years after the initial investment, **R5 000** is withdrawn from the account. How much will be in the account at the end of **5** years?

SOLUTION

Step 1 : Draw a timeline and write down known variables



Remember to show when the withdrawal of **R5 000** was taken out of the account. It is also important to note that the interest rate changes after **18** months ($\mathbf{T_{1\frac{1}{2}}}$).

We break this question down into parts and consider each amount separately.

Step 2 : The initial deposit at t_0

Interest for the first **1**, **5** years is compounded quarterly, therefore:

$$\begin{aligned} n_1 &= 1,5 \times 4 \\ &= 6 \\ and\, i_1 &= \frac{0,07}{4} \end{aligned}$$

Interest for the remaining **3**, **5** years is compounded monthly, therefore:

$$\begin{aligned} n_2 &= 3,5 \times 12 \\ &= 42 \\ and\, i_2 &= \frac{0,05}{12} \end{aligned}$$

Therefore the total growth of the initial deposit over the **5** years is:

$$\begin{aligned} A &= P(1+1_1)^{n_1}(1+i_2)^{n_2} \\ &= 60\,000(1+\frac{0,07}{4})^6(1+\frac{0,05}{12})^42 \end{aligned}$$

Step 3 : The withdrawal at T_3

We calculate the interest that the **R5 000** would have earned if it had remained in the account:

$$n = 2 \times 12$$
$$= 24$$
$$and\, i = \frac{0,05}{12}$$

Therefore we have that:

$$A = P(1+i)^n$$
$$= 5\,000(1 + \frac{0,05}{12})^2 4$$

Step 4 : Determine the total calculation

We subtract the withdrawal and the interest it would have earned from the accumulated amount at the end of the 5 years:

$$A = 60\,000(1 + \frac{0,07}{4})^6 (1 + \frac{0,05}{12})^4 2 - 5\,000(1 + \frac{0,05}{12})^2 4$$
$$= R73\,762,19$$

Step 5 : Write the final answer

The value of the investment at the end of the **5** years is **R73 762**, **19**.

4 NOMINAL AND EFFECTIVE INTEREST RATES

We have seen that although interest is quoted as a percentage per annum it can be compounded more than once a year. We therefore need a way of comparing interest rates. For example, is an annual interest rate of **8**% compounded quarterly higher or lower than an interest rate of **8**% p.a. compounded yearly?

An interest rate compounded more than once a year is called the nominal interest rate. In the investigation above, we determined that the nominal interest rate of **8**% p.a. compounded half-yearly is actually an effective rate of **8**, **16**% p.a.

Given a nominal interest rate $\mathbf{i}^{(}\mathbf{m})$ compounded at a frequency of **m** times per year and the effective interest rate **i**, the accumulated amount calculated using both interest rates will be equal so we can write:

$$P(1+i) = P(1+\frac{i^{(m)}}{m})^m$$
$$\therefore 1+i = (1+\frac{i^{(m)}}{m})^m$$

WORKED EXAMPLE 15: NOMINAL AND EFFECTIVE INTEREST RATES

QUESTION

Interest on a credit card is quoted as **23**% p.a. compounded monthly. What is the effective annual interest rate? Give your answer correct to two decimal places.

SOLUTION

Step 1 : Write down the known variables

Interest is being added monthly, therefore:

$$m = 12$$
$$i^{(12)} = 0,23$$

$$1 + i = (1 + \frac{i^{(m)}}{m})^m$$

Step 2 : Substitute values and solve for i

$$1 + i = (1 + \frac{i^{(m)}}{m})^m$$
$$\therefore i = 1 - (1 + \frac{0,23)}{12})^{12}$$
$$= 25,59\%$$

Step 3 : Write the final answer

The effective interest rate is **25**, **59**% per annum.

WORKED EXAMPLE 16: NOMINAL AND EFFECTIVE INTEREST RATES

QUESTION

Determine the nominal interest rate compounded quarterly if the effective interest rate is **9**% per annum (correct to two decimal places).

