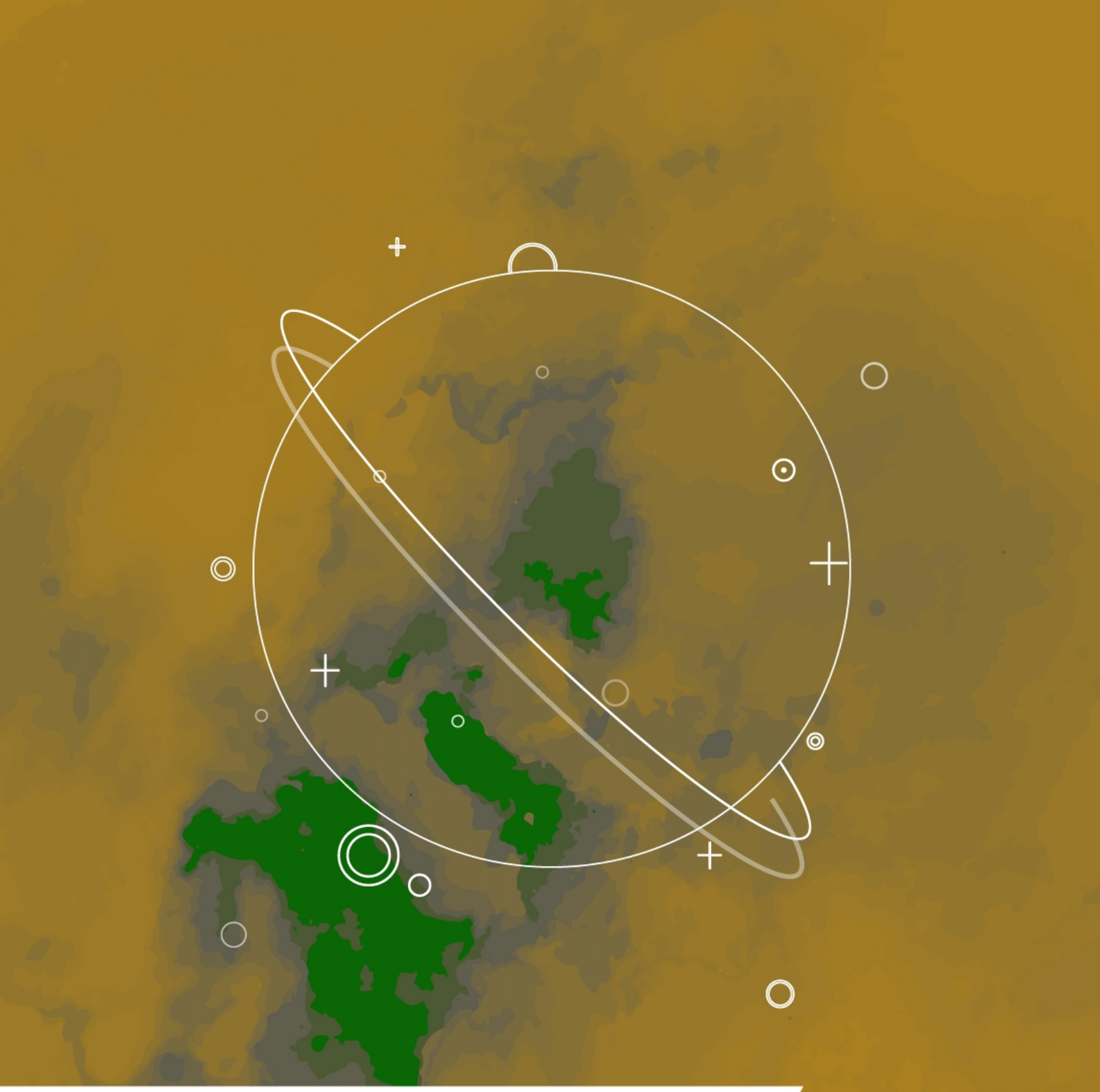


CHAPTER 2

Exponents

CONTENTS

October 19, 2021

In this chapter, you will learn about a very short way to describe calculations like this:

3×3

You already know a short way to describe calculations like this:

$3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3+3$

1 QUICK SQUARES AND CUBES

1.1 Again and again

Note

Instead of saying "ten times ten", we may say "ten squared" and we may write 10^2.

8 squared is 64, and 9 squared is 81.

Instead of saying "10 times 10 times 10", we may say "10 cubed" and we may write 10^3.

5 cubed is 125, and 9 cubed is 729.

2 THE EXPONENTIAL NOTATION

2.1 Repeated multiplication with the same number

Example

$250 = 2 \times 5 \times 5 \times 5$

Note

5 is a **repeated factor** of 250. It is repeated 3 times.

Note

A number that can be expressed as a product of one repeated factor is called a **power** of that number.

Example

32 is a power of 2, because $32 = 2 \times 2 \times 2 \times 2 \times 2$

$100\,000$ is a power of 10, because $10 \times 10 \times 10 \times 10 \times 10 = 100\,000$

Note

Because the factor 2 is repeated 5 times, 32 is called the **fifth power of** 2, or 2 **to the power** 5.

Similarly, 125 is the third power of 5.

125 can also be called "5 to the power 3" or "5 cubed".

Note

Instead of writing "5 to the power 6" we may write 5^6. This is called exponential notation.

5^6 means $5 \times 5 \times 5 \times 5 \times 5 \times 5$.

5×6 means $6 + 6 + 6 + 6 + 6$.

Note

3^5 means $3 \times 3 \times 3 \times 3 \times 3$.

The repeating factor in a power is called the **base**.

The number of repetitions is called the **exponent** or **index**.

3^1 means 3. The base is 3 but there is no repetition.

Any number raised to the power 1 equals the number itself.

2.2 Powers of different numbers

Note

Numbers that follow on each other in a pattern are called **consecutive numbers**.

