CHAPTER 1

Whole Numbers

CONTENTS

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1 PROPERTIES OF WHOLE NUMBERS

1.1 The commutative property of addition and multiplication

Note

We say: **addition and multiplication are commutative**. The numbers can be swopped around and their order does not change the answer. This does not work for subtraction and division, however.

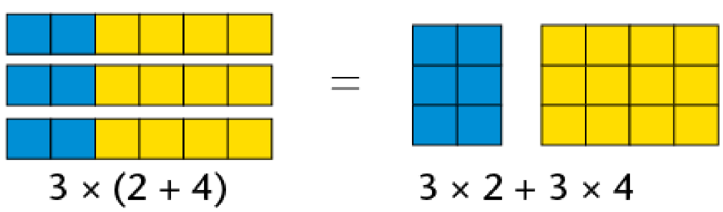
1.2 The associative property of addition and multiplication

Note

If three or more numbers have to be multiplied, it does not matter which two of the numbers are multiplied first.
This is called the **associative property of multiplication**. We also say **multiplication is associative**.

1.3 More conventions and the distributive property

The distributive property is a useful property because it allows us to do this:



Both answers are 18. Notice that we have to use brackets in the first example to show that the addition operation must be done first. Otherwise, we would have done the multiplication first. For example, the expression $3 \times 2 + 4$ means "multiply 3 by 2; then add 4". It does not mean "add 2 and 4; then multiply by 3".

The expression $4 + 3 \times 2$ also means "multiply 3 by 2; then add 4".

Note

If you wish to specify that addition or subtraction should be **done first**, that part of the expression should be enclosed **in brackets**.

The distributive property can be used to break up a difficult multiplication into smaller parts. For example, it can be used to make it easier to calculate 6×204:

6×204 can be rewritten as $6 \times (200 + 4)$ (Remember the brackets!)

$$\begin{aligned} &= 6 \times 200 + 6 \times 4 \\ &= 1200 + 24 \\ &= 1224 \end{aligned}$$

Multiplication can also be distributed over subtraction, for example to calculate 7×96:

$$\begin{aligned} 7 \times 96 &= 7 \times (100 - 4) \\ &= 7 \times 100 - 7 \times 4 \\ &= 700 - 28 \\ &= 672 \end{aligned}$$

Note

Two more properties of numbers are:

- **The additive property of 0**: when we add zero to any number, the answer is that number.
- **The multiplicative property of 1**: when we multiply any number by 1, the answer is that number.

2 CALCULATIONS WITH WHOLE NUMBERS

2.1 Estimating, approximating and rounding

To estimate is to try to get close to an answer without actually doing the required calculations with the given numbers.

Note

An estimate may also be called an **approximation**.

Note

The difference between an estimate and the actual answer is called the **error**.

Calculating with "easy" numbers that are close to given numbers is a good way to obtain approximate answers, for example:

- To approximate $764 + 829$ one may calculate $800 + 80$ to get the approximate answer $1\ 600$, with an error of 7.
- To approximate 84×178 one may calculate 80×200 to get the approximate answer $16\ 000$, with an error of $1\ 048$.

2.2 Rounding off and compensating

Note

The word **compensate** means to do things that will remove damage.

By rounding the numbers off you introduced errors. You then compensated for the errors by making adjustments to your answer.

For example, to calculate R $5\ 362$–R $2\ 687$, you may round R$2\ 687$ up to R$3\ 000$. The calculation can proceed as follows:

- Rounding R $2\ 687$ up to R $3\ 000$ can be done in two steps: $2\ 687 + 13 = 2\ 700$, and $2\ 700 + 300 = 3\ 000$. In total, 313 is added.
- 313 can now be added to $5\ 362$ too: R $5\ 362 + 313 = 5\ 675$.
- Instead of calculating R $5\ 362$–R $2\ 687$, which is a bit difficult, you may calculate R $5\ 675$–R $3\ 000$. This is easy: R $5\ 675$–R $3\ 000 =$ R $2\ 675$.

This means that R $5\ 362$–R $2\ 687 =$ R $2\ 675$, because
R $5\ 362$–R $2\ 687 = ($R $5\ 362 +$ R $313)$–$($R $2\ 687 +$ R $313)$.

2.3 Adding numbers in parts written in columns

Numbers can be added by thinking of their **parts** as we say the numbers. For example, we say $4\ 994$ as four thousand nine hundred and ninety-four. This can be written in expanded notation as $4\ 000 + 900 + 90 + 4$.

Similarly, we can think of $31\ 837$ as $30\ 000 + 1\ 000 + 800 + 30 + 7$.

$31\ 837 + 4\ 994$ can be calculated by working with the various kinds of parts separately. To make this easy, the numbers can be written below each other so that the units are below the units, the tens below the tens and so on, as shown above.

$31\ 837$
$4\ 994$

We write only this: In your mind you can see this:

31837	30000	1000	800	30	7
4994		4000	900	90	4

The numbers in each column can be added to get a new set of numbers.

31837	30000	1000	800	30	7
4994		4000	900	90	4
11					11
120				120	
1700			1700		
5000		5000			
30000	30000				

36831 It is easy to add the new set of numbers to get the answer.

The work may start with the $10\ 000$s or any other parts. Starting with the units as shown above makes it possible to do more of the work mentally, and write less, as shown below.