SOLUTION

Step 1 : Write down the known variables

Interest is being added quarterly, therefore:

$$m = 4$$
$$i = 0,23$$

$$1 + i = (1 + \frac{i^{(m)}}{m})^m$$

Step 2 : Substitute values and solve for i(m)

$$1 + 0,23 = (1 + \frac{i^{(4)}}{4})^4$$
$$\sqrt[4]{1,09} = 1 + \frac{i^{(4)}}{4}$$
$$\sqrt[4]{1,09} - 1 = \frac{i^{(4)}}{4}$$
$$4(\sqrt[4]{1,09} - 1) = i^{(4)}$$
$$\therefore i = 8,71\%$$

Step 3 : Write the final answer

The nominal interest rate is **8, 71**% p.a. compounded quarterly.

5 SUMMARY

- Simple interest: $A = P(1 + in)$
- Compound interest: $A = P(1 + i)n$
- Simple depreciation: $A = P(1–in)$
- Compound depreciation: $A = P(1–i)n$
- Nominal and effective annual interest rates: $1 + i = (1 + \frac{i(m)}{m})^m$

6 EXERCISES

6.1 Exercise 1

1. Determine the value of an investment of R$10\ 000$ at $12{,}1$% p.a. simple interest for 3 years.

2. Calculate the value of R$8\ 000$ invested at $8{,}6$% p.a. compound interest for 4 years. Round to two decimal places.

3. Calculate how much interest John will earn if he invests R$2\ 000$ for 4 years at:

   3.1 $6{,}7$% p.a. simple interest

   3.2 $5{,}4$% p.a. compound interest (round to two decimal places)

4. The value of an investment grows from R$2\ 200$ to R$3\ 850$ in 8 years. Determine the simple interest rate at which it was invested. Give your answer in % and round to two decimal places.

5. James had R$12\ 000$ and invested it for 5 years. If the value of his investment is R$15\ 600$, what compound interest rate did it earn? Give your answer in % and round to two decimal places.

6. Stephan invests R$15\ 000$ for 2 years at a compound interest rate of 8% per annum. Thereafter the interest rate changes to a compound interest rate of $7{,}5$% per annum for the next 3 years. Calculate the value of the investment after 5 years.

7. After 6 years' Thandi's investment has grown to an amount of R$10\ 000$. If the money she invested was invested at an interest rate of $5{,}9$% per annum compound interest, determine how much money she originally invested. Round to two decimal places.

8. Lebho invests a certain amount of money at an interest rate of 7% per annum simple interest. If after 5 years the investment is worth R$6\ 000$, how much did he originally invest? Round to two decimal places.

6.2 Exercise 2

1. A business buys a truck for R$560\ 000$. Over a period of 10 years the value of the truck depreciates to R0 using the straight-line method. What is the value of the truck after 8 years?

2. Harry wants to buy his grandpa's donkey for R 800. His grandpa is quite pleased with the offer, seeing that it only depreciated at a rate of 3% per year using the straight-line method. Grandpa bought the donkey 5 years ago. What did grandpa pay for the donkey then? Round to two decimal places.

3. Seven years ago, Rocco's drum kit cost him R 12 500. It has now been valued at R 2 300. What rate of simple depreciation does this represent? Give your answer in % and round to two decimal places.

4. Fiona buys a DStv satellite dish for R 3 000. Due to weathering, its value depreciates simply at 15% per annum. After how many years will the satellite dish have a book value of zero? Round to the nearest integer.

5. Cornell invests R$5\ 000$ into a savings account at an interest rate of $6{,}2$% per annum compounded monthly for the first 3 years. After the first 3 years the interest rate changes changes to $4{,}9$% per annum compounded quarterly. How much is her investment worth after 6 years? Round to two decimal places.

6. CJ takes out a loan of R$20\ 000$. He has to repay the loan in 8 years as one lump sum. The interest rate for the first 3 years is 5% per annum compounded semi-annually. After 3 years the interest rate goes down by $0{,}5$% for the next 2 years. Thereafter the interest rate goes up by $0{,}25$% for the remainder of the term. Determine how much CJ has to repay after the 8 year period. Round to two decimal places.

7. Phillane decides to start saving for her son's university studies. Her son needs to have R$45\ 000$ saved up in 7 years' time to pay for his first year of university fees. She starts by investing R30000 into a savings account at an interest rate of $7{,}2$% per annum compounded quarterly for the first 3 years. Thereafter the interest rate changes to $7{,}5$% per annum compounded monthly. Will Phillane have enough money to pay for her sons' first year of university? Round to two decimal places.

