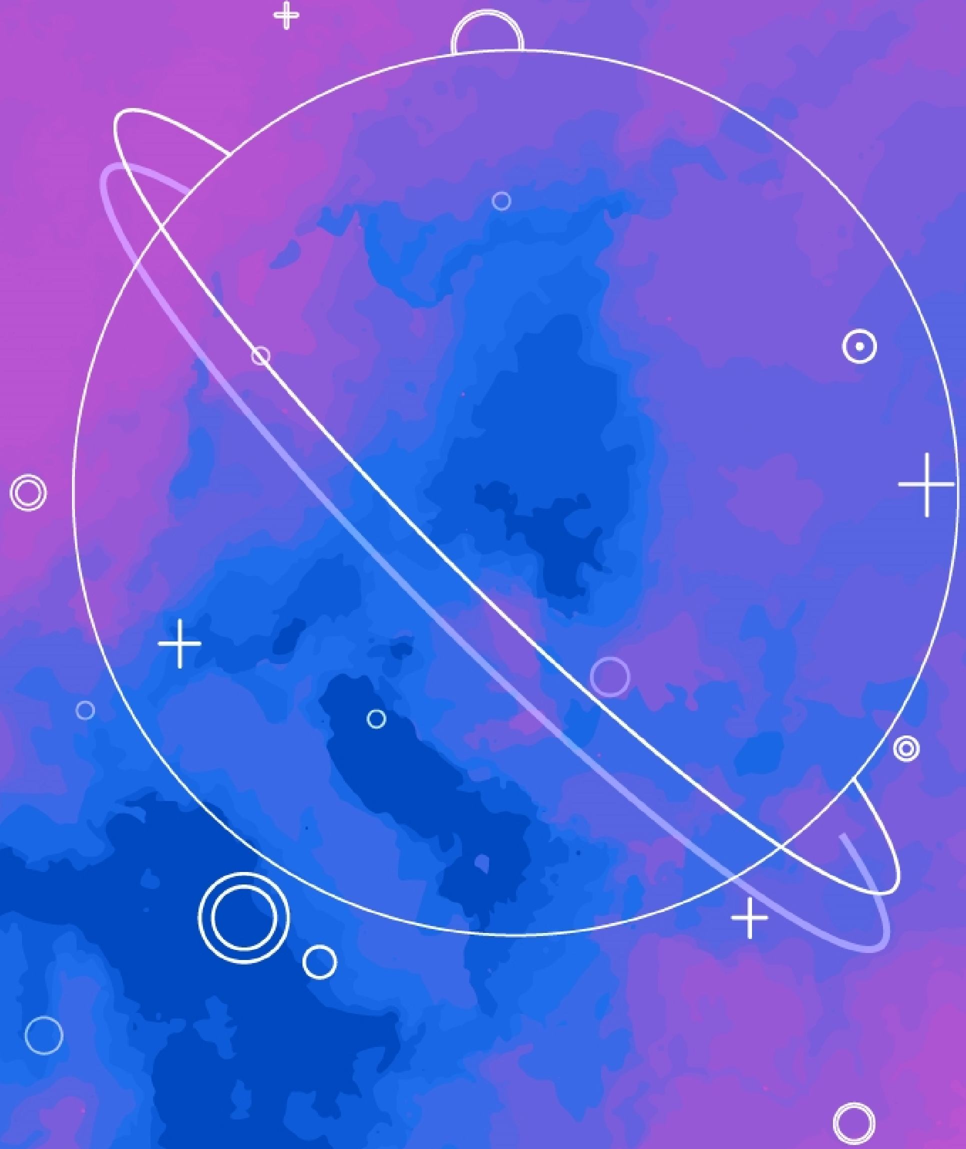

CHAPTER 14

Algebraic Equations 1

MathU Teaching® Emporium

CONTENTS

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1 SOLVING BY INSPECTION

1.1 Number Puzzles

A few word problems will be used to introduce you to equations and the need to solve an equation. Consider the following sentence:

A certain number plus 3 equals 13.

We would like to know what the unknown number is, but how can we do this? Writing the sentence using mathematical language can help us in solving this problem.

If we write the problem mathematically, we get:

$$\text{A certain number} + 3 = 13$$

This mathematical statement is called an equation.In an equation, the equal sign expresses equality; expressions written on either side of the equal sign have the same value. The words *a certain number* are just place holders for the unknown number that will make the sentence and equation true. We can use any letter or symbol as a place holder instead of *a certain number*. A common letter used, is x.