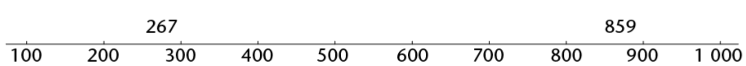
31837
$\underline{4994}$
36831

To achieve this, only the units digit 1 of the 11 is written in the first step. The 10 of the 11 is remembered and added to the 30 and 90 of the tens column, to get 130.

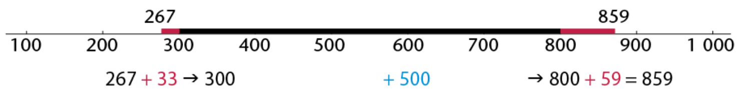
We say the 10 is **carried** from the units column to the tens column. The same is done when the tens parts are added to get 130: only the digit "3" is written (in the tens column, so it means 30), and the 100 is carried to the next step.

2.4 Methods of subtraction

There are many ways to find the difference between two numbers. For example, to find the difference between 267 and 859 one may think of the numbers as they may be written on a number line.



We may think of the distance between 267 and 859 as three steps: from 267 to 300, from 300 to 800, and from 800 to 859. How big are each of these three steps?



The above shows that $859 - 267$ is $33 + 500 + 59$.

Like addition, subtraction can also be done by working with the different parts in which we say numbers. For example, $8764 - 2352$ can be calculated as follows:

$$\begin{aligned} 8 \text{ thousand} - 2 \text{ thousand} &= 6 \text{ thousand} \\ 7 \text{ hundred} - 3 \text{ hundred} &= 4 \text{ hundred} \\ 6 \text{ tens} - 5 \text{ tens} &= 1 \text{ ten} \\ 4 \text{ units} - 2 \text{ units} &= 2 \text{ units} \end{aligned}$$

So, $8\ 764 - 2\ 352 = 6\ 412$

Subtraction by parts is more difficult in some cases, for example $6213 - 2758$:

$$\begin{aligned} 6000 - 2000 &= 4000 \text{ This step is easy, but the following steps cause problems:} \\ 200 - 700 &= ? \\ 10 - 50 &= ? \\ 3 - 8 &= ? \end{aligned}$$

Note

One way to overcome these problems is to work with negative numbers:

$$\begin{aligned} 200 - 700 &= (-500) \\ 10 - 50 &= (-40) \\ 3 - 8 &= (-5) \end{aligned}$$

Fortunately, the parts and sequence of work may be rearranged to overcome these problems, as shown below:

instead of				we may do				but write only this			
6000	200	10	3	5000	1100	100	13	6	2	1	3
2000	700	50	8	2000	700	50	8	2	7	5	8
				3000	400	50	5	3	4	5	5

With some practice, you can learn to subtract using borrowing without writing all the steps. It is convenient to work in columns, as shown below for calculating $6\ 213 - 2\ 758$.

$$\begin{array}{r} 6\ 213 \\ \underline{2\ 758} \\ 5 \\ 50 \\ 400 \\ \underline{3\ 000} \\ 3\ 455 \end{array}$$

In fact, by doing more work mentally, you may learn to save more paper by writing even less as shown below.

$$\begin{array}{r} 6\ 213 \\ \underline{2\ 758} \\ 3\ 455 \end{array}$$

2.5 A method of multiplication

$7 \times 4\ 598$ can be calculated in parts, as shown here:

$$\begin{aligned} 7 \times 4000 &= 28000 \\ 7 \times 500 &= 3500 \\ 7 \times 90 &= 630 \\ 7 \times 8 &= 56 \end{aligned}$$

The four partial products can now be added to get the answer, which is 32186. It is convenient to write the work in vertical columns for units, tens, hundreds and so on, as shown below.

$$\begin{array}{ccccc} & 4 & 5 & 9 & 8 \\ & & & & 7 \\ \hline & & & 5 & 6 \\ & & 6 & 3 & 0 \\ & 3 & 5 & 0 & 0 \\ 2 & 8 & 0 & 0 & 0 \\ \hline 3 & 2 & 1 & 8 & 6 \end{array}$$

The answer can be produced with less writing, by "carrying" parts of the partial answers to the next column, when working from right to left in the columns.

$$\begin{array}{r} 4\ 5\ 9\ 8 \\ 7 \\ \hline 3\ 2\ 1\ 8\ 6 \end{array}$$

Only the 6 of the product 7×8 is written down instead of 56. The 50 is kept in mind, and added to the 630 obtained when 7×90 is calculated in the next step.

2.6 Long division

The municipal gardener wants to work out exactly how many trees, at R27 each, he can buy with the budgeted amount of R9400. His thinking and writing are described below.

Step 1

What he writes:

$$27 \overline{)\,9400}$$

What he thinks: *I want to find out how many chunks of* 27 *there are in* $9\,400$ **Step** 2

What he writes:

$$\begin{array}{r|l} & 300 \\ \hline 27 & 9400 \\ & \underline{8100} \\ & 1300 \end{array}$$

What he thinks: *I think there are at least* 300 *chunks of* 27 *in* 9400. $300 \times 27 = 8100$. *I need to know how much is left over.*

Step 3 (He has to rub out the one "0" of the 300 on top, to make space.)

What he writes:

$$\begin{array}{r|l} & 340 \\ \hline 27 & 9400 \\ & \underline{8100} \\ & 1300 \\ & \underline{1080} \\ & 220 \end{array}$$

What he thinks: *I think there are at least* 40 *chunks of* 27 *in* 1300. $40 \times 27 = 1080$. *I need to know how much is left over. I want to find out how many chunks of* 27 *there are in* 220.*Perhaps I can buy some extra trees.*

Step 4 (He rubs out another "0".)

