CHAPTER 2

Integers

CONTENTS

October 19, 2021

1 INTRODUCTION

Integers are used everywhere in mathematics. It also occurs everywhere in our daily lives. From the depth of planting a tree underground to calculating the positive or negative displacement of an object in science.

In this chapter we will learn how to apply integers correctly in mathematics. This includes how to calculate the depth a submarine can safely descend to under water up to the temperature at which a cake should be baked.

2 WHAT IS BEYOND

In this chapter you will work with whole numbers smaller than 0. These numbers are called negative numbers. The whole numbers larger than 0, 0 itself and the negative whole numbers together are called the integers. Mathematicians have agreed that negative numbers should have certain properties that would make them useful for various purposes. You will learn about these properties and how they make it possible to do calculations with negative numbers. Yes

Do what you can.

$5-0=?$	$5-7=?$	$5+5=?$
$5-1=?$	$5-6=?$	$5+4=?$
$5-2=?$	$5-5=?$	$5+3=?$
$5-3=?$	$5-4=?$	$5+2=?$
$5-4=?$	$5-3=?$	$5+1=?$
$5-5=?$	$5-2=?$	$5+0=?$
$5-6=?$	$5-1=?$	$5+?=?$
$5-7=?$	$5-0=?$	$5+?=?$
$5-8=?$	$5-?=?$	$5+?=?$
$5-9=?$	$5-?=?$	$5+?=?$
$5-10=?$	$5-?=?$	$5+?=?$

Choose a good plan to complete this table.

Plan A: Look at where the number 4 appears in the table. All the 4s lie on a diagonal, going down from left to right. Complete the other diagonals in the same way.

Plan B: Look at the number 13 in the table. It is on the right, in the fourth row from the top. It can be obtained by adding the two numbers indicated by arrows. Complete all the cells by adding numbers from the yellow column and blue row in this way.

Plan C: In each row, add 1 to go right and subtract 1 to go left.

Plan D: In each column, add 1 to go up and subtract 1 to go down.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
					4			7								
						4		6								
-3	-2	-1	0				4	5								13
								4								
-5	-4							3	4							
								2		4						
								1			4					
-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
								-1					4			
								-2						4		
								-3							4	
								-4								4
								-5								
								-6								
								-7							0	
								-8								0

Why people decided to have negative numbers

About 500 years ago, some mathematicians proposed that a "negative number" may be used to describe the result in a situation, where a number is subtracted from a number smaller than itself.

For example, we may say $10 – 20 = (–10)$

This proposal was soon accepted by other mathematicians, and it is now used all over the world.

> **Note**
>
> Mathematicians are people who do mathematics for a living. Mathematics is their profession, like health care is the profession of nurses and medical doctors.

> **Note**
>
> The numbers 1; 2; 3; 4 etc. are called the **natural numbers**. The natural numbers, 0 and the negative whole numbers together are called the **integers**.

The following statement is true if the number is 5:
$15 -$ *(a certain number)* $= 10$

A few centuries ago, some mathematicians decided they wanted to have numbers that will also make sentences like the following true: $15 +$ (a certain number) $= 10$. But to go from 15 to 10 you have to subtract 5.

The number we need to make the sentence $15 +$ (a certain number) $= 10$ true must have the following strange property: If you **add** this number, it should have the same effect as to **subtract** 5.

Now the mathematicians of a few centuries ago really wanted to have numbers for which such strange sentences would be true. So they thought: Let us decide, and agree amongst ourselves, that the number we call negative 5 will have the property that if you add it to another number, the effect will be the same as when you subtract the natural number 5.

This means that the mathematicians agreed that $15 + (–5)$ is equal to $15 – 5$. Stated differently, instead of adding negative 5 to a number, you may subtract 5.

> **Note**
>
> Adding a negative number has the same effect as subtracting a natural number. For example:
> $20 + (–15) = 20 – 15 = 5$

The following statement is true if the number is 5: $15 +$ *(a certain number)* $= 20$

What properties should a number have so that it makes the following statement true?
$15 -$ *(a certain number)* $= 20$

To go from 15 to 20 you have to add 5. The number we need to make the sentence $15 -$ *(a certain number)* $= 20$ true must have the following property: If you **subtract** this number, it should have the **same effect** as to **add 5**.

Let us agree that $15–(-5)$ is equal to $15 + 5$. Stated differently, instead of subtracting negative 5 from a number, you may add 5.

> **Note**
>
> Subtracting a negative number has the same effect as adding a natural number.
> For example: $20–(-15) = 20 + 15 = 35$

You probably agree that:

$$5 + (-5) = 0$$
$$10 + (-10) = 0$$
and
$$20 + (-20) = 0$$

We may say that for each "positive" number there is a **corresponding** or **opposite** negative number. Two positive and negative numbers that correspond, for example 3 and (-3), are called **additive inverses**. They wipe each other out when you add them.