Using x instead of *a certain number*, we get:

$$x + 3 = 13$$

We can do this for many other problems too, that include multiplication, division and subtraction, not just addition. Examples can be seen below:

Number sentence	**Equation**
A certain number multiplied by 6 equals 30.	$x \times 6 = 30$
A certain number divided by 8 equals 2.	$x \div 8 = 2$
A certain number minus 5 equals 10.	$x - 5 = 10$
A certain number plus 7 equals 16.	$x + 7 = 16$

Table 1: Examples of other sentences written mathematically

We can also do this to problems that are tougher and have more than one operation, and any letter can be used as a placeholder, like for the following sentence:

A certain number is multiplied by 5 and 2 is added to the result to give 17

Written mathematically:

$$y \times 5 + 2 = 7$$

The placeholder, *a certain number* or x is called a **variable**. This is because the number that it represents or holds a place for, can change or vary to make the sentence true. Only one value for the variable (or x), will make the number sentence true. When the number sentence is true, we say that the value of the variable, is the solution to the equation.Solving an equation means working out the value of the unknown, for example x, that would make the equation true.

To understand this, lets go back to this number sentence:

A certain number plus 3 equals 13.

To make ths number sentence true, we need to find the value of x in the following equation, that makes the *left hand side* equal the *right hand side* in value:

$$x + 3 = 13$$

What this means is that if we perform all the operations on either side of the equals sign, both numbers should end up being the same. If we were to guess that x was the number 9, then we would have on the left hand side:

$$9 + 3$$

And if we added the two numbers together, we would get the following on the left hand side:

$$12$$

But we remember, the value on the *right hand side* of the equals sign, was actually 13, but we ended up with 12 on the *left hand side*. This means that the *left hand side* doe not equal the *right hand side*, and so 9 is not the solution.

What if we were to try the number 10 as the value of x? We would have:

$$10 + 3$$

Which would evaluate to the following on the *left hand side*:

$$13$$

And if we check again, we see that we end up with the value 13 on both the *left hand side* and *right hand side* of the equation. This means that the correct value of x is in fact 10, and is therefore the solution of the equation as it makes the number sentence true. This means that *a certain number* is actually 10. To summarise this:

Left hand side	**Equals**	**Right hand side**
$x + 3$	$=$	13
$10 + 3$	$=$	13
13	$=$	13

Table 2: Solving a number sentence

1.2 Using Inspection

A useful way to solve number sentences and equations is to solve them by **Inspection**. This means to solve the question immediately. We can do this first asking yourself the following question:

"What must x be so that the left hand side will have the same value as the right hand side?"

And to then determine the value of x without needing to do any working out by writing it out, it is all done by just *inspecting* the problem.

For example, consider the following equation:

$$x - 8 = 10$$

To solve this with inspection, we ask ourselves the following question in our head:

"What must x be so that the left hand side will have the same value as the right hand side?"

And to be more specific:

"What must x be so that when I subtract 8 from it, it's value is 10?"

If we are to think about the problem, it should quickly come to us that the value of x must be 18. This is because if you subtract 8 from 18, you end up with 10:

$$18 - 8 = 10$$

And so we now know that:

$$x = 18$$

EXAMPLE 1: INSPECTION METHOD

QUESTION

Solve the following equation using inspection:

$$x \times 7 = 21$$

SOLUTION

To solve this with inspection, we ask ourselves the following question in our head:

"What must x be so that when I multiply it by 7, it's value is 21?"

By inspection, we should be able to see that the value of x is 3, since multiplying 3 by 7 gives us 21.

$$3 \times 7 = 21$$

And so we now know that for this example:

$$x = 3$$

2 SOLVING BY THE TRIAL AND IMPROVEMENT METHOD

2.1 Trial and Improvement

Sometimes, you cannot see the solution to a number sentence or equation at once, and so we cannot use Inspection to solve the problem. This is true for longer problems like the following:

I am thinking of a number. $6 \times$ the number $- 11 = 43$. What is the number?

It is not easy to inspect this problem and quickly come up with the solution, so we must try another method called **Trial and Improvement**. With it, we try many different possible solutions until you find the correct one. An example of this method is shown below for the previously mentioned number sentence.

