CHAPTER 4

Numeric And Geometric Patters

CONTENTS

April 21, 2021

1 THE TERM-TERM RELATIONSHIP IN A SEQUENCE

1.1 Going from one term to the next

NOTE

A list of numbers which form a pattern is called a **sequence**. Each number in a sequence is called a **term** of the sequence. The first number is the first term of the sequence.

NOTE

Numbers that follow one another are said to be **consecutive**.

1.2 Adding or subtracting the same number

Amanda explains how she figured out how to continue sequence A:

I looked at the first two numbers in the sequence and saw that I needed 3 *to go from* 2 *to* 5*.*

I looked further and saw that I also needed 3 *to go from* 5 *to* 8*. I tested that and it worked for all the next numbers* This gave me a **rule I could use to extend the sequence: add** 3 **to each number to find the next number in the pattern**.

Tamara says you can also find the pattern by working backwards and subtracting 3 each time:

NOTE

When the **differences** between consecutive terms of a sequence are the same, we say the difference is **constant**.

$14 - 3 = 11; 11 - 3 = 8; 8 - 3 = 5; 5 - 3 = 2$

1.3 Multiplying or dividing with the same number

Take a look at the sequence F:$2; 6; 18; 54; 162; 486; \ldots$

Piet explains that he figured out how to continue the sequence F:

I looked at the first two terms in the sequence and wrote$2 \times ? = 6$

When I multiplied the first number by 3, I got the second number:$2 \times 3 = 6$

I then checked to see if I could find the next number if I multiplied 6 *by* 3*:*$6 \times 3 = 18$

I continued checking in this way:$18 \times 3 = 54; 54 \times 3 = 162$

This gave me $aruleIcanusetoextendthesequence$ and my rule was: **multiply each number by 3 to calculate the next number in t**

Zinhle says you can also find the pattern by working backwards and dividing by 3 each time:

$54 \div 3 = 18; 18 \div 3 = 6; 6 \div 3 = 2$

> **NOTE**
>
> The number that we multiply with to get the next term in the sequence is called a **ratio**. If the number we multiply with remains the same throughout the sequence, we say it is a **constant ratio**.

1.4 Neither adding nor multiplying by the same number

> **NOTE**
>
> There are sequences where there is neither a constant difference nor a constant ratio between consecutive terms and yet a pattern still exists, as in the case of sequences B and E.

2 THE POSITION-TERM RELATIONSHIP IN A SEQUENCE

2.1 Using position to make predictions

Sizwe has been thinking about Amanda and Tamara's explanations of how they worked out the rule for sequence A and has drawn up a table. He agrees with them but says that there is another rule that will also work. He explains:

My table shows the terms in the sequence and the difference between consecutive terms:

	1st term	2nd term	3rd term	4th term							
A:	5	8	11	14							
differences	+3	+3	+3	+3	+3	+3	+3	+3	+3		

Sizwe reasons that the following rule will also work:

Multiply the position of the number by 3 and add 2 to the answer.

I can write this rule as a number sentence: **Position of the number**$\times$**3** $+$ **2**

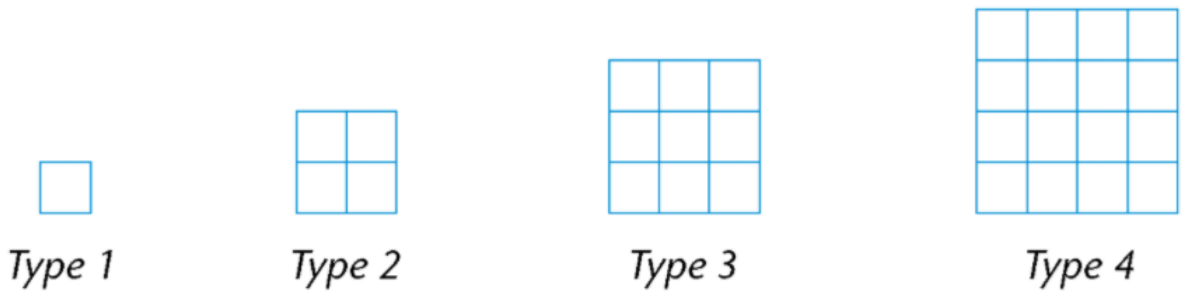
I use my number sentence to check: **1** $\times 3 + 2 = 5;$ **2** $\times 3 + 2 = 8;$ **3** $\times 3 + 2 = 11$