> **Note**
>
> What may each of the following be equal to?
> $(-8) + 5$
> $(-5) + (-8)$

> **Note**
>
> When you add any number to its additive inverse, the answer is 0 (the additive property of 0). For example, $120 + (-120) = 0$.

The idea of additive inverses may be used to explain why $\mathbf{8 + (-5)}$ is equal to 3:

$$\mathbf{8 + (-5) = 3 + \boxed{5 + (-5)} = 3 + 0 = 3}$$

Statements that are true for many different numbers

For how many different pairs of numbers can the following statement be true, if only natural (positive) numbers are allowed?

A number + another number = 10

For how many different pairs of numbers can the statement be true if negative numbers are also allowed?

Basic Rules

- $a + (-a) = 0$
- $a \times \frac{1}{a} = 1$ or $a \div a = 1$
- $a + 0 = a$
- $a \times 1 = a$

3 ADDING AND SUBTRACTING WITH INTEGERS

Adding can make less and subtraction can make more

Note

The numbers $1;\ 2;\ 3;\ 4;$ etc. that we use to count, are called **natural numbers.**

Note

Statements like these are also called number sentences:

An incomplete number sentence, where some numbers are not known at first, is sometimes called an **open number sentence:**

$8 -$ (a number) $= 10$

A **closed number sentence** is where all the numbers are known:

$8 + 2 = 10$

4 MULTIPLYING AND DIVIDING WITH INTEGERS

textbfCommutative Property: A commutative property is a property where the order in which you complete the calculations does not matter. Addition and multiplication is commutative.

$$a + b = b + a, \qquad a \times b = b \times a$$

Please note: Subtraction and division is not commutative.

Note

Multiplication of integers is commutative:
$\mathbf{(-20) \times 5 = 5 \times (-20)}$

Note

The product of two positive numbers is a positive number, for example $\mathbf{5 \times 6 = 30}$.
The product of a positive number and a negative number is a negative number, for example $\mathbf{5 \times (-6) = -30}$.
The product of a negative number and a positive number is a negative number, for example $\mathbf{(-5) \times 6 = -30}$.

Distributive Property: Numbers (or variables) can be added first and then multiplied, or you can first multiply the numbers (or variables) and then add together.

$$a(b + c) = ab + ac$$

You will realise that multiplication with a positive number distributes over addition and subtraction of integers. For example:

$$10 \times (5 + (-3)) = 10 \times 2 = 20 \text{ and } 10 \times 5 + 10 \times (-3) = 50 + (-30) = 20$$
$$10 \times (5-(-3)) = 10 \times 8 = 80 \text{ and } 10 \times 5-10 \times (-3) = 50-(-30) = 80$$

- Calculate: $(-10)\times(5 + (-3))$

Now consider the question of whether multiplication with a negative number distributes over addition and subtraction of integers. For example, would $\mathbf{(-10) \times 5 + (-10) \times (-3)}$ also have the answer -20, as does $\mathbf{(-10) \times (5 + (-3))}$?

- What must $\mathbf{(-10) \times (-3)}$ be equal to, if we want $\mathbf{(-10) \times 5 + (-10) \times (-3)}$ to be equal to -20?

In order to ensure that multiplication distributes over addition and subtraction in the system of integers, we have to agree that **(a negative number)** $\times$ **(a negative number) is a positive number**, for example $(-10) \times (-3) = 30$.

Here is a summary of the properties of integers that make it possible to do calculations with integers:

Note

- When a number is added to its additive inverse, the result is 0, for example $(+12) + (-12) = 0$.
- Adding an integer has the same effect as subtracting its additive inverse. For example, $3 + (-10)$ can be calculated by doing $3 - 10$, and the answer is -7.
- Subtracting an integer has the same effect as adding its additive inverse. For example, $3 - (-10)$ can be calculated by doing $3 + 10$, and the answer is 13.
- The product of a positive and a negative integer is negative, for example $(-15) \times 6 = -90$.
- The product of a negative and a negative integer is positive, for example $(-15) \times (-6) = 90$.

Note

Division is the inverse of multiplication. Hence, if two numbers and the value of their product are known, the answers to two division problems are also known.

When two numbers are multiplied, for example $\mathbf{30 \times 4 = 120}$, the word "product" can be used in various ways to describe the situation:

- An expression that specifies multiplication only, such as $\mathbf{30 \times 4}$, is called a product or a **product** expression.
- The answer obtained is also called the product of the two numbers. For example, 120 is called the **product of 30 and 4**.

An expression that specifies division only, such as $\mathbf{30 \div 5}$, is called a **quotient** or a **quotient expression.** The answer obtained is also called the quotient of the two numbers. For example, 6 is called the **quotient of 30 and 5**.