3 SQUARES AND CUBES

Note

The number 9 is called the **square** of 3 because $3 \times 3 = 9$. The number 3, called the base, is multiplied by itself. 3^2 is read as **three squared** or **three to the power** 2.
The number 27 is called the **cube** of 3 because $3 \times 3 \times 3 = 27$. The base, the number 3, is multiplied by itself and again by itself. 3^3 is read as three cubed or **three to the power** 3.

3.1 Calculating squares and cubes

Squaring the number 2 means that we must multiply 2 by itself. It means we have to calculate 2×2, which has a value of 4, and we write $2 \times 2 = 4$.

Cubing the number 2 means that we must multiply 2 by itself, and again. It means we have to calculate $2 \times 2 \times 2$, which has a value of 8, and we write $2 \times 2 \times 2 = 8$.

The number 64 can be written both as a square and a cube. $64 = 8^2$ and $64 = 4^3$ The number 17 is neither a square nor a cube.

3.2 The square root and the cube root

Note

The number 3 is called the square root of the number 9 because $\sqrt{9} = \sqrt{3 \times 3} = 3$.
The number 3 is called the cubed root of 27 because $\sqrt[3]{27} = \sqrt[3]{3 \times 3 \times 3} = 3$.

3.3 Determining what number was squared

Note

The inverse to finding the square of a number is to find its **square root**.
The question, What is the square root of 25? is the same as the question, What number, when squared, equals 25? The answer to the question is 5 because $5 \times 5 = 25$.

3.4 Determining what number was cubed

Note

The inverse operation to finding the cube of a number is to find its **cube root**.
The question, What number, when cubed, equals 125? is the same as the question, What is the cube root of 125?
The answer to the question above is 5 because $125 = 5 \times 5 \times 5$.

3.5 Calculating square roots and cube roots

Note

The symbol $\sqrt{25}$ can be used to indicate the square root of 25. So we can write $\sqrt{25} = 5$. The symbol $\sqrt[3]{125}$ can be used to indicate the cube root of 125. So we can write $\sqrt[3]{125} = 5$.

By agreement amongst mathematicians, the symbol $\sqrt{}$ means the square root of the number that is written inside the symbol. So we normally write $\sqrt{4}$ instead of $\sqrt[2]{4}$. For the cube root, however, the number 3 outside of the root sign $\sqrt[3]{}$ must be written in order to distinguish the cube root from the square root.

4 COMPARING NUMBERS IN EXPONENTIAL FORM

4.1 Bigger, smaller or equal?

We can use mathematical symbols to indicate whether a number is bigger, smaller or has the same value as another number.

We use the symbol $>$ to indicate that the number on the left-hand side of the symbol is bigger than the one on the right-hand side. **The number 5 is bigger than 3 and we express this in mathematical language as $5 > 3$.**

The symbol $<$ is used to indicate that the number on the left-hand side of the symbol is smaller than the number on the right-hand side. **The number 3 is smaller than 5 and we express this mathematically as $3 < 5$.**

When numbers have the same value we use the equal sign, $=$. **The numbers 2^3 and 8 have the same value and we write this as $2^3 = 8$.**

Note

Two whole numbers that follow on each other, like 4 and 5, are called consecutive numbers. Is the difference between the squares of two consecutive whole numbers always an odd number?

Be smart when doing calculations

Our knowledge of squares can help us to do some calculations much quicker. Suppose you want to calculate 11×12.

11^2 has a value of 121. We know that $11 \times 11 = 121$.

11×12 means that there are 12 elevens in total.

So

$11 \times 12 = 11 \times 11 + 11$

$= 121 + 11$

$= 132$

Suppose you want to calculate 11×17.

$11 \times 17 = 17$ elevens in total $= 11$ elevens $+ 6$ elevens

Well, we know that $11 \times 11 = 121$

So

$11 \times 17 = 11 \times 11 + 6 \times 11$

$= 121 + 66$

$= 187$

4.2 Arranging numbers in ascending and descending order

The numbers $1, 4, 9, 16, 25, \ldots$ are arranged from the smallest to the biggest number. We say that the numbers $1, 4, 9, 16, 25, \ldots$ are arranged in **ascending order**.

The numbers $25, 16, 9, 4, 1, \ldots$ are arranged from the biggest to the smallest number. We say that the numbers $25, 16, 9, 4, 1, \ldots$ are arranged in **descending order**.

5 CALCULATIONS

5.1 The order of operations

When a numerical expression includes multiple operations, for example both multiplication and addition, what you do first makes a difference.

Note

It is important to know the **correct order** in which operations in a numerical expression should be done.

Note

If there are no brackets in a numerical expression, it means that **multiplication and division must be done first, and addition and subtraction only later**. For example, the expression $12 + 3 \times 5$ means "multiply 3 by 5; then add 12". It does not mean "add 12 and 3; then multiply by 5".

If you wish to specify that addition **should be done first** , that part of the expression should be **put in brackets**. For example ,if you wish to say " add 5 and 12; then multiply by 3", the numerical expression should be $3 \times (5 + 12)$ or $(5 + 12) \times 3$.

Here is another example: The expression $10–6 \div 3$ means "divide 6 by 3; then subtract the answer from 10". It does not mean " subtract 6 from 10; then divide by 3".If you wish to specify that subtraction should be done first, that part of the expression should be put in brackets. The numerical expression $(10–6) \div 3$ means " subtract 6 from 10; then divide the answer by 3".