Possible solution	Test	Conclusion
Try 5	$6 \times 5 - 11 = 30 - 11 = 19$	5 is too small
Try 10	$6 \times 10 - 11 = 60 - 11 = 49$	10 is too big
Try 8	$6 \times 8 - 11 = 48 - 11 = 37$	8 is too small
Try 9	$6 \times 9 - 11 = 54 - 11 = 43$	9 is the solution

Table 3: An example on the Trial and Improvement method

TIP

Be careful when calling anything with variables, equations. Some things may be have variables, but are not considered equations. For example: $x \times 8 - 5$
This is called an *expression* and not an equation.
Equations have an equal sign in them, like this: $x \times 8 - 5 = 11$

EXAMPLE 2: TRIAL AND IMPROVEMENT METHOD

QUESTION
Solve the following problem using trial and improvement:

I am thinking of a number. The number $\div 4 + 13 = 17$. What is the number?

SOLUTION
Using the Trial and Improvement method:

Possible solution	Test	Conclusion
Try 12	$12 \div 4 + 13 = 3 + 13 = 16$	12 is too small
Try 20	$20 \div 4 + 13 = 5 + 13 = 18$	12 is too big
Try 16	$16 \div 4 + 13 = 4 + 13 = 17$	16 is the solution

2.2 Solving by Inspection or Trial and Improvement

It is important to be able to tell when you can solve a problem with either Inspection or Trial and Improvement. Fore example, look a the equation below:

$$x + 16 = 20$$

For an equation like this, we should be able to solve this with Inspection, as it can be quickly solved instead of trying Trial and Improvement. By Inspection, we can quickly see that the value of x is 4, since adding 16 to 4 would give us 20.

Now for another example:

$$k \times 4 = 20 - k$$

This is a much harder problem to solve, and the answer cannot be quickly seen using Inspection, and so it must be done using Trial and Improvement.

TIP

When a variable appears more than once in an equation, such as on either side of the equal sign, they will still have the same value. For example:

$$y + 3 = 9 - 2 \times y$$

If we were to use 2 as the value of y, it must be used for both sides like so:

$$2 + 3 = 9 - 2 \times 2$$
$$5 = 9 - 4$$
$$5 = 5$$

Using trial and improvement, we can find the solution as follows, where LHS and RHS refer to the Left Hand Side and Right Hand Side respectively:

Possible solution	Left Hand Side test	Right hand side test	LHS = RHS?	Conclusion
Try 2	$2 \times 4 = 8$	$20 - 2 = 18$	No	2 is not the solution
Try 5	$5 \times 4 = 20$	$20 - 5 = 15$	No	5 is not the solution
Try 3	$3 \times 4 = 12$	$20 - 3 = 17$	No	3 is not the solution
Try 4	$4 \times 4 = 16$	$20 - 4 = 16$	Yes	4 is the solution

Table 4: Using Trial and Improvement method instead of Inspection

3 DESCRIBING PROBLEMS WITH EQUATIONS

Word problems can be described using equations which would aid the process of solving them. Once written as an equation, we can solve the equation and find the answer to the word problem.

The steps used to write a word problem as an equation is as follows:

- Read the problem and think what it is you have to find.
- Write a variable to represent the quantity you have to find.
- Write down what the variable represents.
- Write an equation using the quantities given in the problem.
- Solve the equation.
- Check your solution.

Now that we have the steps to write a word problem as an equation, we can attempt some examples.

EXAMPLE 3: DESCRIBING AND SOLVING A WORD PROBLEM USING EQUATIONS

QUESTION

Consider the problem below:

Sam goes to a restaurant with 7 friends. Sam's friends order a burger for each of them, without checking the price. Sam then orders a pizza for himself which costs R40. If Sam and his friends had to pay R180 in total at the restaurant, how much did each burger cost?

SOLUTION

1. After reading the problem, we can see that we would like to find the cost of each burger.
2. We can choose any variable we would like to represent the price of each of the burgers, so let us use p for the price.
3. p represents the price of each burger.
4. To write the equation, we need to think about the problem:
    - The burger was ordered by 7 people, so it was ordered 7 *times*, so we know that in our equation we will have:

$$7 \times p$$

EXAMPLE 3: DESCRIBING AND SOLVING A WORD PROBLEM USING EQUATIONS (CONTINUED)

- We also know that a pizza was also ordered for R40, which would be added to the price of all the burgers, so we would then have:

$$7 \times p + 40$$

- Finally, we know that the food all came to a total of R180, which means that adding the prices of all the food together would give us the total of R180 which would give us our equation:

$$7 \times p + 40 = 180$$

5. Now that we have our equation, we can begin to solve it. Let us try using the Trial and Improvement method for this equation:

Possible solution	**Test**	**Conclusion**
Try 10	$7 \times 10 + 40 = 70 + 40 = 110$	10 is too small
Try 30	$7 \times 30 + 40 = 210 + 40 = 250$	30 is too big
Try 20	$7 \times 20 + 40 = 140 + 40 = 180$	20 is the solution

6. To finally check our solution, we can think of it in terms of the word problem:

- If each burger costed R20, which is the solution we came to, ordering 7 burgers, or ordering it 7 *times* would give us the following cost of all the burgers:

$$7 \times 20 = 140$$

- Adding the price of the pizza, which was R40, to the total of the burgers should give us the price of all the food:

$$140 + 40 = 180$$

- Now if we compare this to the total of what Sam and his friends had to pay, we see that both are the same value of R180.

Now that we have followed all of the steps, we can say that each burger cost Sam and his friends R20 each.

We can also use equations to make sense of problems, and so we must be able to interpret equations that describe a problem or situation knowing what each variable and constant represents. To understand this, let us try another example.

EXAMPLE 4: INTERPRETING EQUATIONS MADE FROM WORD PROBLEMS

<u>QUESTION</u>

Consider the following:

Jack's parents tell him that they will give him R10 if he helps his parents with the housework. They also tell him that they will give him an extra R5 for every hour he spends helping with housework. Jack helps with the housework and is then paid by his parents. Consider the equation below:

$$10 + 5 \times h = 25$$

We would like to know the following:

1. What does the letter h represent?
2. Why is h multiplied by 5?
3. What does the constant 10 represent?
4. What value of h makes the equation true?

<u>SOLUTION</u>

1. h is the number of hours that Jack helped for.
2. 5 is how much money Jack's parents give him for every hour of work.
3. 10 is the amount of money that Jack's parents give him fro helping for the day.
4. By inspection, we can see that the correct value for h is 3. This is because 3×5 is 15, and adding that to 10 gives us 25.

4 EXERCISES

4.1 Exercise 1

1. I am thinking of a certain number. If I add 3 to that number, the answer is 13. What is the number?
2. I am thinking of a certain number. If I multiply that number by 5, the answer is 30. What is the number?
3. I am thinking of a certain number. If I multiply that number by 3, and then add 4 to the result, the answer is 9.
   - 3.1 Is the number 3? Give a reason for your answer.
   - 3.2 Is the number 4? Give a reason for your answer.
   - 3.3 Is the number 5? Give a reason for your answer.
   - 3.4 Is the number 6? Give a reason for your answer.

4.2 Exercise 2

1. Solve these number sentences (equations) by inspection:
   - 1.1 $x - 8 = 8$
   - 1.2 $x + 7 = 20$
   - 1.3 $\frac{16}{x} = 8$
   - 1.4 $\frac{x}{16} = 2$
   - 1.5 $5 \times x = 40$
   - 1.6 $8 \times x = 40$
2. Solve these number sentences (equations) by inspection:
   - 2.1 $84 \div x = 7$
   - 2.2 $36 \div x = 4$
   - 2.3 $x + 56 = 100$
   - 2.4 $100 - x = 56$

4.3 Exercise 3

Copy the tables below. Solve the equations by means if the trial and improvement method. In each case, the solution is a number between 1 and 20.

1. $2 \times x + 13 = 37$

Possible solution	Test	Conclusion

2. $14 \times x - 21 = 77$

Possible solution	Test	Conclusion

3. $7 \times x + 8 = 71$

Possible solution	Test	Conclusion

4. $4 \times x + 7 = 31$

Possible solution	Test	Conclusion

5. $10 \times x + 11 = 141$

Possible solution	Test	Conclusion

4.4 Exercise 4

1. Solve the following equations by inspection or by the trial and improvement method:

   1.1 $x + 5 = 2 \times x$

   1.2 $k \times 5 = 20 + k$

   1.3 $2 \times q = 18 - q$

   1.4 $3 \times t = t + 22$

2. Solve the following equations by inspection or by the trial and improvement method:

   2.1 $y + 6 = 4 \times y$

   2.2 $5 \times p = 18 + 2 \times p$

   2.3 $4 \times z = 18 + z$

   2.4 $x \times 5 = 20$

   2.5 $42 \div m = 35 - 29$

   2.6 $3 \times x - 2 = x + 6$

4.5 Exercise 5

Write an equation using a letter symbol as a placeholder for the unknown number to describe the problem in each of the situations below.