2.2 More predictions

3 INVESTIGATING AND EXTENDING GEOMETRIC PATTERNS

3.1 Square numbers

A factory makes window frames. Type 1 has one windowpane, type 2 has four windowpanes, type 3 has nine windowpanes, and so on.

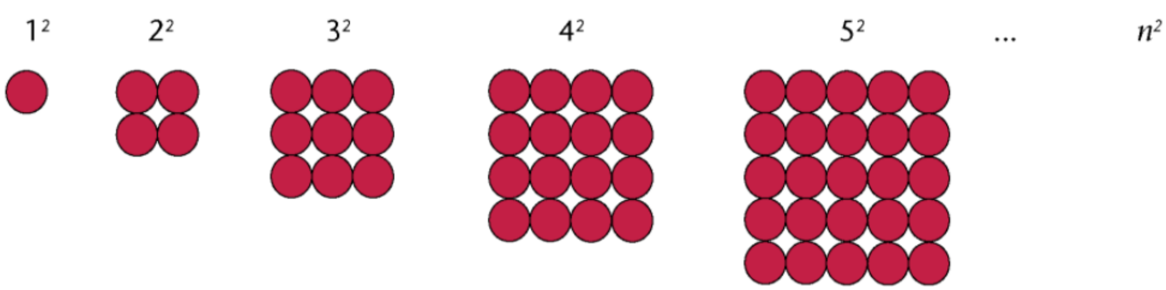


NOTE

The symbol n is used below to represent the position number in the expression that gives the rule (n^2) when generalising.

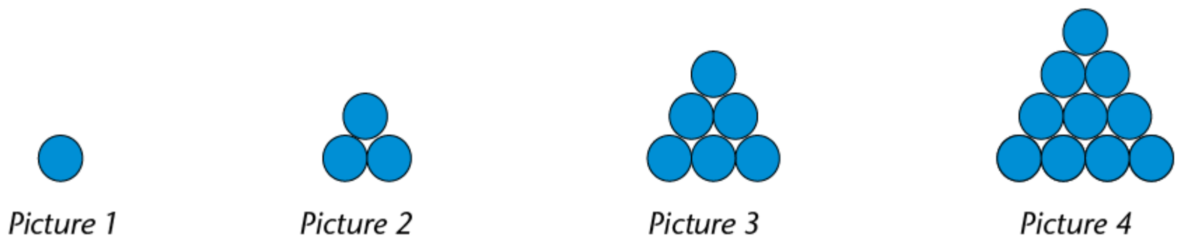
NOTE

In algebra we think of a square as a number that is obtained by multiplying a number by itself. So 1 is also a square because $1 \times 1 = 1$



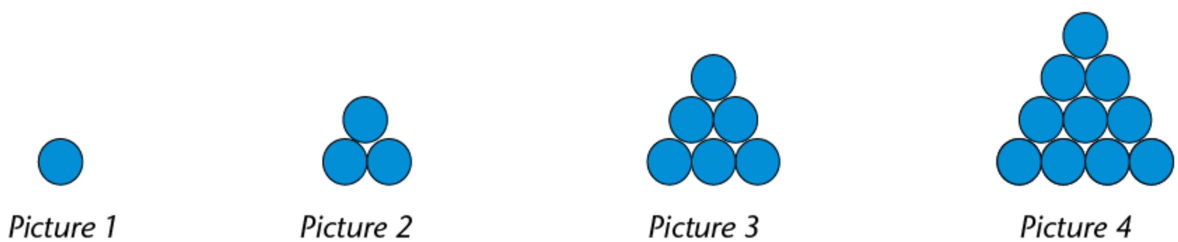
3.2 Triangular numbers

These use circles to form a pattern of triangular shapes:



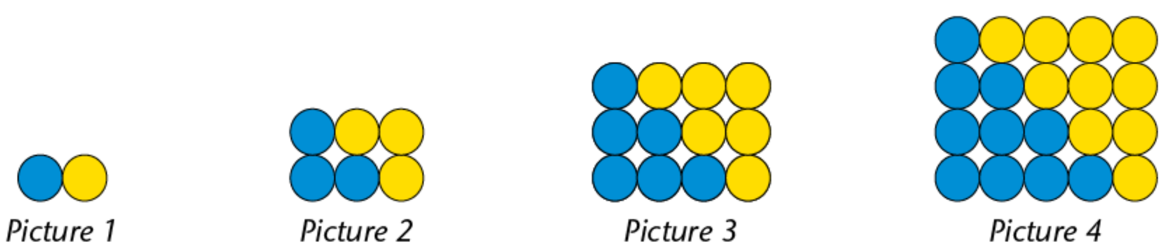
More than 2500 years ago, Greek mathematicians already knew that the numbers 3, 6, 10, 15 and so on could form a triangular pattern. They represented these numbers with dots which they arranged in such a way that they formed equilateral triangles, hence the name **triangular numbers**. Algebraically we think of them as sums of consecutive natural numbers starting with 1.

Let us revisit the activity on triangular numbers that we did in the previous section.



So far, we have determined the number of circles in the pattern by adding consecutive natural numbers. If we were asked to determine the number of circles in picture 200, for example, it would take us a very long time to do so. We need to find a quicker method of finding any triangular number in the sequence.

Consider the arrangement below.



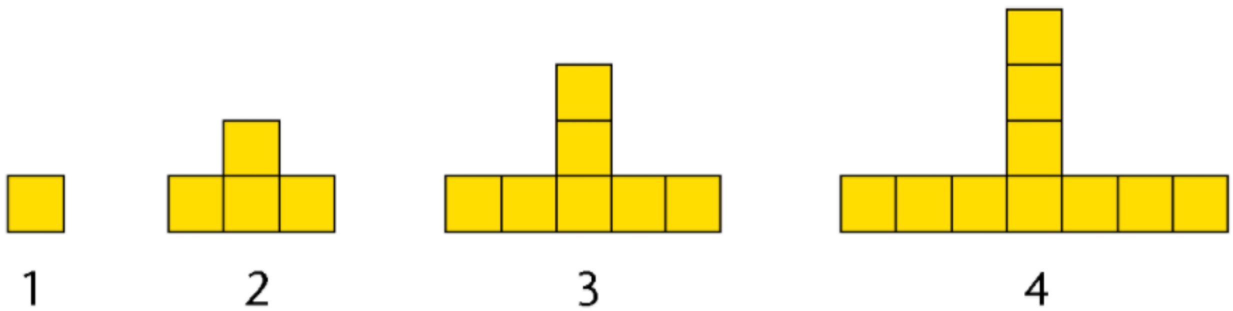
We have added the yellow circles to the original blue circles and then rearranged the circles in such a way that they are in a rectangular form.

Suppose we want to have a quicker method to determine the number of circles in picture 15. We know that picture 15 is 16 circles long and 15 circles wide. This gives a total of $15 \times 16 = 240$ circles. But we must compensate for the fact that the yellow circles were originally not there by halving the total number of circles. In other words, the original figure has $240 \div 2 = 120$ circles.

4 DESCRIBING PATTERNS IN DIFFERENT WAYS

4.1 T-shaped numbers

The pattern below is made from squares.



Below are three different methods or plans to calculate the number of squares for pattern 20. Study each one carefully.

Plan A:

To get from 1 square to 4 squares, you have to add 3 squares. To get from 4 squares to 7 squares, you have to add 3 squares. To get from 7 squares to 10 squares, you have to add 3 squares. So continue to add 3 squares for each pattern until pattern 20.