What he writes:

$$\begin{array}{r|l} & 348 \\ \hline 27 & 9400 \\ & \underline{8100} \\ & 1300 \\ & \underline{1080} \\ & 220 \\ & 216 \\ & 4 \end{array}$$

What he thinks: *I think there are at least* 8 *chunks of* 27 *in* 220. $8 \times 27 = 216$. *So, I can buy* 348 *young trees and will have R*4 *left.*

3 MULTIPLES, FACTORS AND PRIME FACTORS

3.1 Multiples and factors

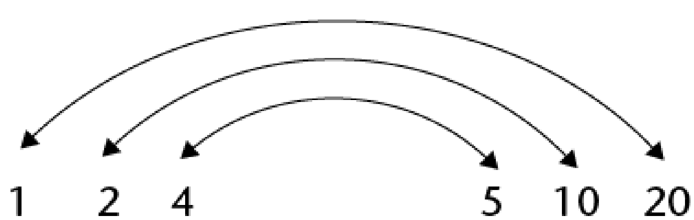
> **Note**
>
> If ***n*** is a natural number, $6n$ represents the multiples of 6.

Of which numbers is 20 a multiple?

$$20 = 1 \times 20 = 2 \times 10 = 4 \times 5 = 5 \times 4 = 10 \times 2 = 20 \times 1$$

> **Note**
>
> 20 is a multiple of 1; 2; 4; 5; 10 and 20 and all of these numbers are factors of 20.

Factors come in pairs. The following pairs are factors of 20:



3.2 Prime numbers and composite numbers

Note

The number 36 can be formed as $2 \times 2 \times 3 \times 3$. Because 2 and 3 are used twice, they are called **repeated factors** of 36.

Note

A number that cannot be expressed as a product of two whole numbers, except as the product $1\times$ the number itself, is called a **prime number**.

Note

Composite numbers are natural numbers with more than two different factors. The sequence of composite numbers is $4; 6; 8; 9; 10; 12; \ldots$

3.3 Prime factorisation

To find all the factors of a number you can write the number as the product of prime factors, first by writing it as the product of two convenient (composite) factors and then by splitting these factors into smaller factors until all factors are prime. Then you take all the possible combinations of the products of the prime factors.

Note

Every composite number can be expressed as the product of prime factors and this can happen in only one way.

Example: Find the factors of 84.

Write 84 as the product of prime factors by starting with different known factors:

$$\begin{aligned} 84 &= 4 \times 21 \\ &= 2 \times 2 \times 3 \times 7 \end{aligned}$$

or

$$\begin{aligned} 84 &= 7 \times 12 \\ &= 7 \times 3 \times 4 \\ &= 7 \times 3 \times 2 \times 2 \end{aligned}$$

or

$$\begin{aligned} 84 &= 2 \times 42 \\ &= 2 \times 6 \times 7 \\ &= 2 \times 2 \times 3 \times 7 \end{aligned}$$

A **more systematic way** of finding the prime factors of a number would be to start with the prime numbers and try the consecutive prime numbers 2; 3; 5; 7; ... as possible factors. The work may be set out as shown below.

2	1 430
5	715
11	143
13	13
	1

$$1\ 430 = 2 \times 5 \times 11 \times 13$$

3	2 457
5	819
11	273
13	91
13	13
	1

$$2\ 457 = 3 \times 3 \times 3 \times 7 \times 13$$

We can use exponents to write the products of prime factors more compactly as products of powers of prime factors.

$$\begin{aligned} 2457 &= 3 \times 3 \times 3 \times 7 \times 13 &= 3^3 \times 7 \times 13 \\ 72 &= 2 \times 2 \times 2 \times 3 \times 3 &= 2^3 \times 3^2 \\ 1500 &= 2 \times 2 \times 3 \times 5 \times 5 \times 5 &= 2^2 \times 3 \times 5^3 \end{aligned}$$

3.4 Common multiples and factors

We use **common multiples** when fractions with different denominators are added.

To add $\frac{2}{3} + \frac{3}{4}$ the common denominator is 3×4, so the sum becomes $\frac{8}{12} + \frac{9}{12}$

In the same way, we could use $6 \times 8 = 48$ as a common denominator to add $\frac{1}{6} + \frac{3}{8}$, but 24 is the **lowest common multiple (LCM)** of 6 and 8.

Prime factorisation makes it easy to find the lowest common multiple or highest common factor. When we simplify a fraction, we divide the same number into the numerator and the denominator. For the simplest fraction, use the **highest common factor (HCF)** to divide into both numerator and denominator.

Note

The HCF is divided into the numerator and the denominator to write the fraction in its **simplest form**.

So $\frac{36}{144} = \frac{2\times2\times3\times3}{2\times2\times2c2\times3\times3} = \frac{1}{4}$

Use prime factorisation to determine the LCM and HCF of 32, 48 and 84 in a systematic way:

$$\begin{aligned} 32 &= 2\times2\times2\times2\times2 &&= 2^5 \\ 48 &= 2\times2\times2\times2\times3 &&= 2^4\times3 \\ 84 &= 2\times2\times3\times7 &&= 2^2\times3\times7 \end{aligned}$$

The LCM is a **multiple**, so all of the factors of all the numbers must divide into it.