4.1 The associative properties of operations with integers

Associative property: The associative property is an addition or multiplication property where grouping or reordering of numbers does not affect the answer.

$$a + b + c = a + (b + c) = (a + b) + c = (a + c) + b$$

OR

$$a \times b \times c = a \times (b \times c) = (a \times b) \times c = (a \times c) \times b$$

Please note: Subtraction and division is not associative.

Multiplication of whole numbers is **associative**. This means that in a product with several factors, the factors can be placed in any sequence, and the calculations can be performed in any sequence. For example, the following sequences of calculations will all produce the same answer:

Note

Multiplication with integers is associative.

The calculation sequence A can be represented in symbols in only two ways:

- $\mathbf{2 \times 3 \times 5 \times 10}$. The convention to work from left to right unless otherwise indicated with brackets ensures that this representation corresponds to A.
- $\mathbf{5 \times (2 \times 3) \times 10}$, where brackets are used to indicate that $\mathbf{2 \times 3}$ should be calculated first. When brackets are used, there are different possibilities to describe the same sequence.

5 SQUARES, CUBES AND ROOTS WITH INTEGERS

Note

10^2 is 100 and $(-10)^2$ is also 100.

Both 10 and (-10) are called **square roots** of 100. 10 may be called the **posistive square root** of 100 and (-10) may be called the **negative square root of 100.**

Note

3^3 is 27 and $(-5)^3$ is -125.

3 is called the **cube root** of 27, because $3^3 = 27$.

-5 is called the cube root of -125 because $(-5)^3 = -125$.

The symbol $\sqrt{}$ is used to indicate "root".

$\sqrt[3]{-125}$ represents the cube root of -125. That means $\sqrt[3]{-125} = -5$.

$\sqrt[2]{36}$ represents the positive square root of 36, and $-\sqrt[2]{36}$ represents the negative square root. The "2" that indicates "square" is normally omitted, so $\sqrt{36} = 6$ and $-\sqrt{36} = -6$

6 EXERCISES

6.1 Exercise 1

Below, you can see how Jimmy prefers to work when doing calculations such as $542 + 253$.

$500 + 200 = 700$
$40 + 50 = 90$
$2 + 3 = 5$
$700 + 90 + 5 = 795$

He tries to calculate $542 - 253$ in a similar way:
$500–200 = 300$
$40–50 =?$

Jimmy clearly has a problem. He reasons as follows:
I can subtract 40 *from* 40*; that gives* 0*. But then there is still* 10 *that I have to subtract.*
He decides to deal with the 10 that he still has to subtract later, and continues:

$500–200 = 300$
$40–50 = 0$, but there is still 10 that I have to subtract.
$2 - 3 = 0$, but there is still 1 that I have to subtract.

1. 1.1 What must Jimmy still subtract, and what will his final answer be?
   1.2 When Jimmy did another subtraction problem, he ended up with this writing at one stage: 600 and $(-)50$ and $(-)7$.
   What do you think is Jimmy's final answer for this subtraction problem?

2. Calculate the following:
   2.1 $16 - 20$
   2.2 $16 - 30$
   2.3 $16 - 40$
   2.4 $16 - 60$
   2.5 $16 - 200$
   2.6 $5 - 1\ 000$

3. Calculate each of the following:

   3.1 $500 + (-300)$

   3.2 $100 + (-20) + (-40)$

   3.3 $500 + (-200) + (-100)$

   3.4 $100 + (-60)$

4. Calculate:

   4.1 $30 - (-10)$

   4.2 $30 + 10$

   4.3 $30 + (-10)$

   4.4 $30 - 10$

   4.5 $30 - (-30)$

   4.6 $30 + 30$

   4.7 $30 + (-30)$

   4.8 $30 - 30$

5. Write the additive inverse of each of the following numbers:

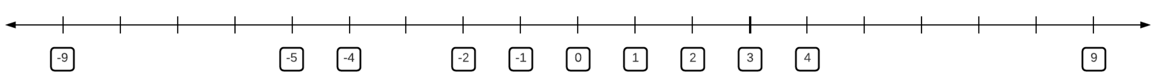
   5.1 24

   5.2 -24

   5.3 -103

   5.4 2348

6. Some numbers are shown on the number line below. Copy the number line and numbers shown, and fill in the missing numbers.



7. Copy and continue the lists of numbers to complete the following table:

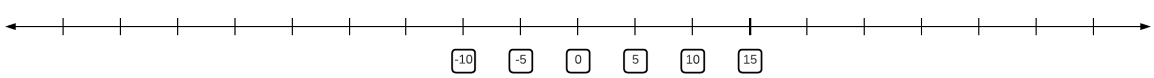
(a)	(b)	(c)	(d)	(e)	(f)	(g)
10	100	3	-3	-20	150	0
9	90	6	-6	-18	125	-5
8	80	9	-9	-16	100	-10
7	70	12	-12	-14	75	-15
6	60	15	-15		50	-20
5	50					-25
4	40					
3	30					
2	20					
1	10					
0	0					
-1	-10					

8. Use the idea of additive inverses to explain why each of these statements is true:

   8.1 $43 + (-30) = 13$

   8.2 $150 + (-80) = 70$

9. Some numbers are shown on the number line below. Copy the number line and numbers shown, and fill in the missing numbers.