Plan B:

Multiply the pattern number by 3, and subtract 2. So pattern 20 will have $20 \times 3 - 2$ squares.

Plan C:

The number of squares in pattern 5 is 13. So pattern 20 will have $13 \times 4 = 52$ squares because $20 = 5 \times 4$.

4.2 ... and some other shapes

5 EXERCISES

5.1 Exercise 1

1.1 Sequence A: $2; 5; 8; 11; 14; 17; 20; 23; \ldots$

1.2 Sequence B : $4; 5; 8; 13; 20; 29; 40; \ldots$

1.3 Sequence C : $1; 2; 4; 8; 16; 32; 64; \ldots$

1.4 Sequence D : $3; 5; 7; 9; 11; 13; 15; 17; 19; \ldots$

1.5 Sequence E: $4; 5; 7; 10; 14; 19; 25; 32; 40; \ldots$

1.6 Sequence F : $2; 6; 18; 54; 162; 486; \ldots$

1.7 Sequence G : $1; 5; 9; 13; 17; 21; 25; 29; 33; \ldots$

1.8 Sequence H : $2; 4; 8; 16; 32; 64; \ldots$

5.2 Exercise 2

1. Which sequences above are of the same kind as sequence A? Explain your answer.
2. Provide a rule to describe the relationship between the numbers in the sequence. Use this rule to calculate the missing numbers in each sequence.

   2.1 $1; 8; 15; \ldots$

   2.2 $10\ 020; \ldots; \ldots; \ldots; 9\ 980; 9\ 970; \ldots; \ldots; 9\ 940; 9\ 930$

   2.3 $1,5; 3,0; 4,5; \ldots$

   2.4 $2,2; 4,0; 5,8; \ldots$

   2.5 $45\frac{3}{4}\ ; 46\frac{1}{2}\ ; 47\frac{1}{4}\ ; 48\ ; \ldots$

   2.6 $\ldots; 100,49; 100,38; 100,27; \ldots; \ldots; 99,94; 99,83; 99,72$

3. Find the values for $a - g$.

Input number	1	2	3	4	5	d	12	f	n
Input number $+7$	8	a	b	11	c	15	e	30	g

5.3 Exercise 3

1. Piet has proposed a way of continuing Sequence A

   Sequence A : $2; 6; 18; 54; 162; 486; \ldots$

   Sequence B : $2; 4; 8; 16; 32; 64; \ldots$

   Piet: " I looked at the first two terms in the sequence and wrote $2\times? = 6$. When I multiplied the first number by 3, I got the second number: $2 \times 3 = 6$. I then checked to see if I could find the next number if I multiplied 6 by $3 : 6 \times 3 = 18$. I continued checking in this way: $18 \times 3 = 54;\ 54 \times 3 = 162$ and so on. This gave me a rule I can use to extend the sequence and my rule was: multiply each number by 3 to calculate the next number in the sequence."

   1.1 Check whether Piet's reasoning works for Sequence B

2. Describe, in words, the rule for finding the next number in the following sequence. Also, determine the next five terms of the sequence.