All of the factors that are present in the three numbers must also be factors of the LCM, even if it is a factor of only one of the numbers. But because it has to be the lowest common multiple, no unnecessary factors are in the LCM.

The highest power of each factor is in the LCM, because then all of the other factors can divide into it. In 32, 48 and 84, the highest power of 2 is 2^5, the highest power of 3 is 3 and the highest power of 7 is 7.

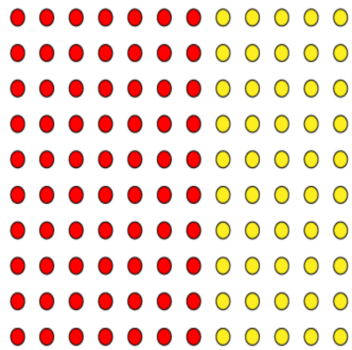
LCM $= 2^5\times3\times7 = 672$

The HCF is a common factor. Therefore, for a factor to be in the HCF, it must be a factor of *all* of the numbers. 2 is the only number that appears as a factor of all three numbers. The lowest power of 2 is 2^2, so the HCF is 2^2.

4 EXERCISES

4.1 Exercise 1

1. Consider the figure below:



1.1 Which of the following calculations would you choose to calculate the number of yellow beads in this pattern? Do not do any calculations now, just make a choice.

(a) $7 + 7 + 7 + 7 + 7$
(b) $10 + 10 + 10 + 10 + 10 + 10 + 10$
(c) $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5$
(d) $5 + 5 + 5 + 5 + 5 + 5 + 5$
(e) $7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7$
(f) $10 + 10 + 10 + 10 + 10$

1.2 How many red beads are there in the pattern?

1.3 How many yellow beads are there?

1.4 How many beads are there in the pattern in total?

1.5 Which expression describes what you did to calculate the total number of beads: $70 + 50$ or $50 + 70$? Does it make a difference?

1.6 Which expression describes what you did to calculate the number of red beads: 7×10 or 10×7? Does it make a difference?

2. Calculate:

   2.1 5×8

   2.2 10×8

   2.3 12×8

   2.4 8×12

   2.5 6×8

   2.6 3×7

   2.7 6×7

   2.8 7×6

4.2 Exercise 2

1. Lebogang and Nathi both have to calculate 25×24. Lebogang calculates 25×4 and then multiplies by 6. Nathi calculates 25×6 and then multiplies by 4.
   Will they get the same answers or not?

2. Calculate:
   Do not use a calculator.

   2.1 $4 + 7 + 5 + 6$

   2.2 $7 + 6 + 5 + 4$

   2.3 $6 + 5 + 7 + 4$

   2.4 $7 + 5 + 4 + 6$

3. Is addition associative?

4. Find the value of the expression by working in the easiest possible way:

   4. 1 $2 \times 17 \times 5$

   4. 2 $4 \times 7 \times 5$

   4. 3 $75 + 37 + 25$

   4. 4 $60 + 87 + 40 + 13$

5. What must you add to the number to get 100?

   5.1 82

5.2 44

5.3 56

5.4 78

5.5 24

5.6 86

5.7 77

6. What must you multiply the number by to get $1\ 000$?

6.1 250

6.2 125

6.3 25

6.4 500

6.5 200

6.6 50

7. Calculate:
Note that you can make the work very easy by deciding how to group the operations.

7.1 $82 + 54 + 18 + 46 + 237$

7.2 $24 + 89 + 44 + 76 + 56 + 11$

7.3 $25 \times (86 \times 4)$

7.4 32×125

4.3 Exercise 3

1. Here is an expression with the answer. Rewrite it with brackets to make the answer correct.

1.1 $8 + 6 \times 5 = 70$

1.2 $8 + 6 \times 5 = 38$

1.3 $5 + 8 \times 6 - 2 = 52$

1.4 $5 + 8 \times 6 - 2 = 76$

1.5 $5 + 8 \times 6 - 2 = 51$

1.6 $5 + 8 \times 6 - 2 = 37$

2. Calculate:

2.1 $100 \times (10 + 7)$

2.2 $100 \times 10 + 100 \times 7$

2.3 $100 \times (10 - 7)$

2.4 $100 \times 10 - 100 \times 7$

3. Copy and Complete the table:

X	**8**	**5**	**4**	**9**	**7**	**3**	**6**	**2**	**10**	**11**	**12**
7											
3											
9											
5											
8											
6											
4											
2											
10											
12											
11											

4. Use the various mathematical conventions for numerical expressions to make the calculation easier. Show how you work them out.

4.1 18×50

4.2 125×28

4.3 39×220

4.4 $443 + 2\,100 + 557$

4.5 $318 + 650 + 322$

4.6 $522 + 3\,003 + 78$

4.4 Exercise 4

1. Copy and complete the following statement by giving the answer to the question, without doing any calculations with the given number

1.1 Is 8×117 more than $2\,000$ or less than $2\,000$?

1.2 Is 27×88 more than $3\,000$ or less than $3\,000$?

1.3 Is 18×117 more than $3\,000$ or less than $3\,000$?

1.4 Is 47×79 more than $3\,000$ or less than $3\,000$?

2. The numbers $1\ 000, 2\ 000, 3\ 000, 4\ 000, 5\ 000, 6\ 000, 7\ 000, 8\ 000, 9\ 000$ and $10\ 000$ are all multiples of a thousand. Write down the multiple of $1\ 000$ that you think is closest to the answer. The number you write down is called an estimate.