6.2 Exercise 2

1. Calculate each of the following:

   1.1 $10 + 4 + (-4)$

   1.2 $10 + (-4) + 4$

   1.3 $3 + 8 + (-8)$

   1.4 $3 + (-8) + 8$

2. Calculate each of the following:

  2.1 $18 + 12$

  2.2 $12 + 18$

  2.3 $2 + 4 + 6$

  2.4 $6 + 4 + 2$

  2.5 $2 + 6 + 4$

  2.6 $4 + 2 + 6$

  2.7 $4 + 6 + 2$

  2.8 $6 + 2 + 4$

  2.9 $6 + (-2) + 4$

  2.10 $4 + 6 + (-2)$

  2.11 $4 + (-2) + 6$

  2.12 $(-2) + 4 + 6$

  2.13 $6 + 4 + (-2)$

  2.14 $(-2) + 6 + 4$

  2.15 $(-6) + 4 + 2$

3. Calculate each of the following:

  3.1 $(-5) + 10$

  3.2 $10 + (-5)$

  3.3 $(-8) + 20$

  3.4 $20 - 8$

  3.5 $30 + (-10)$

  3.6 $30 + (-20)$

  3.7 $30 + (-30)$

  3.8 $10 + (-5) + (-3)$

  3.9 $(-5) + 7 + (-3) + 5$

  3.10 $(-5) + 2 + (-7) + 4$

4. In each case, find the number that makes the statement true. Give your answer by writing a closed number sentence.

   4.1 $20 +$ (an unknown number) $= 50$

   4.2 $50 +$ (an unknown number) $= 20$

   4.3 $20 +$ (an unknown number) $= 10$

   4.4 (an unknown number) $+ (-25) = 50$

   4.5 (an unknown number) $+ (-25) = -50$

5. Calculate the following:

   5.1 $80 + (-60)$

   5.2 $500 + (-200) + (-200)$

6. Calculate the following:

   6.1 $20-20$

   6.2 $50-20$

   6.3 $(-20)-(-20)$

   6.4 $(-50)-(-20)$

7. Calculate.

   7.1 $20-(-10)$

   7.2 $100-(-100)$

   7.3 $20 + (-10)$

   7.4 $100 + (-100)$

   7.5 $(-20)-(-10)$

   7.6 $(-100)-(-100)$

   7.7 $(-20) + (-10)$

   7.8 $(-100) + (-100)$

8. In each case, state whether the statement is true or false and give a numerical example to demonstrate your answer.

   8.1 Subtracting a positive number from a negative number has the same effect as adding the additive inverse of the positive number.

   8.2 Adding a negative number to a positive number has the same effect as adding the additive inverse of the negative number.

8.3 Subtracting a negative number from a positive number has the same effect as subtracting the additive inverse of the negative number.

8.4 Adding a negative number to a positive number has the same effect as subtracting the additive inverse of the negative number.

8.5 Adding a positive number to a negative number has the same effect as adding the additive inverse of the positive number.

8.6 Adding a positive number to a negative number has the same effect as subtracting the additive inverse of the positive number.

8.7 Subtracting a positive number from a negative number has the same effect as subtracting the additive inverse of the positive number.

8.8 Subtracting a negative number from a positive number has the same effect as adding the additive inverse of the negative number.

9. Complete the table as far as you can.

(a)	(b)	(c)
$5 - 8 =$	$5 + 8 =$	$8 - 3 =$
$5 - 7 =$	$5 + 7 =$	$7 - 3 =$
$5 - 6 =$	$5 + 6 =$	$6 - 3 =$
$5 - 5 =$	$5 + 5 =$	$5 - 3 =$
$5 - 4 =$	$5 + 4 =$	$4 - 3 =$
$5 - 3 =$	$5 + 3 =$	$3 - 3 =$
$5 - 2 =$	$5 + 2 =$	$2 - 3 =$
$5 - 1 =$	$5 + 1 =$	$1 - 3 =$
$5 - 0 =$	$5 + 0 =$	$0 - 3 =$
$5 - (-1) =$	$5 + (-1) =$	$(-1) - 3 =$
$5 - (-2) =$	$5 + (-2) =$	$(-2) - 3 =$
$5 - (-3) =$	$5 + (-3) =$	$(-3) - 3 =$
$5 - (-4) =$	$5 + (-4) =$	$(-4) - 3 =$
$5 - (-5) =$	$5 + (-5) =$	$(-5) - 3 =$
$5 - (-6) =$	$5 + (-6) =$	$(-6) - 3 =$

6.3 Exercise 3

1. Fill <, > or = into the block to make the relationship between the numbers true:

   1.1 $-103 \square -99$

   1.2 $-699 \square -701$

   1.3 $30 \square -30$

   1.4 $10-7 \square -(10-7)$

   1.5 $-121 \square -200$

   1.6 $-12-5 \square -(12+5)$

   1.7 $-199 \square -110$

2. At 5 a.m. in Bloemfontein the temperature was $-5°$C. At 1 p.m., it was $19°$C. By how many degrees did the temperature rise?