8. Marius invests R$75\ 000$ into a saving account at an interest rate of $6{,}7$% per annum compounded monthly for the first 2 years. After 2 years the interest rate changes to $6{,}9$% per annum compounded quarterly for the next 2 years. Thereafter, the interest rate changes to $7{,}1$% per annum compounded semi-annually. How much will Marius have in his account after 7 years? Round to two decimal places.

9. After saving up for 9 years, Armand has R$100\ 000$ saved up. When he invested the money the interest rate was $5{,}25$% per annum compounded quarterly for the first 4 years, thereafter the interest rate changed to $5{,}9$% per annum compounded monthly. How much did Armand initially invest? Round to two decimal places.

10. Marco has an amount of R$55\ 000$ saved up. 6 years ago, he invested an amount of money at an interest rate of 9% per annum compounded semi-annually. After 2 years the interest rate changed to 10% per annum compounded monthly for the next 3 years. For the final year the interest rate increased to $10{,}5$% per annum compounded monthly. What amount did Marco initially invest? Round to two decimal places.

11. Christine invests R$1\ 500$ into a savings account. After 3 years she withdraws R500 from the account. The interest rate for the first 2 years in 10% per annum compounded semi-annually. Thereafter, the interest rate changes to 9% per annum compounded quarterly. How much does Christine have in total after 5 years? Round to two decimals.

12. Robin invest R$5\ 000$. After 2 years she adds R$1\ 000$ and another year after that she withdraws R$1\ 500$ from the account. If the interest rate is $8{,}5$% per annum compounded monthly, how much does she have in total after 6 years? Round to two decimals.

13. Levi invests R$4\ 500$. 18 months later he adds R500 to the investment. Another 12 months after that another R500 is invested. The interest rate for the first 2 years is 8% per annum compounded semi-annually, thereafter the interest rate changes to $8{,}5$% per annum compounded semi-annually. How much does Levi have after 4 years? Round to two decimal places.

14. Cassandra has R$10\ 000$ saved up after 6 years. After saving up for 2 years, Cassandra withdrew R$1\ 000$ from the account. If the interest rate for the first 3 years was $6{,}9$% per annum compounded quarterly and changed to $5{,}7$% per annum compounded monthly after that, determine how much Cassandra initially invested. Round to two decimals.

6.3 Exercise 3

1. Cornell invests R$5\ 000$ into a savings account at an interest rate of $6,2$% per annum compounded monthly for the first 3 years. After the first 3 years the interest rate changes changes to $4,9$% per annum compounded quarterly. How much is her investment worth after 6 years?

2. CJ takes out a loan of R$20\ 000$. He has to repay the loan in 8 years as one lump sum. The interest rate for the first 3 years is 5% per annum compounded semi-annually. After 3 years the interest rate goes down by $0,5$% for the next 2 years. Thereafter the interest rate goes up by $0,25$% for the remainder of the term. Determine how much CJ has to repay after the 8 year period.

3. Phillane decides to start saving for her son's university studies. Her son needs to have R$45\ 000$ saved up in 7 years' time to pay for his first year of university fees. She starts by investing R30000 into a savings account at an interest rate of $7,2$% per annum compounded quarterly for the first 3 years. Thereafter the interest rate changes to $7,5$% per annum compounded monthly. Will Phillane have enough money to pay for her sons' first year of university?

4. Marius invests R$75\ 000$ into a saving account at an interest rate of $6,7$% per annum compounded monthly for the first 2 years. After 2 years the interest rate changes to $6,9$% per annum compounded quarterly for the next 2 years. Thereafter, the interest rate changes to $7,1$% per annum compounded semi-annually. How much will Marius have in his account after 7 years?

5. After saving up for 9 years, Armand has R$100\ 000$ saved up. When he invested the money the interest rate was $5,25$% per annum compounded quarterly for the first 4 years, thereafter the interest rate changed to 5.9% per annum compounded monthly. How much did Armand initially invest?

6. Marco has an amount of R$55\ 000$ saved up. 6 years ago, he invested an amount of money at an interest rate of 9% per annum compounded semi-annually. After 2 years the interest rate changed to 10% per annum compounded monthly for the next 3 years. For the final year the interest rate increased to $10,5$% per annum compounded monthly. What amount did Marco initially invest?

6.4 Exercise 4

1. Christine invests R$1\ 500$ into a savings account. After 3 years she withdraws R500 from the account. The interest rate for the first 2 years in 10% per annum compounded semi-annually. Thereafter, the interest rate changes to 9% per annum compounded quarterly. How much does Christine have in total after 5 years?