   2.1 8×117

   2.2 27×88

   2.3 18×117

   2.4 47×79

3. Calculate the error between the true value and the given estimate. (The difference between the actual answer and an estimate is called the error)

   3.1 8×117 estimated as $1\ 000$

   3.2 27×88 estimated as $2\ 000$

   3.3 18×117 estimated as $2\ 000$

   3.4 47×79 estimated as $4\ 000$

4. Estimate the answers for each of the following products and sums. Try to approximate the answers for the products to the nearest thousand, and for the sums to the nearest hundred.

   4.1 84×178

   4.2 $677 + 638$

   4.3 124×93

   4.4 $885 + 473$

   4.5 79×84

   4.6 $921 + 367$

   4.7 56×348

   4.8 $764 + 829$

5. Use a calculator to find the exact answer for the question and calculate the error in the approximation.

   5.1 84×178 estimated as $15\ 000$

   5.2 $677 + 638$ estimated as $1\ 300$

   5.3 124×93 estimated as $12\ 000$

   5.4 $885 + 473$ estimated as $1\ 300$

   5.5 79×84 estimated as $7\ 000$

   5.6 $921 + 367$ estimated as $1\ 300$

   5.7 56×348 estimated as $19\ 000$

5.8 $764 + 829$ estimated as $1\ 600$

6. Calculating with "easy" numbers that are close to given numbers is a good way to obtain approximate answers, for example: -To approximate $764 + 829$, you may calculate $800 + 800$ to get the approximate answer $1\,600$; with an error of 7. To approximate 84×178, you may calculate 80×200 to get the approximate answer $16\ 000$; with an error of $1\ 048$. Calculate with "easy" numbers close to the given numbers to produce an approximate answer for the product. Do not use a calculator. When you have made your approximation, use a calculator to work out the precise answer.

   6.1 78×46

   6.2 67×88

   6.3 34×276

   6.4 78×178

7. Consider $386 + 3\,435$

   7.1 Approximate the answer for $386 + 3\,435$, by rounding both numbers off to the nearest hundred, and then adding the rounded numbers.

   7.2 When you round 386 up to 400, you introduce an error of 14 into your approximate answer. What error do you introduce by rounding $3\,435$ down to $3\,400$?

   7.3 What combined (total) error do you introduce by rounding both numbers off before calculating?

   7.4 Use the total error caused by rounding both numbers off, to find the correct answer

8. Round off and compensate to calculate the following accurately:

   8.1 $473 + 638$

   8.2 $677 + 921$

4.5 Exercise 5

1. Calculate, without using a calculator:

   1.1 $4\,638 + 2\,667$

   1.2 $748 + 7\,246$

2. Impilo Enterprises plans a new computerized training facility in their existing building. The training manager has to keep the total expenditure budget under R 1 million. This is what she has written so far:

Architects and builders	R $102\ 700$
Painting and carpeting	R $42\ 600$
Security doors and blinds	R $52\ 000$
Data projector	R $4\ 800$
25 new secretary chairs	R $50\ 400$
24 desks for work stations	R $123\ 000$
1 desk for presenter	R $28\ 000$
25 new computers	R $300\ 000$
12 colour laser printers	R $38\ 980$

3. Calculate, without using a calculator:

    3.1 $7\ 828 + 6\ 284$

    3.2 $7\ 826 + 888 + 367$

    3.3 $657 + 32\ 890 + 6\ 542$

    3.4 $6\ 666 + 3\ 333 + 1$

4. Calculate $33 + 500 + 59$ to find the answer for $859 - 267$.

5. Calculate the following.
You may think of working out the distance between the two numbers - or use any other method you prefer.
Do not use a calculator

    5.1 $823 - 456$

    5.2 $1\ 714 - 829$

    5.3 $3\ 045 - 2\ 572$

    5.4 $5\ 131 - 367$

6. Calculate:
$6\ 213 - 2\ 758$

7. Calculate the following:

    7.1 $7\ 342 - 3\ 877$

    7.2 $8\ 653 - 1\ 856$

    7.3 $5\ 671 - 4\ 528$

8. Estimate the difference between the two car prices, to the nearest R$1\ 000$ or closer, then calculate the difference.

    8.1 R$102\ 365$ and R$98\ 128$

    8.2 R$63\ 378$ and R$96\ 889$

9. First estimate the answer to the nearest $100\ 000$, $10\ 000$ or $1\ 000$. Then calculate accurately.

    9.1 $238\ 769 - 141\ 453$

    9.2 $856\ 333 - 739\ 878$

    9.3 $65\ 244 - 39\ 427$

4.6 Exercise 6

1. Calculate the following.
   Do not use a calculator.

    1.1 27×649

    1.2 $75 \times 1\ 756$

    1.3 348×93

2. Calculate the following.
   Do not use a calculator.

    2.1 67×276

    2.2 84×178

3. The municipal head gardener wants to buy young trees to plant along the main street of the town. The young trees cost R 27 each, and an amount of R $9\ 400$ has been budgeted for trees.

    3.1 He needs 324 trees. Do you think he has enough money?

    3.2 How much will 300 trees cost?

    3.3 How much money will be left if 300 trees are bought?

    3.4 The gardener buys 300 trees but decides he will need more. How much money will be left if 20 more trees are bought?

4. Determine the following. Do not use a calculator.

    4.1 Graham bought 64 goats, all at the same price. He paid R$5\ 440$ in total. What was the price for each goat? You can start by working out how much he would have paid if he paid R10 per goat. You can start with a bigger step if you wish.