6 EXERCISES

6.1 Exercise 1

1. How much is each of the following?

   1.1 2×2

   1.2 3×3

   1.3 4×4

   1.4 5×5

   1.5 6×6

   1.6 7×7

   1.7 8×8

   1.8 9×9

   1.9 10×10

   1.10 11×11

   1.11 12×12

   1.12 1×1

2. Copy and complete the following tables

   2.1

2×2
2^2
2 squared

   2.2

5^2
25

   2.3

10 cubed
100

2.4

12×12

2.5

4^2

2.6

8×8
64

2.7

11 squared
121

2.8

7^2

2.9

1×1

2.10

9×9
9^2

2.11

3 squared

2.12

36

3. 8 squared is 64 and 9 squared is 81

   3.1 What number squared is 25?

   3.2 What number squared is 100?

   3.3 What number squared is 64?

4. Calculate the following

   4.1 $10^2 + 5^2 + 2^2$

   4.2 $5 \times 10^2 + 7 \times 10 + 3$

   4.3 $7 \times 10^2 + 3 \times 10 + 6$

   4.4 $2 \times 10^2 + 9 \times 10 + 6$

5. How much is each of the following?

   5.1 $2 \times 2 \times 2$

   5.2 $3 \times 3 \times 3$

   5.3 $4 \times 4 \times 4$

   5.4 $5 \times 5 \times 5$

   5.5 $6 \times 6 \times 6$

   5.6 $7 \times 7 \times 7$

   5.7 $8 \times 8 \times 8$

   5.8 $9 \times 9 \times 9$

   5.9 $10 \times 10 \times 10$

   5.10 $11 \times 11 \times 11$

   5.11 $12 \times 12 \times 12$

   5.12 $13 \times 13 \times 13$

   5.13 $1 \times 1 \times 1$

6. Copy and complete the following tables like the example

$4 \times 4 \times 4$
4^3
4 cubed
64

6.1

$7 \times 7 \times 7$

6.2

11^3

6.3

2 cubed

6.4

216

6.5

1000

6.6

$8 \times 8 \times 8$

6.7

12 cubed

6.8

1

6.9

9^3

6.10

3 cubed

6.11

125

7. If 5 cubed is 125 and 9 cubed is 729, what is the answer to the following questions?

   7.1 What number cubed is 27?

   7.2 What number cubed is $1\ 000$?

   7.3 What number cubed is 8?

   7.4 What number cubed is 1?

   7.5 What number cubed is 216?

   7.6 What number cubed is 343?

8. Calculate the following

   8.1 $3 \times 10^3 + 7 \times 10^2 + 5 \times 10 + 6$

8.2 $7 \times 10^3 + 7 \times 10^2 + 7 \times 10 + 7$

8.3 $8 \times 10^3 + 1 \times 10^2 + 4 \times 10 + 2$

8.4 $4 \times 10^3 + 3 \times 10^2 + 4 \times 10 + 9$

8.5 10×10^2

8.6 $10^2 \times 10^2$

9. Can you think of two numbers, so that the square of the one number is equal to the cube of the other number?

10. Can you think of two numbers, so that when you add their squares, you get the square of another number?

11. Copy and complete the following tables by squaring each element in Set A and writing it as an element in Set B

11.1

Set A	Set B
$\{1; 2; 3; 4; 5; 6; 7; 8\}$	

11.2

Set A	Set B
$\{1; 3; 5; 7; 9; 11; 13\}$	

11.3

Set A	Set B
$\{10; 20; 30; 40; 50\}$	

11.4

Set A	Set B
$\{2; 4; 6; 8; 10; 12; 14\}$	

11.5

Set A	Set B
$\{5; 10; 15; 20; 25\}$	

11.6

Set A	Set B
$\{15; 12; 9; 6; 3\}$	

12. Cube the following numbers

12.1 $1, 2$ and 3

12.2 $5, 10$ and 4

13. Complete Set B by cubing each element in Set A for the following sets

13.1 Set A : $\{1; 2; 3; 4; 5; 6; 7; 8\}$

13.2 Set A : $\{10; 20; 30; 40; 50\}$

14. Complete the following questions

    14.1 Write down the squares of the first 15 natural numbers

    14.2 For the result of the square of the first 15 natural numbers, what do you observe about the last digit in each answer.

    14.3 Give a number that ends in $0, 1, 4, 5, 6$ or 9 that is not a square

15. Are the following numbers squares, cubes, both or neither?

    15.1 64

    15.2 1

    15.3 121

    15.4 $1\ 000$

    15.5 512

    15.6 400

    15.7 65

    15.8 216

    15.9 169

16. What number, when squared, equals 9?

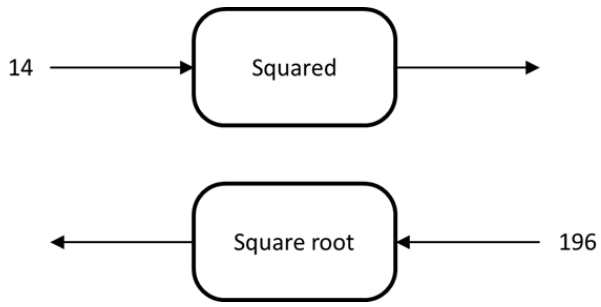
17. What is the square root of 49? Explain

18. What number, when squared, equals 81?

19. What number, when squared, equals 225?

20. What is the square root of 121?

21. Copy and complete the following flow diagram



22. What number, when cubed, equals 27?

23. What is the cube root of 343?

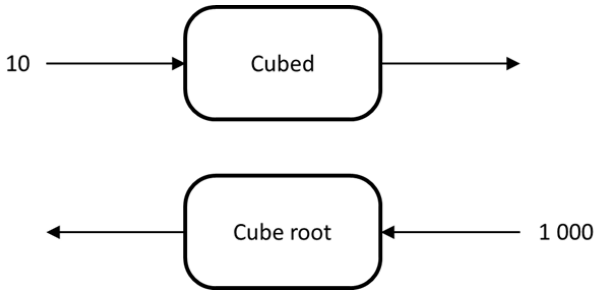
24. What number, when cubed, equals 8?

25. What is the cube root of $1\,000$?

26. What number, when cubed, equals 512?

27. What number produces the same answer when it is squared and when it is cubed?

28. Copy and complete the following diagram



6.2 Exercise 2

1. Express each number below as a product of prime factors. Example: $250 = 2 \times 5 \times 5 \times 5$

   1.1 35

   1.2 70

   1.3 140

   1.4 280

   1.5 81

   1.6 625

2. For the numbers given below, say which has repeated factors. In each case, state what number is repeated and how many times it is repeated.

