CHAPTER 11

Numeric And Geometric Patterns

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CONTENTS

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1 NUMBER PATTERNS IN SEQUENCES

A set of numbers in a given order is called a **number sequence**. In some cases each number in a sequence can be formed from the previous number by performing the same or a similar action. In such a case, we can say there is a **pattern** in the sequence. The elements of a sequence are called **terms** and are in order, which means they follow on from each other. Terms that follow one another are said to be **consecutive**.
To make a number sequence we need the following:

- a number to start with
- a rule to make more terms: rules like $+4$ (add 4 to each term), $\times 2$ (multiply each term by 2) or $\div 2$ (divide each term by 2).

NOTE

Sometimes terms do not differ by a constant value or there is not a constant ratio between terms, but the difference between terms increases by a fixed amount.

A sequence can be formed by repeatedly adding or subtracting the same number. In this case the **difference** between one term and the next is constant.

A sequence can be formed by repeatedly multiplying or dividing by the same number. In this case the **ratio** between one term and the next is constant.

A sequence can also be formed in such a way that neither the difference nor the ratio between one term and the next is constant.

NOTE

The word "recur" means "to happen again". The extension of a number sequence by repeatedly performing the same or similar action is called **recursion**. The rule that describes the relationship between consecutive terms is called a **recursive rule**.

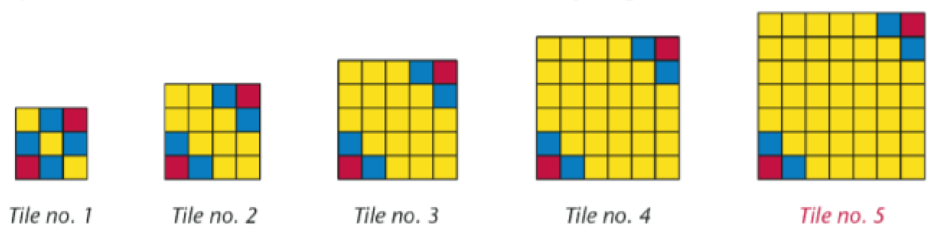
1.1 Relationships between dependent and independent variables

Look at the following example:
Mr Twala pays a fee to park his car in a parking lot every day. He has to pay R3 to enter the parking lot and then a further R2 for every hour that he leaves his car there.

In the above example, there is a relationship between the number of hours and the cost of the parking. The cost depends on the number of hours parked. These two quantities are the variables in the relationship.
The number of hours is the **independent variable**, and the parking cost is the **dependent variable** because the amount that needs to be paid *depends* on the number of hours parked.

2 GEOMETRIC PATTERNS

When we study the following picture, there are some conclusions we can make:



The number of red tiles is constant and the number of blue tiles is constant. It is clear that there is always a red tile in the top right corner, and also in the bottom left corner, and that the red tiles are always "bordered" by two blue tiles each. So the number of red and blue tiles is **constant** in this situation.
The number of yellow tiles in the arrangement varies. The number of yellow tiles is a **variable** in this situation.

NOTE

The words **geometric patterns** in this section refer to the drawings and do not necessarily mean the patterns are geometric sequences (where there is a common ratio between terms).

3 EXERCISES

3.1 Exercise 1

1. What may the next three numbers in each of these sequences be?

   1.1. $4;\ 8;\ 12;\ 16;\ 20;$ ...; ...; ...

   1.2. $4;\ 8;\ 16;\ 32;\ 64;$ ...; ...; ...

   1.3. $4;\ 8;\ 14;\ 22;\ 32;$ ...; ...; ...

   1.4. $5;\ 7;\ 4;\ 8;\ 3;\ 9;\ 2;$ ...; ...; ...

2. Write down the next number in each of these sequences:

   2.1. $4;\ 7;\ 10;\ 13;\ 16;$ ...; ...; ...

   2.2. $5;\ 10;\ 20;\ 40;\ 80;$ ...; ...; ...

   2.3. $2;\ 5;\ 10;\ 17;\ 26;$ ...; ...; ...

3. Write down the next five terms in each of the sequences below: In each case, describe the relationships between consecutive terms.

   3.1. $100;\ 95;\ 90;\ 85;$ ...; ...; ...; ...; ...

   3.2. $0,3;\ 0,5;\ 0,7;\ 0,9;$ ...; ...; ...; ...; ...