    4.2 Mary has R$2\ 850$ and she wants to buy candles for her sister's wedding reception. The candles cost R48 each. How many candles can she buy?

5. Calculate the following.
   Do not use a calculator.

   5.1 $7\ 234 \div 48$

   5.2 $3\ 267 \div 24$

   5.3 $9\ 500 \div 364$

   5.4 $8\ 347 \div 24$

4.7 Exercise 7

1. The numbers $6; 12; 18; 24; \ldots$ are multiples of 6
   The numbers $7; 14; 21; 28; \ldots$ are multiples of 7

   1.1 What is the hundredth number in each sequence?

   1.2 Is 198 a number in the first sequence?

   1.3 Is 175 a number in the second sequence?

2. A rectangle has an area of 30 cm. What are the possible lengths of the sides of the rectangle in centimeters if the lengths of the sides are natural numbers?

3. Are $4, 8, 12$ and 16 factors of 48?

4. Simon says that all multiples of 4 smaller than 48 are factors 48. Is he right?

5. Consider the number 210

   5.1 Are $2, 3, 5$ and 7 factors of 210?

   5.2 Are $2 \times 3, 3 \times 5, 5 \times 7, 2 \times 5$ and 2×7 factors of 210?

   5.3 Are $2 \times 3 \times 5, 3 \times 5 \times 7$ and $2 \times 5 \times 7$ factors of 210?

6. Is 20 a factor of 60 ? What factors of 20 are also factors of 60?

7. Express the number as a product of as many factors as possible, including repeated factors. Do not use 1 as a factor.

   7.1 66

   7.2 67

   7.3 68

   7.4 69

   7.5 70

   7.6 71

7.7 72

7.8 73

8. Which of the following numbers cannot be expressed as a product of two whole numbers, except $1 \times$ the number itself
(A number that cannot be expressed as a product of two whole numbers, except as the product $1 \times$ the number itself, is called a prime number.)
$66; 67; 68; 69; 70; 71; 72; 73$

9. Is the statement True or False? If your answer is "False", explain why.

9.1 All prime numbers are odd numbers.

9.2 All composite numbers are even numbers.

9.3 1 is a prime number

9.4 If a natural number is not prime, then it is composite.

9.5 2 is a composite number.

9.6 785 is a prime number.

9.7 A prime number can only end in $1, 3, 7$ or 9.

9.8 Every composite number is divisible by at least one prime number.

10. We can find out if a given number is prime by systematically checking whether the primes $2; 3; 5; 7; 11; 13; \ldots$ are factors of the given number or not.
To find possible factors of 131 , we need to consider only the primes $2; 3; 5; 7$ and 11 . Why not $13; 17; 19; \ldots?$

11. Determine whether the number is prime or composite. If the number is composite, write down at least two factors of the number (besides 1 and the number itself).

11.1 221

11.2 713

12. Express the number as the product of powers of primes:

12.1 792

12.2 444

13. Find the prime factors of the number:

    13.1 28

    13.2 32

    13.3 124

    13.4 36

    13.5 42

    13.6 345

    13.7 182

4.8 Exercise 8

1. Is 4×5 a multiple of 4? is 4×5 a multiple of 5?

2. Comment on the following statement: The product of numbers is a multiple of each of the numbers in the product.

3. Determine the LCM (Lowest Common Denominator) and the HCF (Highest Common Factor) of:

    3.1 $24; 28; 42$

    3.2 $17; 21; 35$

    3.3 $75; 120; 200$

    3.4 $18; 30; 45$

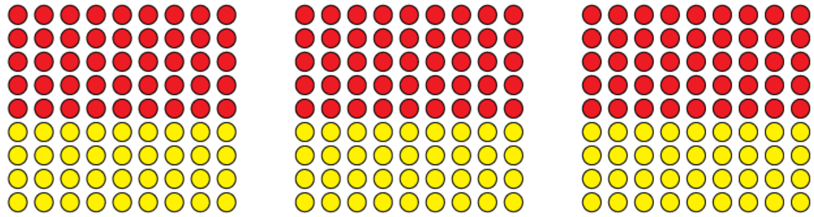
4.9 Exercise 9

1. Tree plantations in the Western Cape are to be cut down in favour of natural vegetation. There are roughly $3\ 000\ 000$ trees on plantations in the area and it is possible to cut them down at a rate of $15\ 000$ trees per day with the labour available. How many working days will it take before all the trees will be cut down?

2. 1 car travels a distance of 180 km in two hours on a straight road. How many kilometres can it travel in three hours at the same speed?

3. Thobeka wants to order a book that costs $56{,}67$ dollars.
   The rand-dollar exchange rate is R$13{,}79$ to a dollar. What is the price of the book in rands?

4. In pattern A below, there are five red beads for every four yellow beads. Describe patterns B and C in the same way.



Pattern A Pattern B Pattern C

5. A company has two machines that produce screws in different periods of time.

   5.1 Copy and complete the following table:

Number of hours	1	2	3	5	8
Number of screws at machine A	1 800				
Number of screws at thachine B	2 700				

   5.2

Number of hours	1	2	3	5	8
Number of screws at machine A	1 800	3 600	5 400	9 000	14 400
Number of screws at thachine B	2 700	5 400	8 100	13 500	21 600

   How much faster is machine B than machine A?