2. Robin invest R$5\ 000$. After 2 years she adds R$1\ 000$ and another year after that she withdraws R$1\ 500$ from the account. If the interest rate is $8,5\%$ per annum compounded monthly, how much does she have in total after 6 years?

3. Levi invests R$4\ 500$. 18 months later he adds R500 to the investment. Another 12 months after that another R500 is invested. The interest rate for the first 2 years is 8% per annum compounded semi-annually, thereafter the interest rate changes to $8,5\%$ per annum compounded semi-annually. How much does Levi have after 4 years?

4. Cassandra has R$10\ 000$ saved up after 6 years. After saving up for 2 years, Cassandra withdrew R$1\ 000$ from the account. If the interest rate for the first 3 years was $6,9\%$ per annum compounded quarterly and changed to $5,7\%$ per annum compounded monthly after that, determine how much Cassandra initially invested.

6.5 Exercise 5

1. Give all answers in % and round to one decimal place. Determine the effective annual interest rate if the nominal interest rate is:

   1.1 12% p.a. compounded quarterly.

   1.2 14,5% p.a. compounded weekly.

   1.3 20% p.a. compounded daily.

2. Consider the following:
   - 16,8% p.a. compounded annually.
   - 16,4% p.a. compounded monthly.
   - 16,5% p.a. compounded quarterly.

   2.1 Determine the effective annual interest rate of the 1st nominal rate listed above. Give your answer in % and round to one decimal place.

   2.2 Determine the effective annual interest rate of the 2nd nominal rate listed above. Give your answer in % and round to one decimal place.

   2.3 Determine the effective annual interest rate of the 3rd nominal rate listed above. Give your answer in % and round to one decimal place.

2.4 Which of these is the best interest rate for an investment? Give your answer in % and round to one decimal place.

2.5 Which is the best interest rate for a loan? Give your answer in % and round to one decimal place.

3. Calculate the effective annual interest rate equivalent to a nominal interest rate of $8{,}75$% p.a. compounded monthly. Give your answer in % and round to one decimal place.

4. Cebela is quoted a nominal interest rate of $9{,}15$% per annum compounded every four months on her investment of R$85\ 000$. Calculate the effective rate per annum. Give your answer in % and round to one decimal place.

5. Determine the effective rate per annum of the following rates. Give your answer in % and round to two decimal places.

   5.1 9,1% p.a. compounded quarterly

   5.2 9% p.a. compounded monthly

   5.3 9,3% p.a. compounded half-yearly

6. Miranda invests R 8 000 for 5 years for her son's study fund. Determine how much money she will have at the end of the period if the nominal interest of 6% is compounded as follows: (Round to two decimal places)

   6.1 Yearly

   6.2 Half-Yearly

   6.3 Quarterly

   6.4 Monthly

7 ANSWERS FOR EXERCISES

7.1 Exercise 1

1. R 13 630
2. R 10 246,59

3.1 R 2 536

3.2 R 2 468,27

4. 9,38%
5. 4,56%
6. R 21 735,23
7. R 7 089,64
8. R 4 444,44

7.2 Exercise 2

1. R 112 000
2. R 941,18
3. 11,66%
4. 7 years
5. R 6 066,17
6. R 29 187,03
7. R 50 116,12
8. R 129 932,35
9. R 60 476,76
10. R 30 813,89
11. R 1 783,85
12. R 7 780,81
13. R 7 329,32
14. R 5 995,07

7.3 Exercise 3

1. R 6 066,17
2. R 29 187,03
3. R 50 116,12
4. R 129 932,35
5. R 60 476,76
6. R 30 813,89

7.4 Exercise 4

1. R 1 783,85
2. R 7 780,81
3. R 7 329,32
4. R 5 995,07

7.5 Exercise 5

1.1 12,6%

1.2 15,5%

1.3 22,1%

2.1 16,8%

2.2 17,7%

2.3 17,5%

2.4 17,7% yields the highest return

2.5 16,8%

3. 9,1%

4. 9,4%

5.1 9,42%

5.2 9,38%

5.3 9,52%

6.1 R 10 705,80

6.2 R 10 751,33

6.3 R 10 774,84

6.4 R 10 790,80