35	**70**
140	**280**
81	**625**

3. Express each number as a power of $2, 3, 5$ or 10

   3.1 125

   3.2 64

   3.3 100

   3.4 $1\,000$

4. Calculate each of the following

   4.1 $2 \times 2 \times 2 \times 2$

   4.2 $2 \times 2 \times 2 \times 2 \times 2$

   4.3 $2 \times 2 \times 2 \times 2 \times 2 \times 2$

   4.4 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

   4.5 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

   4.6 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

   4.7 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

   4.8 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

   4.9 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

   4.10 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

5. The seventh power of 2 is shown by

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

What power of 2 is shown in each of the following parts

5.1 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

5.2 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

5.3 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

5.4 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

6. What power of what number is shown in each case below?

6.1 $15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15$

6.2 $12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12$

7. Write each of the numbers in question 3 in exponential notation

7.1 125

7.2 64

7.3 100

7.4 $1\,000$

8. Write each of the following in exponential notation

8.1 $2 \times 2 \times 2 \times 2$

8.2 $2 \times 2 \times 2 \times 2 \times 2$

8.3 $2 \times 2 \times 2 \times 2 \times 2 \times 2$

8.4 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

8.5 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

8.6 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

8.7 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

8.8 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

8.9 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

8.10 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

9. In each case write the number in exponential notation

9.1 The fifth power of 5

9.2 The sixth power of 5

9.3 The third power of 4

9.4 6 to the power of 4

9.5 4 to the power of 6

9.6 5 to the power of 15

10. In each case below, some information about a number is given. Each number can be expressed as a power. What is the number in each case?

10.1 The base is 5 and the index is 3

10.2 The base is 10 and the exponent is 4

10.3 The base is 20 and the exponent is 3

11. Calculate each of the following

11.1 $5 \times 5 \times 5$

11.2 $5 \times 5 \times 5 \times 5 \times 5$

11.3 $5 + 5 + 5$

11.4 $5 + 5 + 5 + 5 + 5$

11.5 5×3

11.6 5^3

12. Copy and complete the following tables

12.1

Exponent	1	2	3	4	5	6	7	8	9
Power of 2	2	4	8	16					

12.2

Exponent	10	11	12	13	14
Power of 2	1024				

13. Calculate each of the following

13.1 $2^2 - 2^1$

13.2 $2^3 - 2^2$

13.3 $2^4 - 2^3$

13.4 $2^5 - 2^4$

13.5 $2^6 - 2^5$

13.6 $2^7 - 2^6$

13.7 $2^8 - 2^7$

14. Describe what you notice about the differences between consecutive powers of 2

$$2^2 - 2^1 = 2$$
$$2^3 - 2^2 = 4$$
$$2^4 - 2^3 = 8$$
$$2^5 - 2^4 = 16$$

15. If you calculate the differences between consecutive powers of 3 as in $3^2 - 3^1, 3^3 - 3^2 \ldots$ Do you think these differences will be the consecutive powers of 3 again?

16. Copy and complete the following tables

16.1

Exponent	1	2	3	4	5
Power of 3	3	9			

16.2

Exponent	6	7	8	9
Power of 3	729			

16.3

Exponent	10	11	12	13	14
Power of 3	$59\,049$				

17. Calculate each of the following

17.1 $3^2 - 3^1$

17.2 $3^3 - 3^2$

17.3 $3^4 - 3^3$

17.4 $3^5 - 3^4$

17.5 $3^6 - 3^5$

17.6 $3^7 - 3^6$

17.7 $3^8 - 3^7$

18. Calculate each of the following and divide the answer by 2. Do you observe anything interesting?

$$3^2 - 3^1 =$$
$$3^3 - 3^2 =$$
$$3^4 - 3^3 =$$
$$3^5 - 3^4 =$$
$$3^6 - 3^5 =$$
$$3^7 - 3^6 =$$
$$3^8 - 3^7 =$$

19. Investigate the differences between consecutive powers of 4 for exponents of 1 to 10. Comment on what you observe

20. Investigate the differences between consecutive powers of 10 for exponents between 1 to 10

6.3 Exercise 3

1. Which one of the two is bigger?

   1.1 2^5 or 5^2

   1.2 3^4 or 4^3

   1.3 2^3 or 6^1

2. Use the symbols $=, <$ or $>$ to make the following true

   2.1 $\sqrt[3]{64} \ldots \sqrt{16}$

   2.2 $3^3 \ldots 4^2$

   2.3 $6 \ldots \sqrt{36}$

   2.4 $\sqrt[3]{125} \ldots \sqrt{100}$

   2.5 $3^3 \ldots \sqrt[3]{216}$

   2.6 $2^4 \ldots 3^4$

   2.7 $2^3 \ldots 3^2$

   2.8 $\sqrt[3]{1} \ldots \sqrt{1}$

   2.9 $9 \ldots 3^3$

   2.10 $100 \ldots 15^2$

3. Which is bigger, 1^{100} or 100^1? Explain.

4. What is the biggest number you can make with the numbers 4 and 2?

5. Two whole numbers that follow on each other, like 4 and 5 , are called consecutive numbers. Is the difference between the squares of two consecutive whole numbers always an odd number?