   3.3. $6;\ 18;\ 54;\ 162;$ ...; ...; ...; ...; ...

   3.4. $1;\ 3;\ 6;\ 10;\ 15;$ ...; ...; ...; ...; ...

   3.5. $20;\ 31;\ 42;\ 53;$ ...; ...; ...; ...; ...

   3.6. $10;\ 9,7;\ 9,4;\ 9,1;$ ...; ...; ...; ...; ...

   3.7. $18\,000;\ 1\,800;\ 180;\ 18;$ ...; ...; ...; ...; ...

   3.8. $\frac{1}{48};\ \frac{1}{24};\ \frac{1}{12};\ \frac{1}{6};$ ...; ...; ...; ...; ...

   3.9. $1;\ 4;\ 9;\ 16;$ ...; ...; ...; ...; ...

   3.10. $625;\ 125;\ 25;\ 5;$ ...; ...; ...; ...; ...

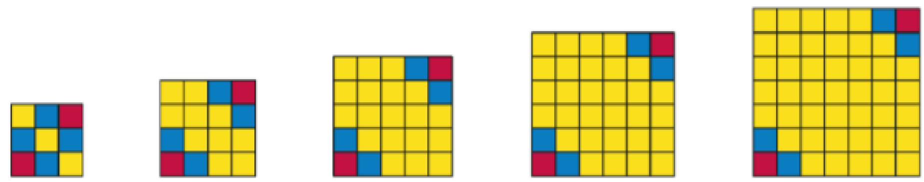
4. Complete the table:

Term Number	1	2	3	4	5	6	7	8	50
Term	15	19	23	27	31				

   4.1. How did you calculate term number 50?

3.2 Exercise 2

1. Study the figure and answer the questions:



Tile no. 1 *Tile no. 2* *Tile no. 3* *Tile no. 4* *Tile no. 5*

1.1. Complete the following table:

	Tile no. 1	Tile no. 2	Tile no. 3	Tile no. 4	Tile no. 5	Tile no. 10
No. of yellow tiles						
No. of red tiles						
No. of blue tiles						

1.2. How many red tiles are there in each bigger tile?

1.3. How many yellow tiles are there in each bigger tile? Some of the quantities in this situation are variables and some are constants. Which are variables and which are constants?

1.4. Was it possible to predict the pattern on tile no.2 by looking at tile no.1.

3.3 Exercise 3

1. Study the pattern of matches shown below:

Figure 1 *Figure 2* *Figure 3*

1.1. Explain how the pattern is formed.

1.2. Complete the table:

Figure Number	1	2	3	4	5	6	7	8
Number of matches	3	5	7					

1.3. What rule did you use to complete the table?

1.4. How many matches are needed to form Figure number 9?

1.5. How many matches are needed to form Figure number 17? Explain.

1.6. Can you link the number of matches added each time to the number that you multiply by in the flow diagram?

2. Another pattern with matches is shown below:

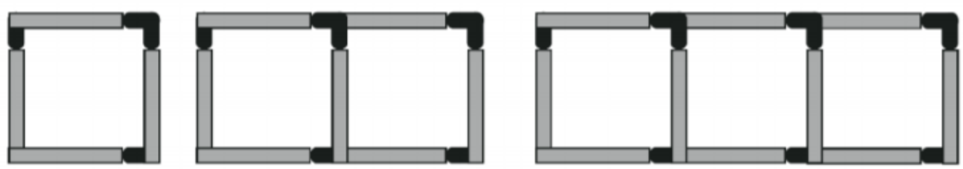


Figure 1 *Figure 2* *Figure 3*

1.1. Explain how the pattern is formed.

1.2. Complete the table:

Figure Number	1	2	3	4	5	6	7	8
Number of matches	4							

1.3. What rule did you use to complete the table?

1.4. How many matches are needed for Figure 9?

1.5. How many matches are needed for figure 20?

1.6. Compare the way in which the number of matches increases in question 1 to the way in which it increases in question 2. What is the same and what is different?