   2.1 $1; 10; 100; 1\ 000; \ldots$

   2.2 $16; 8; 4; 2; \ldots$

   2.3 $7\ ;\ \ -21;\ 63;\ \ -189; \ldots$

   2.4 $3;\ 12;\ 48; \ldots$

   2.5 $2\ 187; -729;\ 243; -81; \ldots$

3. Find the values for $a - f$.

Input numbers	1	2	3	4	5	d	12	x
Output numbers	6	a	b	24	c	36	e	f

5.4 Exercise 4

1. Consider sequences A to H and answer the following question:

   Sequence A: $2; 5; 8; 11; 14; 17; 20; 23; \ldots$

   Sequence B: $4; 5; 8; 13; 20; 29; 40; \ldots$

   Sequence C: $1; 2; 4; 8; 16; 32; 64; \ldots$

   Sequence D: $3; 5; 7; 9; 11; 13; 15; 17; 19; \ldots$

   Sequence E: $4; 5; 7; 10; 14; 19; 25; 32; 40; \ldots$

   Sequence F: $2; 6; 18; 54; 162; 486; \ldots$

   Sequence G: $1; 5; 9; 13; 17; 21; 25; 29; 33; \ldots$

   Sequence H: $2; 4; 8; 16; 32; 64; \ldots$

1.1 Which other sequence(s) is/are of the same kind as Sequence B? Explain.

1.2 In what way are Sequences B and E different from the other sequences?

2. Consider the sequence: $10; 17; 26; 37; 50; \ldots$

   2.1 Write down the next five numbers in the sequence.

3. Consider the sequence: $10; 17; 26; 37; 50; \ldots$
   Eric observed that he can get the next term in the sequence as follows:
   $10 + 7 = 17;\ 17 + 9 = 32;\ 26 + 11 = 33$
   Use Eric's method to check if the next terms of the sequence are the following:
   $65; 82; 101; 122; 145$

4. Consider the sequence: $10; 17; 26; 37; 50; \ldots$
   Jens observed that he can calculate the next terms in the sequence as follows:
   $10 + 7 = 17;\ 17 + 9 = 26;\ 26 + 11 = 37.$
   Which of the statement below can Jens use to describe the relationship between the numbers in the sequence shown above. Test the rule for the first three terms of the sequence and then simply respond "yes" or "no".

   4.1 Increase the difference between consecutive terms by two each time.

   4.2 Increase the difference between consecutive terms by one each time.

   4.3 Add two more than you added to get the previous term.

5. Provide a rule to describe the relationship between the numbers in the sequence below. Use your rule to provide the next five numbers in each sequence.

   5.1 $1; 4; 9; 16; 25; \ldots$

   5.2 $2; 13; 26; 41; 58; \ldots$

   5.3 $4; 14; 29; 49; 74; \ldots$

   5.4 $5; 6; 8; 11; 15; 20; \ldots$

5.5 Exercise 5

1. Which sequence(s) are of the same kind as sequence A? Explain.
   Sequence A: $2; 5; 8; 11; 14; 17; 20; 23; \ldots$
   Sequence B: $4; 5; 8; 13; 20; 29; 40; \ldots$
   Sequence C: $1; 2; 4; 8; 16; 32; 64; \ldots$
   Sequence D: $3; 5; 7; 9; 11; 13; 15; 17; 19; \ldots$
   Sequence E: $4; 5; 7; 10; 14; 19; 25; 32; 40; \ldots$
   Sequence F: $2; 6; 18; 54; 162; 486; \ldots$

Sequence G: $1; 5; 9; 13; 17; 21; 25; 29; 33; \ldots$

Sequence H: $2; 4; 8; 16; 32; 64; \ldots$

2. Amanda and Tamara have proposed that Sequence A can be calculated by adding 3 to the previous term in the sequence. Sizwe agrees with Amanda and Tamara's explanation but suggests another method for determining Sequence A. Sizwe reasons that the following rule will also work: Multiply the position of the number by 3 and add 2 to the answer. I can write this rule as a number sentence: Position of the number $\times 3 + 2$. I use my number sentence to check: **1** $\times 3 + 2 = 5$; **2** $\times 3 + 2 = 8$; **3** $\times 3 + 2 = 11$.

   2.1 What do the numbers in bold in Sizwe's number sentence stand for?

   2.2 What does the number 3 in Sizwe's number sentence stand for?

3. Consider the sequence $5; 8; 11; 14; \ldots$ Sizwe has proposed that the terms in the sequence can be calculated as follows:

   Position of the number $\times 3 + 2$

   Apply Sizwe's rule to the sequence and determine:

   3.1 term number 7 of the sequence

   3.2 term number 10 of the sequence

   3.3 the hundredth term of the sequence

4. Consider the sequence: $3; 5; 7; 9; 11; 13; 15; 17; 19; \ldots$
   Sizwe has proposed that the terms in the sequence can be calculated as follows:

   Position of the number $\times x + a$, where x and a are both constants

   4.1 Use Sizwe's explanation to find a rule for this sequence.

   4.2 Determine the 28th term of the sequence.

5.6 Exercise 6

1. Copy and complete the following tables by calculating the missing terms:

   1.1 Find the values for a and b

Position in sequence	1	2	3	4	10	54
Term	4	7	10	13	a	b

1.2 Find the values for a and b

Position in sequence	1	2	3	4	8	16
Term	4	9	14	19	a	b

1.3 Find the values for a, b and c

Position in sequence	1	2	3	4	7	30
Term	3	15	27	a	b	c

2. Find the values for $a - e$. Use the rule position in the sequence $\times$ (position in the sequence +1) to determine it.

Position in sequence	1	2	3	4	5	6
Term	2	a	b	c	d	e

5.7 Exercise 7

1. A factory makes window frames. Type 1 has one windowpane, type 2 has four windowpanes, type 3 has nine windowpanes, and so on.

   1.1 How many windowpanes will there be in type 5?

   1.2 How many windowpanes will there be in type 6?

   1.3 How many windowpanes will there be in type 7?

   1.4 How many windowpanes will there be in type 12? Explain.

2. Find the values for a and b. Show your calculations.

Frame type	1	2	3	4	15	20
Number of windowpanes	1	4	9	16	a	b

5.8 Exercise 8

1. Therese uses circles to form a pattern of triangular shapes. If the pattern is continued, how many circles must Therese have.

1.1 In the bottom row of picture 5?

1.2 In the second row from the bottom of picture 5?

1.3 In the third row from the bottom of picture 5?

1.4 In the second row from the top of picture 5?

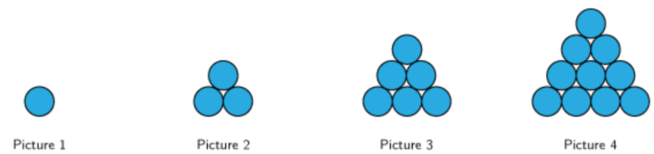
1.5 In the top row of picture 5?

1.6 In total in picture 5? Show your calculations.

1.7 To form triangle picture 7? Show the calculation.

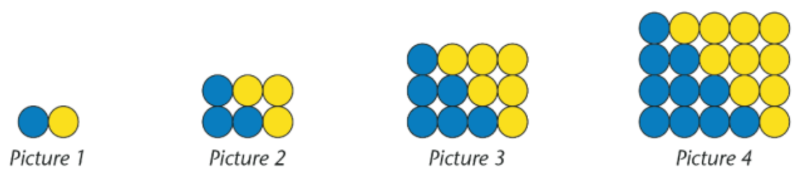
1.8 To form triangle picture 8? Show the calculation.

2. Find the values $a - d$. Show all your work.



Picture number	1	2	3	4	5	6	12	15
Number of circles	1	3	6	10	a	b	c	d

3. Consider the arrangement in the figure. We have added the yellow circles to the original blue circles and then rearranged the circles in such a way that they are in a rectangular form. Picture 2 is three circles long and two circles wide. Copy and complete the following sentences:



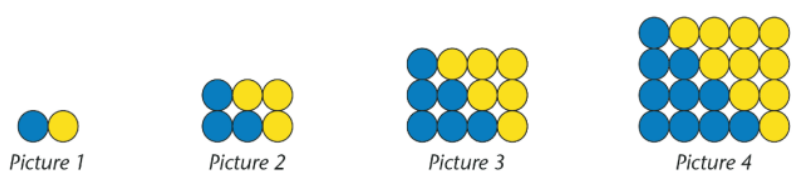
3.1 Picture 3 is ...... circles long and......circles wide.

3.2 Picture 1 is......circles long and......circle wide.

3.3 Picture 4 is ...... circles long and ...... circles wide.

3.4 Picture 5 is ...... circles long and ...... circles wide.

4. Consider the arrangement below. Picture 2 is three circles long and two circles wide and consists of 6 circles. Using this information, determine how many circles will there be in a picture that is:



4.1 ten circles long and nine circles wide?