6. Arrange the following numbers in ascending order

   6.1 $\sqrt[3]{64}$; 3^2 ; $\sqrt{64}$; $\sqrt{36}$

   6.2 $\sqrt{225}$; $\sqrt[3]{729}$; $\sqrt[3]{1000}$; 2^2

   6.3 $\sqrt[3]{1}$; 0 ; 100 ; 10^3

   6.4 1^2 ; 2^3 ; 4^2 ; 5^2

7. Arrange the following numbers in descending order

7.1 $\sqrt[3]{216}$; $\sqrt[3]{10^3}$; 2^5 ; 20

7.2 10^3 ; $\sqrt[3]{20^3}$; $\sqrt{144}$; 12^2

7.3 $\sqrt{121}$; $\sqrt[3]{125}$; 11^2 ; 5^3

7.4 1^5 ; 2^4 ; 7^2 ; 6^3 ; 5^3

6.4 Exercise 4

1. Write each of the following numerical expressions in words

   1.1 $5 \times 2^2 + 3$

   1.2 $5^2 \times (2 + 3)^2$

   1.3 $\sqrt{36 + 64} + 3^3$

   1.4 $\sqrt{16} + \sqrt{9}$

   1.5 $10^3 - 9^3$

   1.6 $(18 \div \sqrt{9})^2$

   1.7 $\frac{26-\sqrt{4}}{6}$

2. Do the following calculation without a calculator

   2.1 $2^4 + 1^4$

   2.2 $(2 + 1)^4$

   2.3 $2^3 + 3^3 + 4^3$

   2.4 $2^3 + 5^3 \times 3$

   2.5 $12^2 \div 2^3$

   2.6 $\frac{12+2\times3^2}{4^2-1^3}$

3. Say whether the following has the same value as 2^5

   3.1 $2^3 + 2^2$

   3.2 $2^3 \times 2^2$

4. Determine if the following is equal to 5^4

   4.1 $5^3 + 5^1$

   4.2 $5^3 \times 5^1$

5. Determine if the following is equal to 8^4

   5.1 $2^4 \times 4^4$

   5.2 $8^3 \times 8$

6. Calculate the following

   6.1 $4^2 + 3^2$

   6.2 $12^2 + 5^2$

7. Answer the following question regarding the exponents of 2

   7.1 Continue this list to find the values of the "powers of 2^n" from 2^1 to 2^{12} :
   $2^1 = 2;\ 2^2 = 4;\ 2^3 = 8;\ 2^4 = 16$

   7.2 For the given list of exponents of 2, is there a pattern for the last digits of the values?
   $2^1 = 2\ ;\ 2^2 = 4\ ;\ 2^3 = 8\ ;\ 2^4 = 16\ ;\ 2^5 = 32$
   $2^6 = 64\ ;\ 2^7 = 128\ ;\ 2^8 = 256\ ;\ 2^9 = 512$
   $2^{10} = 1024\ ;\ 2^{11} = 2048\ ;\ 2^{12} = 4096$

   7.3 Using the values for the exponents of 2 , find the last digit for 2^{20}
   $2^1 = 2\ ;\ 2^2 = 4\ ;\ 2^3 = 8\ ;\ 2^4 = 16\ ;\ 2^5 = 32$
   $2^6 = 64\ ;\ 2^7 = 128\ ;\ 2^8 = 256\ ;\ 2^9 = 512$
   $2^{10} = 1024\ ;\ 2^{11} = 2048\ ;\ 2^{12} = 4096$

   7.4 Using the values for the exponents of 2, find the last digit for $2^{1\,002}$
   $2^1 = 2\ ;\ 2^2 = 4\ ;\ 2^3 = 8\ ;\ 2^4 = 16\ ;\ 2^5 = 32$
   $2^6 = 64\ ;\ 2^7 = 128\ ;\ 2^8 = 256\ ;\ 2^9 = 512$
   $2^{10} = 1024\ ;\ 2^{11} = 2048\ ;\ 2^{12} = 4096$