3.4 Exercise 4

1. Consider the figures below, formed with red dots:

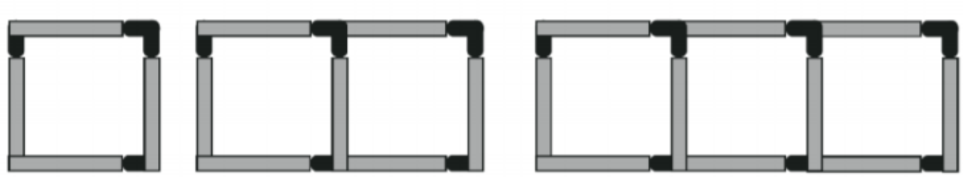


Figure 1 *Figure 2* *Figure 3*

1.1. How many dots are used to form Figure 5?

1.2. Complete the table:

Figure Number	1	2	3	4	5	6	7	8
Number of matches	7	12	17					

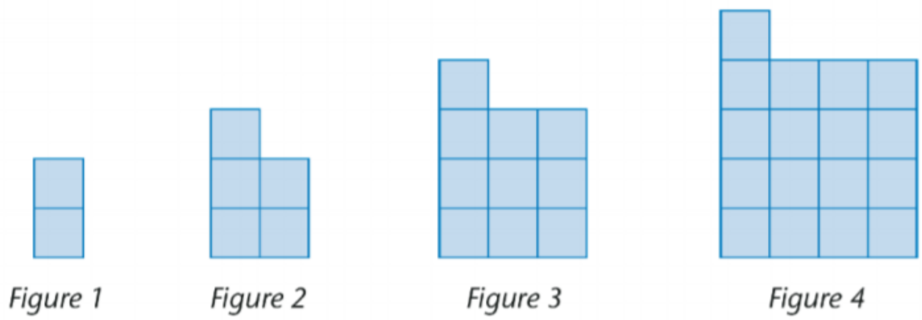
1.3. What rule did you use to complete the table? Describe your rule.

1.4. Can you think of another rule to complete the table? Describe your rule.

1.5. Name the dependent variable and the independent variable in this situation.

3.5 Exercise 5

1. Study the following set of figures and answer the questions:



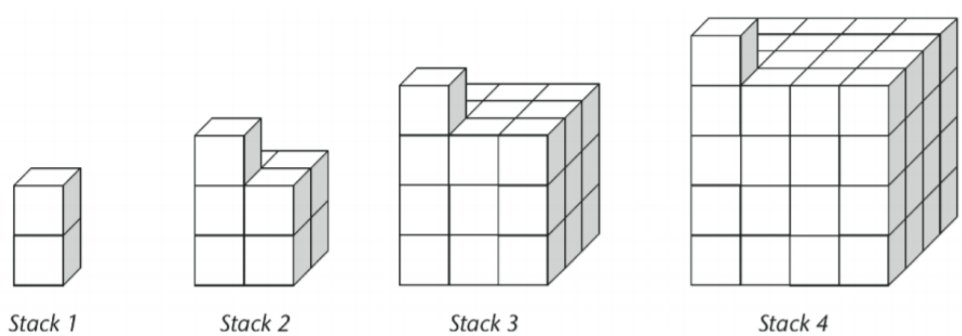
1.1. Complete the table

Figure Number	1	2	3	4	5	6	7	8
Number of matches	2	5						

1.2. Describe the recursive rule that you can use to extend the pattern in words.

1.3. Write a rule to calculate the number of squares for any figure number.

2. Identical cubes are arranged to form stacks of cubes in the following way:



2.1. Complete the table:

Figure Number	1	2	3	4	5	6	7	8
Number of matches	2	9	28					

2.2. Describe the way in which you completed the table.

2.3. David looked carefully at the structure of stacks and did the following to link the stack number with the number of cubes in a stack. Complete the pattern.

Stack $1 : 1 \times 1 \times 1 + 1 = 1 + 1 = 2$

Stack $2 : 2 \times 2 \times 2 + 1 = 8 + 1 = 9$

Stack $3 : 3 \times 3 \times 3 + 1 = 27 + 1 = 28$

Stack $4 : 4 \times 4 \times 4 + 1 = 64 + 1 = 2$

Stack 5 :...

Stack 6 :...

Stack 7 :...

Stack 8 :...

Stack 9 :...

Stack 10 :...

2.4. How many cubes will there be in stack 50?

2.5. Write the rule that you used to calculate the number of cubes in stack 50 in words.

2.6. Write your rule in the previous question in terms of *n* where *n* is the symbol for any stack number.