3. A diver swims 150 m below the surface of the sea. She moves 75 m towards the surface. How far below the surface is she now?

4. One trench in the ocean is 800 m deep and another is $2\ 200$ m deep. What is the difference in their depths?

5. An island has a mountain which is $1\ 200$ m high. The surrounding ocean has a depth of 860 m. What is the difference in height?

6. On a winter's day in Upington the temperature rose by $19°$C. If the minimum temperature was $-4°$C, what was the maximum temperature?

6.4 Exercise 4

1. Calculate the following:

   1.1 $-5 + -5 + -5 + -5 + -5 + -5 + -5 + -5 + -5 + -5$

   1.2 $-10 + -10 + -10 + -10 + -10$

   1.3 $-6 + -6 + -6 + -6 + -6 + -6 + -6 + -6$

   1.4 $-8 + -8 + -8 + -8 + -8 + -8$

   1.5 $-20 + -20 + -20 + -20 + -20 + -20 + -20$

2. In each case, state whether the given statement is true or false. If false, correct the statement.

   2.1 $10 \times (-5) = 50$

   2.2 $8 \times (-6) = (-8) \times 6$

   2.3 $(-5) \times 10 = 5 \times (-10)$

2.4 $6 \times (-8) = -48$

2.5 $(-5) \times 10 = 10 \times (-5)$

2.6 $8 \times (-6) = 48$

2.7 $4 \times 12 = -48$

2.8 $(-4) \times 12 = -48$

3. Calculate the following:

3.1 $20 \times (-10)$

3.2 $(-5) \times 4$

3.3 $(-20) \times 10$

3.4 $4 \times (-25)$

3.5 $29 \times (-20)$

3.6 $(-29) \times (-2)$

4. Calculate.

4.1 $10 \times 50 + 10 \times (-30)$

4.2 $50 + (-30)$

4.3 $10 \times (50 + (-30))$

4.4 $(-50) + (-30)$

4.5 $10 \times (-50) + 10 \times (-30)$

4.6 $10 \times ((-50) + (-30))$

5. Four numerical expressions are given below. Which expressions would you expect to have the same answer? Do not do the calculations.

a)	$14 \times (23 + 58)$	b)	$23 \times (14 - 58)$
c)	$14 \times 23 + 14 \times 58$	d)	$14 \times 23 + 58$

6. What property of operations is demonstrated by the fact that two of the above expressions have the same value?

7. Three numerical expressions are given below. Which expressions would you expect to have the same answer? Do not do the calculations.

a)	$10 \times ((-50) - (-30))$
b)	$10 \times (-50) - (-30)$
c)	$10 \times (-50) - 10 \times (-30)$

8. Calculate the following:

    8.1 $10 \times ((-50) - (-30))$

    8.2 $10 \times (-50) - (-30)$

    8.3 $10 \times (-50) - 10 \times (-30)$

    8.4 $(-10) \times (5 + (-3))$

9. What must x be equal to if we want $(-10) \times 5 + x$ to be equal to -20?

10. Calculate the following:

    10.1 $(-10) \times (-5)$

    10.2 $(-10) \times 5$

    10.3 10×5

    10.4 $10 \times (-5)$

    10.5 $(-20) \times (-10) + (-20) \times (-6)$

    10.6 $(-20) \times ((-10) + (-6))$

    10.7 $(-20) \times (-10) - (-20) \times (-6)$

    10.8 $(-20) \times ((-10) - (-6))$

6.5 Exercise 5

1. Calculate the following:

    1.1 25×8

    1.2 $200 \div 25$

    1.3 $200 \div 8$

2. Calculate the following:

    2.1 $25 \times (-8)$

    2.2 $(-125) \times 8$

3. Calculate the following:

    3.1 $(1\ 000) \div (-125)$

    3.2 $(-1\ 000) \div 8$

    3.3 $(-200) \div 25$

    3.4 $(-200) \div 8$

    3.5 $(-100) \div (-25)$

4. In each case, state whether you agree or disagree with the statement, and give an example to illustrate your answer.

   4.1 The quotient of a positive and a negative integer is negative.

   4.2 The quotient of a positive and a positive integer is negative.

   4.3 The quotient of a negative and a negative integer is negative.

   4.4 The quotient of a negative and a negative integer is positive.

5. Do the necessary calculations to enable you to provide the values of the quotients.

   5.1 $(-500) \div (-20)$

   5.2 $(-144) \div 6$

   5.3 $(-1\,440) \div (-6)$

   5.4 $-14\,400 \div 600$

   5.5 $500 \div (-20)$

6.6 Exercise 6

1. A, B, C, and D are given below:

   A) $\mathbf{2} \times \mathbf{3}$, the answer of $\mathbf{2} \times \mathbf{3}$ multiplied by 5, the new answer multiplied by 10.