1. There are 30 learners in a class. x learners are absent and 19 are present.
2. There are 70 passengers on a bus. At a bus stop m passengers get off. There are now 23 passengers on the bus.
3. A boy buys a bicycle for R1260 on lay-by. How many payments of R90 each must he make to pay for the bicycle? Let x be the number of payments to be made.
4. Five people share a total cost of R240 equally amongst themselves. Let c be the cost per person.
5. A school charges R100 a day for the use of its training facilities for athletes plus R30 per athlete per day for food and use of equipment. A team of athletes paid R400 for a day's practice. Let x be the number of athletes attending the training.
6. Bennie has R54 with which to buy chocolates for his friends. Each chocolate costs R6. How many chocolates can he buy for that amount? Let x be the number of chocolates that Bennie can buy.
7. Write an equation to calculate the area of a rectangle with length $2,5$ cm and breadth 2 cm. Let A represent the area of the rectangle.
8. There are 38 girls in Grade 7 . This is 6 more than double the number of boys.
9. Janine is 12 years old. Her father's age is 7 years plus three times Janine's age.

4.6 Exercise 6

1. Rajbansi Taxi Service charges R10 per kilometre travelled and a standard charge of R30 per trip. Consider the equation below about a taxi trip:

$$10 \times t + 30 = 80$$

   1.1 Explain what each number and letter symbol stands for in the equation.

   1.2 Why is t multiplied by 10 in the equation?

2. The cost of an adult's ticket for a music concert is four times the cost of a child's ticket. An adult's ticket costs R240. The equation below represents this problem:

$$4 \times x = 240$$

   2.1 What does x represent?

2.2 Why is x multiplied by 4?

2.3 Solve the equation by inspection.

2.4 How much does a child's ticket cost?

3. There are 12 eggs in a carton. Consider the equation below:

$$12 \times c = 72$$

3.1 What does the letter symbol c represent in the equation?

3.2 What value of c makes the equation true?

3.3 What does the number 72 represent?

5 ANSWERS TO EXERCISES

5.1 Exercise 1

1. 10
2. 6
3. No, because $3 \times 3 + 4 = 9 + 4 = 13$
3. No, because $3 \times 4 + 4 = 12 + 4 = 16$
3. Yes, because $3 \times 5 + 4 = 15 + 4 = 19$
3. No, because $3 \times 6 + 4 = 18 + 4 = 22$

5.2 Exercise 2

1.1 $x = 16$

1.2 $x = 13$

1.3 $x = 2$

1.4 $x = 32$

1.5 $x = 8$

1.6 $x = 5$

2.1 $x = 12$

2.2 $x = 9$

2.3 $x = 44$

2.4 $x = 44$

5.3 Exercise 3

1. $x = 37$
2. $x = 77$
3. $x = 71$
4. $x = 31$
5. $x = 141$

5.4 Exercise 4

1.1 $x = 5$

1.2 $k = 6$

1.3 $q = 6$

1.4 $t = 11$

2.1 $y = 2$

2.2 $p = 6$

2.3 $z = 6$

2.4 $x = 4$

2.5 $m = 7$

2.6 $x = 4$

5.5 Exercise 5

1. $30 - x = 19$ or $19 + x = 30$ or $30 - 19 = x$
2. $70 - m = 23$ or $m + 23 = 70$ or $70 - 23 = m$
3. $90 \times x = 1260$ or $x = 1260 \div 90$
4. $5 \times c = 240$ or $c = 240 \div 5$
5. $30 \times x + 100 = 400$ or $x = (400 - 100) \div 30$
6. $6 \times x = 54$ or $x = 54 \div 6$ or $54 \div x = 6$
7. $A = 2,5 \times 2$

8. $2 \times x + 6 = 38$

9. $x = 3 \times 12 + 7$ or $x = 7 + 3 \times 12$

5.6 Exercise 6

1.1 80 is the total amount charged for the trip.
t is the number of kilometres travelled.
10 is the cost per kilometre.
30 is the standard charge.

1.2 Because each kilometre travelled costs R10.

2.1 The cost of a child's ticket

2.2 Because the cost of an adult's ticket is four times that of a child's ticket.

2.3 $4 \times x = 240; 4 \times 60 = 240 \div 4$ so $x = 60$

2.4 R60

3.1 The number of cartons.

3.2 $c = 6$

3.3 The total number of eggs in c cartons.