6.5 Exercise 5

1. Calculate the following without a calculator

   1.1 $\sqrt{64} + \sqrt{36}$

   1.2 $\sqrt{9 + 16}$

   1.3 $\sqrt{25}$

   1.4 $\sqrt{100}$

   1.5 $\sqrt{64 + 36}$

   1.6 $\sqrt{9} + \sqrt{16}$

2. Determine if the following is true or false and give a reason for your answer

   2.1 $\sqrt{64 + 36} = \sqrt{64} + \sqrt{36}$

   2.2 $\sqrt{16} + \sqrt{9} = \sqrt{16 + 9}$

   2.3 $\sqrt{100} = \sqrt{64} + \sqrt{36}$

   2.4 $\sqrt{25} \neq \sqrt{9} + \sqrt{16}$

2.5 $\sqrt{9 \times 9} = 9$

2.6 $\sqrt[3]{2 \times 2 \times 2} = 2$

2.7 $\sqrt{169} - \sqrt{25} = 8$

2.8 $\sqrt{169 - 25} = 12$

3. Calculate the following without a calculator

3.1 $2 + \sqrt[3]{8} + (3 + 2)^2$

3.2 $2 + \sqrt[3]{8} + 3^2 + 2^2$

3.3 $2 + \sqrt[3]{8} + 2^5 - 2^3$

3.4 $\frac{5+4\times\left(\sqrt{169}-2^3\right)}{5}$

3.5 $(15 - \sqrt{25})^3$

3.6 $\frac{28-24\div\sqrt{4}}{(\sqrt[3]{27}+1)^2}$

4. Complete the following tables as in the example

Number	Cube root	Check your answer
8	2	$2 \times 2 \times 2$

4.1

Number	Cube root	Check your answer
27		

4.2

Number	Cube root	Check your answer
64		

4.3

Number	Cube root	Check your answer
125		

4.4

Number	Cube root	Check your answer
216		

4.5

Number	Cube root	Check your answer
1 331		

4.6

Number	Cube root	Check your answer
1 000		

4.7

Number	Cube root	Check your answer
512		

4.8

Number	Cube root	Check your answer
8 000		

5. Complete the following tables as in the example

Number	Cube root	Check your answer
9	3	$3 \times 3 = 9$

5.1

Number	Cube root	Check your answer
1 600		

5.2

Number	Cube root	Check your answer
144		

5.3

Number	Cube root	Check your answer
196		

5.4

Number	Cube root	Check your answer
625		

5.5

Number	Cube root	Check your answer
900		

5.6

Number	Cube root	Check your answer
16		

5.7

Number	Cube root	Check your answer
400		

5.8

Number	Cube root	Check your answer
121		

6. What mathematic symbol can be used to represent the following?

6.1 The square root of 169

6.2 The cube root of 343

6.3 The square root of $2\ 500$

6.4 The cube root of 729

6.5 The cube of 25

6.6 The square of 25

7. Copy the following tables and complete them as was done in the example

	Value	Check your answer
$\sqrt{64}$	8	$8 \times 8 = 64$

7.1

	Value	Check your answer
$\sqrt{49}$		

7.2

	Value	Check your answer
$\sqrt{36}$		

7.3

	Value	Check your answer
$\sqrt{784}$		

7.4

	Value	Check your answer
$\sqrt{2\ 025}$		

7.5

	Value	Check your answer
$\sqrt{324}$		

8. Copy and complete the following tables as was done in the example

	Value	Check your answer
$\sqrt[3]{8}$	2	$2 \times 2 \times 2 = 8$

8.1

	Value	Check your answer
$\sqrt[3]{64}$		

8.2

	Value	Check your answer
$\sqrt[3]{512}$		

8.3

	Value	Check your answer
$\sqrt[3]{1}$		

8.4

	Value	Check your answer
$\sqrt[3]{216}$		

8.5

	Value	Check your answer
$\sqrt[3]{125}$		

6.6 Exercise 6

1. Write in expanded form 6^6
2. Write in exponential form 14 to the power 9
3. Rewrite the numbers from smallest to largest
   3^4 ; 2^5 ; 4^3 ; 10
4. Say whether the following is true or false. Explain your answer
   4.1 $\sqrt{64+36} = \sqrt{64} + \sqrt{36}$
   4.2 $\sqrt{25} + \sqrt{9} = \sqrt{59+5}$
5. Calculate the following
   5.1 $3^3 \times 2^2$
   5.2 $\sqrt{144} + \sqrt{81}$
   5.3 $11^2 + 5^2 - \sqrt{144}$
   5.4 $(14-12)^4 \div \sqrt[3]{8}$
   5.5 $9^2 - 4^2 \times 3$
   5.6 $7 + \sqrt[3]{125} + 1^4 - 2^3$
   5.7 $(\sqrt[3]{27} + \sqrt{64})^2$
   5.8 $\left(\sqrt{16+9} \div 5^1\right) \times 93$
   5.9 $\frac{9^2+12^2+5^3+650}{\sqrt[3]{125}\times 10^2}$
   5.10 $\frac{6^3-(\sqrt{169})^2+\sqrt[3]{8}}{7^2\times 1^9}$