   B) $\mathbf{2} \times \mathbf{5}$, the answer of $\mathbf{2} \times \mathbf{5}$ multiplied by 10, the new answer multiplied by 3.

   C) $\mathbf{10} \times \mathbf{5}$, the answer of $\mathbf{10} \times \mathbf{5}$ multiplied by 3, the new answer multiplied by 2.

   D) $\mathbf{3} \times \mathbf{5}$, the answer of $\mathbf{3} \times \mathbf{5}$ multiplied by 2, the new answer multiplied by 10

   1.1 Do the four sets of calculations given to check whether they really produce the same answers.

   1.2 If the numbers 3 and 10 in the calculation sequences A, B, C and D are replaced with -3 and -10, do you think the four answers will still be the same?

   1.3 Express the calculation sequences B, C and D given above symbolically, without using brackets.

2. Calculate the following:

   2.1 $80 - 30 + 40 - 20$

   2.2 $80 + (-30) + 40 + (-20)$

   2.3 $30 - 80 + 20 - 40$

   2.4 $(-30) + 80 + (-20) + 40$

   2.5 $20 + 30 - 40 - 80$

3. Calculate the following:

   3.1 $-3 \times 4 + (-7) \times 9$

   3.2 $-20(-4-7)$

   3.3 $20 \times (-5)-30 \times 7$

   3.4 $-9(20-15)$

   3.5 $-8 \times (-6)-8 \times 3$

   3.6 $(-26-13) \div (-3)$

   3.7 $-15 \times (-2) + (-15) \div (-3)$

   3.8 $-15(2-3)$

   3.9 $(-5 + -3) \times 7$

   3.10 $-5 \times (-3 + 7) + 20 \div (-4)$

4. Calculate the following:

   4.1 $20 \times (-15 + 6)-5 \times (-2-8)-3 \times (-3-8)$

   4.2 $40 \times (7 + 12-9) + 25 \div (-5)-5 \div 5$

   4.3 $-50(20-25) + 30(-10 + 7)-20(-16 + 12)$

   4.4 $-5 \times (-3 + 12-9)$

   4.5 $-4 \times (30-50) + 7 \times (40-70)-10 \times (60-100)$

   4.6 $-3 \times (-14 + 6) \times (-13 + 7) \times (-20 + 5)$

   4.7 $20 \times (-5) + 10 \times (-3) + (-5) \times (-6)-(3 \times 5)$

   4.8 $-5(-20-5) + 10(-7-3)-20(-15-5) + 30(-40-35)$

   4.9 $(-50 + 15-75) \div (-11) + (6-30 + 12) \div (-6)$

6.7 Exercise 7

1. Calculate the following:

   1.1 20×20

   1.2 $20 \times (-20)$

   1.3 -20×-20

2. Complete the table.

x	1	-1	2	-2	5	-5	10	-10
x^2 which is $x \times x$								
x^3								

In each case, state for which values of x the given statement is true.

2.1 x^3 is a negative number

2.2 x^2 is a negative number

2.3 $x^2 > x^3$

2.4 $x^2 < x^3$

3. Ben thinks of a number. He adds 5 to it, and his answer is 12. Give all the possible numbers that Ben could have thought of.

4. Lebo also thinks of a number. She multiplies the number by itself and gets 25. State all the numbers Lebo could have thought of.

5. Mary thinks of a number and calculates (the number) $\times$ (the number) $\times$ (the number). Her answer is 27. What number did Mary think of?

6. Write the positive square root and the negative square root of each number.

6.1 64

6.2 9

7. Complete the table

Number	1	4	9	16	25	36	49	64
Positive square root								
Negative square root								

8. Complete the table:

x	1	2	3	4	5	6	7	8
x^3								

9. Complete the table:

x	-1	-2	-3	-4	-5	-6	-7	-8
x^3								

10. Complete the table:

Number	-1	8	-27	-64	-125	-216	1000
Cube root							

11. Complete the table:

$\sqrt[3]{-8}$	$\sqrt{121}$	$\sqrt[3]{-64}$	$-\sqrt{64}$	$\sqrt{64}$	$\sqrt[3]{-1}$	$-\sqrt{1}$	$\sqrt[3]{-216}$