   5.3

Number of hours	1	2	3	5	8
Number of screws at machine A	1 800	3 600	5 400	9 000	14 400
Number of screws at thachine B	2 700	5 400	8 100	13 500	21 600

   How many screws will machine B produce in the same time that it takes machine A to make 100 screws?

6. Nathi, Paul and Tim worked in Mr Setati's garden. Nathi worked for five hours, Paul for four hours and Tim for three hours. Mr Setati gave the boys R600 for their work. How should they divide the R600 among the three of them?

7. Ntabi uses three packets of jelly to make a pudding for eight people. How many packets of jelly does she need to make a pudding for 16 people? And for 12 people?

8. Which rectangle is more like a square: a 3×5 rectangle or a 6×8 rectangle

9. Increase 56 in the ratio $2 : 3$

10. Decrease 72 in the ratio $4 : 3$

11. Divide 840 in the ratio $3 : 4$

12. Divide 360 in the ratio $1 : 2 : 3$

13. Look at the following data about the performance of different athletes during a walking event. Investigate the data to find out who walks the fastest and who walk: the slowest. Arrange the athletes from the fastest walker to the slowest walker.

Athlete	A	B	C	D	E	F
Distance walked in metres	2 480	4 283	3 729	6 209	3 112	5 638
Time taken in minutes	17	43	28	53	24	45

4.10 Exercise 10

1. Consider an amount of R800
   1. 1 How much is one-eighth of R800
   1. 2 How much is one hundredth of R800
   1. 3 How much is seven eighths of R800?
2. Rashid is a furniture dealer. He buys a couch for R2 420. He displays the couch in his showroom with the price marked as R3 200. Rashid offers a discount of R320 to customers who pay cash.
   2. 1 What is the cost price of the couch in Rashid's furniture shop?
   2. 2 What is the marked price?
   2. 3 What is the selling price for a customer who pays cash?
   2. 4 How much is ten hundredths of R3 200?
3. Calculate a discount of 6% on each of the following marked prices of articles
   3. 1 R3 600
   3. 2 R9 360
4. Coats-galore is a store that sells coats.
   4. 1 How much is one hundredth of R700?
   4. 2 A customer pays cash for a coat marked at R700. He is given a R63 discount. How many hundredths of R700 is this?
   4. 3 A customer is given R63 discount on a R700 coat. What is the percentage discount?
5. A client buys a blouse marked at R300 and she is given R36 discount for paying cash. What percentage discount was she given?
6. A dealer buys an article for R7 500 and makes the price 30% higher. The article is sold at a 20% discount. You may use a calculator to answer.

6.1 What is the selling price of the article?

6.2 What is the dealer's percentage profit?

7. When a person borrows money from a bank or some other institution, he or she normally has to pay for the use of the money. This is called interest. Sam borrows R$7\ 000$ from a bank at 14% interest for one year. How much does he have to pay back to the bank at the end of the period?

8. Jabu invests R$5\ 600$ for one year at 8% interest. You may use a calculator.

   8.1 What will the value of his investment be at the end of that year?

   8.2 At the end of the year, Jabu's investment is worth R$6\ 048$, he does not withdraw the investment or the interest earned; instead, he reinvests it for another year. How much will it be worth at the end of the second year?

   8.3 What will the value of Jabu's investment be after five years?

5 ANSWERS FOR EXERCISES

5.1 Exercise 1

1.1 (c) $5+5+5+5+5+5+5+5+5+5$
(f) $10+10+10+10+10$

1.2 70

1.3 50

1.4 120

1.5 No, it does not.
$70+50=120$ and $50+70=120$

1.6 No
$7 \times 10=70$ and $10 \times 7=70$

2.1 40

2.2 80

2.3 96

2.4 96

2.5 48

2.6 21

2.7 42

2.8 42

5.2 Exercise 2

1. Yes

2.1 22

2.2 22

2.3 22

2.4 22

3. Yes

4.1 170

4.2 140

4.3 137

4.4 200

5.1 18

5.2 56

5.3 44

5.4 22

5.5 76

5.6 14

5.7 23

6.1 4

6.2 8

6.3 40

6.4 2

6.5 5

6.6 20

7.1 437

7.2 300

7.3 8 600

7.4 4 000

5.3 Exercise 3

1.1 $(8+6)\times 5 = 70$

1.2 $8+(6\times 5) = 38$

1.3 $(5+8)\times(6-2) = 52$

1.4 $(5+8)\times 6-2 = 76$

1.5 $5+(8\times 6)-2 = 51$

1.6 $5+8\times(6-2) = 37$

2.1 1 700

2.2 1 700

2.3 300

2.4 300

3.