7 ANSWERS FOR EXERCISES

7.1 Exercise 1

1.1 4

1.2 9

1.3 16

1.4 25

1.5 36

1.6 49

1.7 64

1.8 81

1.9 100

1.10 121

1.11 144

1.12 1

2.1

2×2
2^2
2 squared
4

2.2

5×5
5^2
5 squared
25

2.3

10×10
10^2
10 squared
100

2.4

12×12
12^2
12 squared
144

2.5

4×4
4^2
4 squared
16

2.6

8×8
8^2
8 squared
64

2.7

11×11
11^2
11 squared
121

2.8

7×7
7^2
7 squared
49

2.9

1×1
1^2
1 squared
1

2.10

9×9
9^2
9 squared
81

2.11

3×3
3^2
3 squared
9

2.12

6×6
6^2
6 squared
36

3.1 5

3.2 10

3.3 8

4.1 129

4.2 573

4.3 736

4.4 296

5.1 8

5.2 27

5.3 64

5.4 125

5.5 216

5.6 343

5.7 512

5.8 729

5.9 $1\,000$

5.10 $1\,331$

5.11 $1\,728$

5.12 $2\,197$

5.13 1

6.1

$7 \times 7 \times 7$
7^3
7 cubed
343

6.2

$11 \times 11 \times 11$
11^3
11 cubed
$1\,331$

6.3

$2 \times 2 \times 2$
2^3
2 cubed
8

6.4

$6 \times 6 \times 6$
6^3
6 cubed
216

6.5

$10 \times 10 \times 10$
10^3
10 cubed
1000

6.6

$8 \times 8 \times 8$
8^3
8 cubed
512

6.7

$12 \times 12 \times 12$
12^3
12 cubed
$1\ 728$

6.8

$1 \times 1 \times 1$
1^3
1 cubed
1

6.9

$9 \times 9 \times 9$
9^3
9 cubed
729

6.10

$3 \times 3 \times 3$
3^3
3 cubed
27

6.11

$5 \times 5 \times 5$
5^3
5 cubed
125

7.1 3

7.2 10

7.3 2

7.4 1

7.5 6

7.6 7

8.1 $3\ 756$

8.2 $7\ 777$

8.3 $8\,142$

8.4 $4\,349$

8.5 $1\,000$

8.6 $10\,000$

9 Numbers 8 and 4: the square of $8 = 64$ and the cube of $4 = 64$

10 $3^2 + 4^2 = 5^2$;
also $6^2 + 8^2 = 10^2$; $5^2 + 12^2 = 13^2$

11.1

Set A	Set B
$\{1; 2; 3; 4; 5; 6; 7; 8\}$	$\{1; 4; 9; 16; 25; 36; 49; 64\}$

11.2

Set A	Set B
$\{1; 3; 5; 7; 9; 11; 13\}$	$\{1; 9; 25; 49; 81; 121; 169\}$

11.3

Set A	Set B
$\{10; 20; 30; 40; 50\}$	$\{100; 400; 900; 1\,600; 2\,500\}$

11.4

Set A	Set B
$\{2; 4; 6; 8; 10; 12; 14\}$	$\{4; 16; 36; 64; 100; 144; 196\}$

11.5

Set A	Set B
$\{5; 10; 15; 20; 25\}$	$\{25; 100; 225; 400; 625\}$

11.6

Set A	Set B
$\{15; 12; 9; 6; 3\}$	$\{225; 144; 81; 36; 9\}$

12.1 $1^3 = 1$
$2^3 = 8$
$3^3 = 27$

12.2 $5^3 = 125$
$10^3 = 1\,000$
$4^3 = 64$

13.1 Set B : $\{1; 8; 27; 64; 125; 216; 343; 512\}$

13.2 Set B:$\{1\ 000; 8\ 000; 27\ 000; 64\ 000; 125\ 000\}$

14.1

$$\begin{aligned}
1^2 &= 1\\
2^2 &= 4\\
3^2 &= 9\\
4^2 &= 16\\
5^2 &= 25\\
6^2 &= 36\\
7^2 &= 49\\
8^2 &= 64\\
9^2 &= 81\\
10^2 &= 100\\
11^2 &= 121\\
12^2 &= 144\\
13^2 &= 169\\
14^2 &= 196\\
15^2 &= 225
\end{aligned}$$

14.2 The last digit is always either $0, 1, 4, 5, 6$ or 9

14.3 $20, 11, 14, 35, 26, 39\ldots$

15.1 Both

15.2 Both

15.3 Square

15.4 Cubed

15.5 Cubed

15.6 Square

15.7 Neither

15.8 Cubed

15.9 Square

16 3

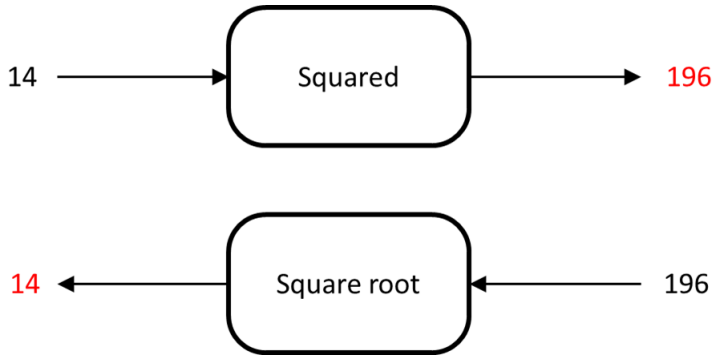
17 7

18 9

19 15

20 11

21 Completed diagram:



22 3

23 7

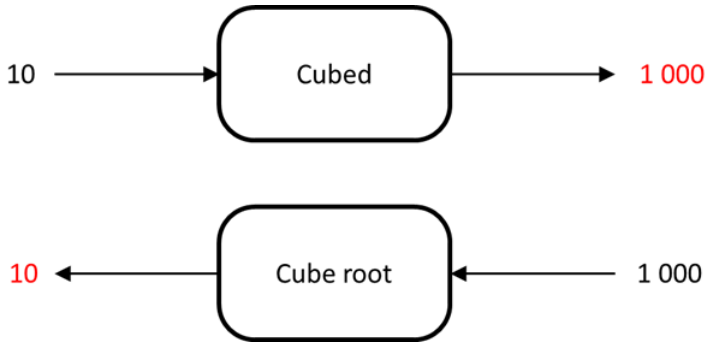
24 2

25 10

26 8

27 1 and 0

28 Completed figure:



7.2 Exercise 2

1.1 $35 = 5 \times 7$

1.2 $70 = 2 \times 5 \times 7$

1.3 $140 = 2 \times 2 \times 5 \times 7$

1.4 $280 = 2 \times 2 \times 2 \times 5 \times 7$

1.5 $81 = 3 \times 3 \times 3 \times 3$

1.6 $625 = 5 \times 5 \times 5 \times 5$

21 2 is repeated two times as a factor of 140
2 is repeated three times as a factor of 280
3 is repeated four times as a factor of 81
5 is repeated four times as a factor of 625