6.8 Exercise 8

1. Use the numbers $-8, -5$ and -3 to demonstrate each of the following:

   1.1 Multiplication with integers distributes over addition.

   1.2 Multiplication with integers distributes over subtraction.

   1.3 Multiplication with integers is associative.

   1.4 Addition with integers is associative.

2. Calculate the following without using a calculator:

   2.1 $5 \times (-2)^3$

   2.2 $3 \times (-5)^2$

   2.3 $2 \times (-5)^3$

   2.4 $10 \times (-3)^2$

3. Use a calculator to work out the following:

   3.1 $24 \times (-53) + (-27) \times (-34) - (-55) \times 76$

   3.2 $64 \times (27 - 85) - 29 \times (-47 + 12)$

4. Use a calculator to work out each of the following:

   4.1 $-24 \times 53 + 27 \times 34 + 55 \times 76$

   4.2 $64 \times (-58) + 29 \times (47 - 12)$

7 ANSWERS FOR EXERCISES

7.1 Exercise 1

1. 1.1 He must still subtract 11.
   His final answer will be $300 - 11 = 289$

   1.2 543

2. 2.1 -4

   2.2 -14

   2.3 -24

   2.4 -44

   2.5 -184

   2.6 -995

3. 3.1 200

   3.2 40

   3.3 200

   3.4 40

4. 4.1 40

   4.2 40

   4.3 20

   4.4 20

   4.5 60

   4.6 60

   4.7 0

   4.8 0

5. 5.1 -24

   5.2 24

   5.3 103

   5.4 $-2\ 348$

6. The numbers are increasing by 1.
   The missing numbers are:
   $-8; -7; -6; -3; 5; 6; 7; 8$

7.

(a)	(b)	(c)	(d)	(e)	(f)	(g)
10	100	3	-3	-20	150	0
9	90	6	-6	-18	125	-5
8	80	9	-9	-16	100	-10
7	70	12	-12	-14	75	-15
6	60	15	-15	-12	50	-20
5	50	18	-18	-10	25	-25
4	40	21	-21	-8	0	-30
3	30	24	-24	-6	-25	-35
2	20	27	-27	-4	-50	-40
1	10	30	-30	-2	-75	-45
0	0	33	-33	0	-100	-50
-1	-10	36	-36	2	-125	-55
-2	-20	39	-39	4	-150	-60
-3	-30	42	-42	6	-175	-65
-4	-40	45	-45	8	-200	-70

8. 8.1 13

   8.2 70

9. The numbers are increasing by 5 each time.

   The missing numbers are:

   $-45; -40; -35; -30; -25; -20; -15; 20; 25; 30; 35; 40; 45$

7.2 Exercise 2

1. 1.1 10

   1.2 10

   1.3 3

   1.4 3

2. 2.1 30

   2.2 30

   2.3 12

   2.4 12

   2.5 12

   2.6 12

2.7 12

2.8 12

2.9 8

2.10 8

2.11 8

2.12 8

2.13 8

2.14 8

2.15 0

3. 3.1 5

   3.2 5

   3.3 12

   3.4 12

   3.5 20

   3.6 10

   3.7 0

   3.8 2

   3.9 4

   3.10 -6

4. 4.1 30

   4.2 -30

   4.3 -10

   4.4 75

   4.5 -25

5. 5.1 20

   5.2 100

6. 6.1 0

   6.2 30

   6.3 0

   6.4 -30

7. 7.1 30

7.2 200

7.3 10

7.4 0

7.5 -10

7.6 0

7.7 -30

7.8 -200

8. 8.1 True

   8.2 False

   8.3 False

   8.4 True

   8.5 False

   8.6 True

   8.7 False

   8.8 True

9.

(a)	(b)	(c)
$5-8=-3$	$5+8=13$	$8-3=5$
$5-7=-2$	$5+7=12$	$7-3=4$
$5-6=-1$	$5+6=11$	$6-3=3$
$5-5=0$	$5+5=10$	$5-3=2$
$5-4=1$	$5+4=9$	$4-3=1$
$5-3=2$	$5+3=8$	$3-3=0$
$5-2=3$	$5+2=7$	$2-3=-1$
$5-1=4$	$5+1=6$	$1-3=-2$
$5-0=5$	$5+0=5$	$0-3=-3$
$5-(-1)=6$	$5+(-1)=4$	$(-1)-3=-4$
$5-(-2)=7$	$5+(-2)=3$	$(-2)-3=-5$
$5-(-3)=8$	$5+(-3)=2$	$(-3)-3=-6$
$5-(-4)=9$	$5+(-4)=1$	$(-4)-3=-7$
$5-(-5)=10$	$5+(-5)=0$	$(-5)-3=-8$
$5-(-6)=11$	$5+(-6)=-1$	$(-6)-3=-9$