X	8	5	4	9	7	3	6	2	10	11	12
7	56	35	28	63	49	21	42	14	70	77	84
3	24	15	12	27	21	9	18	6	30	33	36
9	72	45	36	81	63	27	54	18	90	99	108
5	40	25	20	45	35	15	30	10	50	55	60
8	64	40	32	72	56	24	48	16	80	88	96
6	48	30	24	54	42	18	36	12	60	66	72
4	32	20	16	36	28	12	24	8	40	44	48
2	16	10	8	18	14	6	12	4	20	22	24
10	80	50	40	90	70	30	60	20	100	110	120
12	96	60	48	108	84	36	72	24	120	132	144
11	88	55	44	99	77	33	66	22	110	121	132

4.1 90

4.2 3 500

4.3 8 580

4.4 3 100

4.5 1 290

4.6 3 603

5.4 Exercise 4

1.1 Less

1.2 Less

1.3 Less

1.4 More

2.1 1 000

2.2 2 000

2.3 2 000

2.4 4 000

3.1 64

3.2 376

3.3 106

3.4 287

4.1 15 000

4.2 1 300

4.3 12 000

4.4 1 300

4.5 7 000

4.6 1 300

4.7 19 000

4.8 1 600

5.1 48

5.2 15

5.3 468

5.4 58

5.5 364

5.6 12

5.7 488

5.8 7

6.1 Approx: 4 000
Exact: 3 588

6.2 Approx: 6 300
Exact: 5 896

6.3 Approx: 8 400
Exact: 9 384

6.4 Approx: 14 400
Exact: 13 884

7.1 3 800

7.2 35

7.3 21

7.4 3 821

8.1 1 111

8.2 1 598

5.5 Exercise 5

1.1 7 305

1.2 7 994

2. 742 480

3.1 14 112

3.2 9 081

3.3 40 089

3.4 10 000

4. 592

5.1 367

5.2 885

5.3 473

5.4 4 764

6. 3 455

7.1 3 465

7.2 6 797

7.3 6 797

8.1 Approx: R4 000 or R4 300
Exact: R4 237

8.2 Approx: R34 000 or R33 500
Exact: R33 511

9.1 97 316

9.2 116 455

9.3 25 817

5.6 Exercise 6

1.1 $17\,532$

1.2 $131\,700$

1.3 $32\,364$

2.1 $18\,492$

2.2 $14\,952$

3.1 Yes

3.2 R$8\,100$

3.3 R$1\,300$

3.4 R760

4.1 R85

4.2 59

5.1 150 remainder 34

5.2 136 remainder 3

5.3 26 remainder 36

5.4 347 remainder 19

5.7 Exercise 7

1.1 600 and 700 respectively

1.2 Yes

1.3 Yes

2. 1 and 30; 2 and 15; 3 and 10; 5 and 6

3. Yes

4. No, 28 is not a factor of 48

5.1 Yes

5.2 Yes

5.3 Yes

6. Yes and $1; 2; 4; 5; 10$ are also factors of 60.

7.1 $2 \times 33; 3 \times 22; 6 \times 11; 2 \times 3 \times 11$

7.2 No other factors

7.3 $2 \times 34; 4 \times 17; 2 \times 2 \times 17$

7.4 3×23

7.5 $2 \times 35; 5 \times 14; 7 \times 10; 2 \times 5 \times 7$

7.6 No other factors

7.7 $2 \times 36; 3 \times 24; 4 \times 18; 6 \times 12; 8 \times 9; 2 \times 2 \times 2 \times 3 \times 3$

7.8 No other factors

8. $67; 71; 73$

9.1 False. The number 2 is a prime number and it is even.

9.2 False. The number 15, for example, is an odd composite number.

9.3 False. 1 only has one factor.

9.4 True, except for 1, which is neither prime nor composite.

9.5 False. It is a prime number because it has only two factors: 1 and itself.

9.6 False. Any whole number with 5 as the last digit has 5 as a factor.

9.7 False. The prime number 2 is an exception.

9.8 True.

10. $13 \times 11 = 143$; already to large.

11.1 Composite, $13; 17$ are factors.

11.2 Composite, $23; 31$ are factors

12.1 $2^3 \times 3^2 \times 11$

12.2 $2^2 \times 3 \times 37$

13.1 $2 \times 2 \times 7$

13.2 $2 \times 2 \times 2 \times 2 \times 2$

13.3 $2 \times 2 \times 31$

13.4 $2 \times 2 \times 3 \times 3$

13.5 $2 \times 3 \times 7$

13.6 $3 \times 5 \times 23$

13.7 $2 \times 7 \times 13$

5.8 Exercise 8

1. Both answers are Yes.

2. The statement is true.

3.1 HCF = 2
LCM = 168

3.2 HCF = None
LCM = 255

3.3 HCF = 5
LCM = 600

3.4 HCF = 3
LCM = 90

5.9 Exercise 9

1. 200 days

2. 270 km

3. R$781,48$

4. In pattern B, there are six red beads for every three yellow beads.
In pattern C, there are seven beads for every two yellow beads.

5.1

Number of hours	1	2	3	5	8
Number of screws at machine A	1 800	3 600	5 400	9 000	14 400
Number of screws at thachine B	2 700	5 400	8 100	13 500	21 600

5.2 B is one and a half times faster than A

5.3 150

6. Nathi gets $5 \times 50 =$ R250
   Paul gets $4 \times 50 =$ R200
   Tim gets $3 \times 50 =$ R150

7. 6 packets for 16 people; $4,5$ packets for 12 people.

8. A 6×8 rectangle

9. 84

10. 54

11. $360 : 480$

12. $60 : 120 : 180$

13. Fastest to slowest: A, C, E, F, D, B

5.10 Exercise 10

1.1 R100

1.2 R8

1.3 R700

2.1 R$2\ 420$

2.2 R$3\ 200$

2.3 R$2\ 880$

2.4 R320

3.1 R216

3.2 R$561,60$

4.1 7

4.2 9 hundredths

4.3 9%

5. 12%

6.1 R$7\ 800$

6.2 4%

7. R$7\,980$

8.1 R$6\,048$

8.2 R$6\,531,84$

8.3 R$8\,228,24$