3.1 $125 = 5 \times 5 \times 5$

3.2 $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

3.3 $100 = 10 \times 10$

3.4 $1\,000 = 10 \times 10 \times 10$

4.1 16

4.2 32

4.3 64

4.4 128

4.5 256

4.6 512

4.7 $1\,024$

4.8 $2\,048$

4.9 $4\,096$

4.10 $8\,192$

5.1 Thirteenth power

5.2 Twelfth power

5.3 Eleventh power

5.4 Ninth power

6.1 Eighth power of 15

6.2 Twelfth power or 12

7.1 5^3

7.2 2^6

7.3 10^2

7.4 10^3

8.1 2^4

8.2 2^5

8.3 2^6

8.4 2^7

8.5 2^8

8.6 2^9

8.7 2^{10}

8.8 2^{11}

8.9 2^{12}

8.10 2^{13}

9.1 5^5

9.2 5^6

9.3 4^3

9.4 6^4

9.5 4^6

9.6 5^{15}

10.1 $5^3 = 125$

10.2 $10^4 = 10\,000$

10.3 $20^3 = 8\,000$

11.1 125

11.2 $3\,125$

11.3 15

11.4 25

11.5 15

11.6 125

12.1

Exponent	1	2	3	4	5	6	7	8	9
Power of 2	2	4	8	16	32	64	128	256	512

12.2

Exponent	10	11	12	13	14
Power of 2	1 024	2 048	4 096	8 192	16 384

13.1 2

13.2 4

13.3 8

13.4 16

13.5 32

13.6 64

13.7 128

14 They are the powers of 2 again.

15 No

16.1

Exponent	1	2	3	4	5
Power of 3	3	9	27	81	243

16.2

Exponent	6	7	8	9
Power of 3	729	2 187	6 561	19 683

16.3

Exponent	10	11	12	13	14
Power of 3	59 049	177 147	531 441	1 594 323	4 782 969

17.1 6

17.2 18

17.3 54

17.4 162

17.5 486

17.6 $1\,458$

17.7 $4\,374$

18. The answers is all powers of 3

19. All the answers is 3 times a power of 4

20. The difference between the numbers are 9 times a power of 10

7.3 Exercise 3

1.1 2^5

1.2 3^4

1.3 2^3

2.1 $\sqrt[3]{64} = \sqrt{16}$

2.2 $3^3 > 4^2$

2.3 $6 = \sqrt{36}$

2.4 $\sqrt[3]{125} < \sqrt{100}$

2.5 $3^3 > \sqrt[3]{216}$

2.6 $2^4 < 3^4$

2.7 $2^3 < 3^2$

2.8 $\sqrt[3]{1} = \sqrt{1}$

2.9 $9 < 3^3$

2.10 $100 < 15^2$

3. 100^1 is bigger, because $1^{100} = 1$ and $100^1 = 100$

4. 4^2

5. Yes

6.1 $\sqrt[3]{64}; \sqrt{36}; \sqrt{64}; 3^2$

6.2 $2^2; \sqrt[3]{729}; \sqrt[3]{1000}; \sqrt{225}$

6.3 $0; \sqrt[3]{1}; 100; 10^3$

6.4 $1^2; 2^3; 4^2; 5^2$

7.1 $2^5; 20; \sqrt[3]{10^3}; \sqrt[3]{216}$

7.2 $10^3; 12^2; \sqrt[3]{20^3}; \sqrt{144}$

7.3 $5^3; 11^2; \sqrt{121}; \sqrt[3]{125}$

7.4 $6^3; 5^3; 7^2; 2^4; 1^5$

7.4 Exercise 4

1.1 Multiply 2 with itself and then with 5; then add 3

1.2 Add 2 and 3 together then square the answer; multiply it by 5 twice or 25

1.3 Add 36 and 64; then take the square root of the answer. Add this to 3 multiplied by itself two times $(3\times3\times3)$

1.4 Take the square root of 16 and the square root of 9 and add them together

1.5 Multiply 10 by 10 and 10 again. Multiply 9 by 9 and 9 again. Subtract your second answer from your first answer

1.6 Divide 18 by the square root of 9 and then multiply the answer with itself

1.7 26 minus the square root of 4 and divide the answer by 6

2.1 17

2.2 81

2.3 99

2.4 378

2.5 18

2.6 2

3.1 It is not equal since it is 12

3.2 Yes it is equal

4.1 No it is not equal since it is 130

4.2 Yes they are equal

5.1 Yes they are equal

5.2 Yes they are equal

6.1 25

6.2 169

7.1 $2^1 = 2; 2^2 = 4; 2^3 = 8; 2^4 = 16; 2^5 = 32;$
$2^6 = 64; 2^7 = 128; 2^8 = 256; 2^9 = 512;$
$2^{10} = 1024; 2^{11} = 2048; 2^{12} = 4096$

7.2 The last digit cycles through the values $2; 4; 8$ and 6

7.3 6

7.4 6

7.5 Exercise 5

1.1 14

1.2 5

1.3 5

1.4 10

1.5 10

1.6 7

2.1 $\sqrt{64 + 36} = 10$
$\sqrt{64} + \sqrt{36} = 14$
$\therefore$ False

2.2 $\sqrt{16} + \sqrt{9} = 7$
$\sqrt{16 + 9} = 5$
$\therefore$ False

2.3 $\sqrt{100} = 10$
$\sqrt{64} + \sqrt{36} = 14$
$\therefore$ False

2.4 $\sqrt{25} = 5$

$\sqrt{9} + \sqrt{16} = 7$

$\therefore$ True

2.5 $\sqrt{9 \times 9} = 9$

$\therefore$ True

2.6 $\sqrt[3]{2 \times 2 \times 2} = 2$

$\therefore$ True

2.7 $\sqrt{169} - \sqrt{25} = 8$

$\therefore$ True

2.8 $\sqrt{169 - 25} = 12$

$\therefore$ True

3.1 29

3.2 17

3.3 28

3.4 5

3.5 $1\,000$

3.6 1

4.1

Number	Cube root
27	3

4.2

Number	Cube root
64	4

4.3

Number	Cube root
125	5

4.4

Number	Cube root
216	6

4.5

Number	Cube root
1 331	11

4.6

Number	Cube root
1 000	10

4.7

Number	Cube root
512	8

4.8

Number	Cube root
$8\ 000$	20

5.1

Number	Square root
$1\ 600$	40

5.2

Number	Square root
144	12

5.3

Number	Square root
196	14

5.4

Number	Square root
625	25

5.5

Number	Square root
900	30

5.6

Number	Square root
16	4

5.7

Number	Square root
400	20

5.8

Number	Square root
121	11

6.1 $\sqrt{169}$

6.2 $\sqrt[3]{343}$

6.3 $\sqrt{2\ 500}$

6.4 $\sqrt[3]{729}$

6.5 25^3

6.6 25^2

7.1

	Value
$\sqrt{49}$	7

7.2

	Value
$\sqrt{36}$	6

7.3

	Value
$\sqrt{784}$	28

7.4

	Value
$\sqrt{2\,025}$	45

7.5

	Value
$\sqrt{324}$	18

8.1

	Value
$\sqrt[3]{64}$	4

8.2

	Value
$\sqrt[3]{512}$	8

8.3

	Value
$\sqrt[3]{1}$	1

8.4

	Value
$\sqrt[3]{216}$	6

8.5

	Value
$\sqrt[3]{125}$	5

7.6 Exercise 6

1 $6 \times 6 \times 6 \times 6 \times 6 \times 6$

2. 14^9

3. $10; 2^5; 4^3; 3^4$

4.1 $\sqrt{64 + 36} = 10$
$\sqrt{64} + \sqrt{36} = 14$
$\therefore$ False

4.2 $\sqrt{25} + \sqrt{9} = 8$
$\sqrt{59 + 5} = 8$
$\therefore$ True

5.1 108

5.2 21

5.3 134

5.4 8

5.5 33

5.6 5

5.7 121

5.8 93

5.9 2

5.10 1