7.3 Exercise 3

1. 1.1 $-103 < -99$

   1.2 $-699 > -701$

   1.3 $30 > -30$

   1.4 LHS $= 3$, RHS $= -3$
   $\therefore$ LHS $>$ RHS

   1.5 $-121 > -200$

   1.6 LHS $= 7$, RHS $= -17$
   $\therefore$ LHS $>$ RHS

   1.7 $-199 < -110$

2. 24°C

3. She is now 75 m below the surface.

4. $1\ 400$m

5. $2\ 060$m

6. 15°C

7.4 Exercise 4

1. 1.1 -50

   1.2 -50

   1.3 -48

   1.4 -48

   1.5 -140

2. 2.1 False

   2.2 True

   2.3 True

   2.4 True

   2.5 True

   2.6 False

   2.7 False

   2.8 True

3. 3.1 -200
   3.2 -20
   3.3 -200
   3.4 -100
   3.5 -580
   3.6 58
4. 4.1 200
   4.2 20
   4.3 200
   4.4 -80
   4.5 -800
   4.6 -800
5. A and C are the same.
6. The distributive property
7. A and C
8. 8.1 -200
   8.2 -470
   8.3 -200
   8.4 -20
9. $x = 30$
10. 10.1 50
    10.2 -50
    10.3 50
    10.4 -50
    10.5 320
    10.6 320
    10.7 80
    10.8 80

7.5 Exercise 5

1. 1.1 200

   1.2 8

   1.3 25

2. 2.1 -200

   2.2 $-1\ 000$

3. 3.1 -8

   3.2 -125

   3.3 -8

   3.4 -25

   3.5 4

4. 41 True. For example
   $(-100) \div 2 = -50$

   42 False. For example:
   $14 \div 7 = 2$
   The quotient is positive.

   43 False. For example:
   $(-8) \div (-2) = 4$
   The quotient is positive.

   44 True. For example:
   $(-20) \div (-5) = 4$
   The quotient is positive

5. 5.1 25

   5.2 -24

   5.3 240

   5.4 -24

   5.5 -25

7.6 Exercise 6

1. 1.1 A, B, C, and D. All four statements are equal to 300:

   1.2 Yes. All four sequences will still be equal.

1.3 $B) 2 \times 5 \times 10 \times 3$
$C) 10 \times 5 \times 3 \times 2$
$D) 3 \times 5 \times 2 \times 10$

2. 2.1 70
   2.2 70
   2.3 -70
   2.4 70
   2.5 -70

3. 3.1 -75
   3.2 220
   3.3 -310
   3.4 -45
   3.5 24
   3.6 13
   3.7 35
   3.8 15
   3.9 -56
   3.10 -25

4. 4.1 -97
   4.2 394
   4.3 240
   4.4 0
   4.5 270
   4.6 $2\ 160$
   4.7 -115
   4.8 $-1\ 825$
   4.9 12

7.7 Exercise 7

1. 1.1 400
   1.2 -400

1.3 400

2.

x	1	-1	2	-2	5	-5	10	-10
x^2 which is $x \times x$	1	1	4	4	25	25	100	100
x^3	1	-1	8	-8	125	-125	$1\,000$	$-1\,000$

2.1 The value of x^3 is always negative if x is negative.

2.2 Never, x^2 is always positive.

2.3 If x is negative.

2.4 If x is positive and is greater than 1.

3. 7

4. 5 and -5

5. 3

6. 6.1 8 and -8

   6.2 3 and -3

7. Completed table:

Number	1	4	9	16	25	36	49	64
Positive square root	1	2	3	4	5	6	7	8
Negative square root	-1	-2	-3	-4	-5	-6	-7	-8

8. Completed table:

x	1	2	3	4	5	6	7	8
x^3	1	8	27	64	125	216	343	512

9. Completed table:

x	-1	-2	-3	-4	-5	-6	-7	-8
x^3	-1	-8	-27	-64	-125	-216	-343	-512

10. Completed table:

Number	-1	8	-27	-64	-125	-216	1 000
Cube root	-1	2	-3	-4	-5	-6	10

11. Completed table:

$\sqrt[3]{-8}$	$\sqrt{121}$	$\sqrt[3]{-64}$	$-\sqrt{64}$	$\sqrt{64}$	$\sqrt[3]{-1}$	$-\sqrt{1}$	$\sqrt[3]{-216}$
-2	11	-4	-8	8	-1	1	-6

7.8 Exercise 8

1. 1.1 $-8[-5+(-3)] = -8(-5)+(-8)(-3)$

   1.2 $-8[-5-(-3)] = -8(-5)-(-8)(-3)$

   1.3 $-8\times(-3)\times(-5) = -5\times(-8)\times(-3)$

   1.4 $-8+(-3)+(-5) = -5+(-8)+(-3)$

2. 2.1 -40

   2.2 75

   2.3 -250

   2.4 90

3. 3.1 $3\ 826$

   3.2 $-2\ 697$

4. 4.1 $3\ 826$

   4.2 $-2\ 697$