4.2 seven circles long and six circles wide?

4.3 six circles long and five circles wide?

4.4 20 circles long and 19 circles wide?

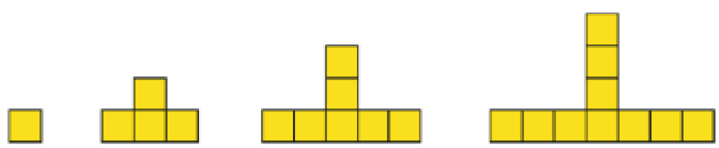
5. Consider the arrangement in the figure. We have added the yellow circles to the original blue circles and then rearranged the circles in such a way that they are in a rectangular form. Suppose we want to use a quick method to determine the number of circles in picture 15. We know that picture 15 is 16 circles long and 15 circles wide. This gives a total of $15 \times 16 = 240$ circles. But we must compensate for the fact that the yellow circles were originally not there by halving the total number of circles. In other words, the original figure has $240 \div 2 = 120$ circles. Use the above reasoning to calculate the number of circles in:

5.1 Picture 20

5.2 Picture 35

5.9 Exercise 9

1. The pattern below is made from squares.



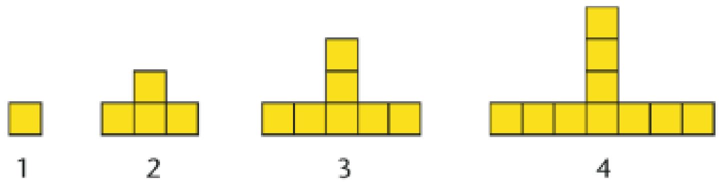
1.1 How many squares will there be in pattern 5?

1.2 How many squares will there be in pattern 15?

1.3 Find the values of $a - c$.

Pattern number	1	2	3	4	5	6	20
Number of squares	1	4	7	10	a	b	c

2. The pattern below is made from squares.



You can use the following three plans (or methods) to calculate the number of squares for pattern 20. Study each one carefully.

Plan A :

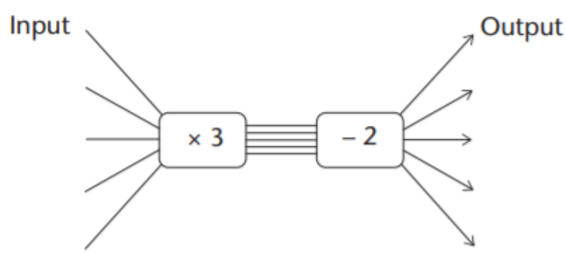
To get from one square to four squares, you have to add three squares. To get from four squares to seven squares, you have to add three squares. To get from seven squares to ten squares, you have to add three squares. Continue to add three squares for each pattern until pattern 20 .

Plan B : Multiply the pattern number by three and subtract two. Pattern 20 will therefore have $20 \times 3 - 2$ squares.

Plan C : The number of squares in pattern 5 is 13 . Pattern 20 will therefore have $13 \times 4 = 52$ squares because $20 = 5 \times 4$

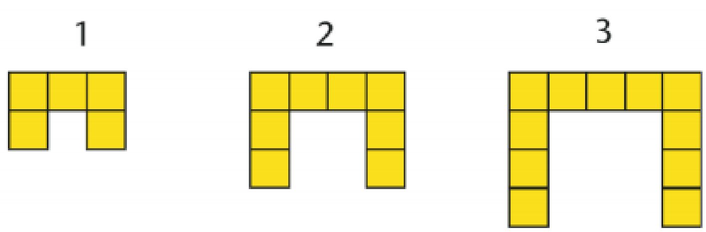
2.1 Which method or plan (A, B or C) will give the right answer? Explain why.

2.2 Can this flow diagram be used to calculate the number of squares?



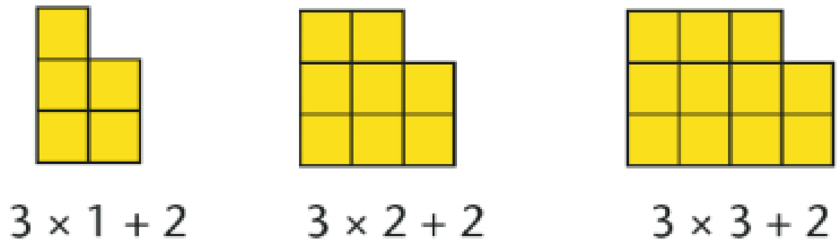
5.10 Exercise 10

1. Three figures are given below.



1.1 If the pattern is continued, how many tiles will there be in the 17th figure?
Answer this question by analysing what happens.

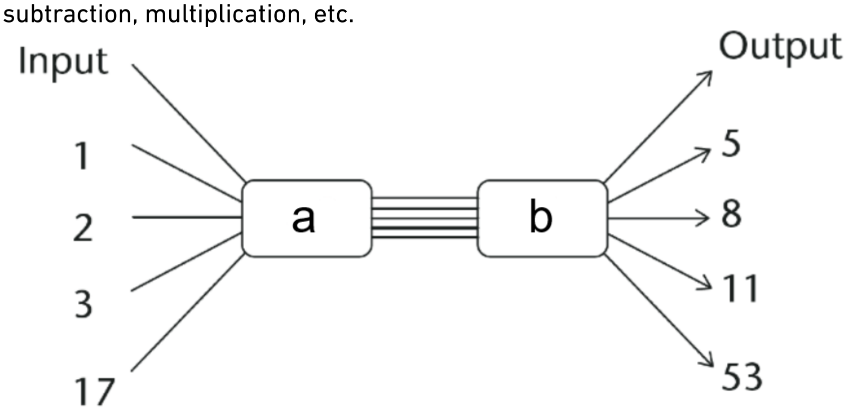
1.2 Thato decides that it easier for him to see the pattern when the tiles are rearranged as shown below:



Use Thato's method to determine the number of tiles in the 23rd figure.

1.3 How many tiles will there be in the 50th figure if the pattern is continued?

1.4 Complete the following flow diagram by choosing appropriate operators for (a) and (b), so that it can be used to calculate the number of tiles in any figure of the pattern. Operators refers to addition, subtraction, multiplication, etc.



5.11 Exercise 11

1. Write down the next four terms in each sequence. Also explain, in each case, how you figured out what the terms are.

   1.1 $2; 4; 8; 14; 22; 32; 44; \ldots$

   1.2 $2; 6; 18; 54; 162; \ldots$

   1.3 $1; 7; 13; 19; 25; \ldots$

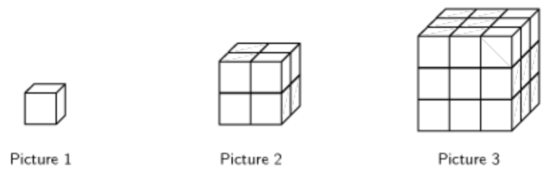
2. Consider the following table:

Position in sequence	1	2	3	4	5	7	10
Term	3	10	17	a	b	c	d

   2.1 Determine the values for $a - d$

   2.2 Write the rule to calculate the term from the position in the sequence in words.

3. Consider the stacks below.



3.1 How many cubes will there be in stack 5?

3.2 Determine the values for $a - d$

Stack number	1	2	3	4	5	6	10
Number of cubes	1	8	27	a	b	c	d

3.3 Write down the rule to calculate the number of cubes for any stack number.

6 ANSWERS FOR EXERCISES

6.1 Exercise 1

1.1 $26; 29; 32$ – Add 3 each time

1.2 $53; 68; 85$ - Add consecutive odd numbers: 1 then 3 then 5 and so on.

1.3 $128; 256; 512$ - Multiply by 2 each time.

1.4 $21; 23; 25 -$ Add 2 each time or count in odd numbers.

1.5 $49; 59; 70 -$ Add consecutive natural numbers: 1 then 2 then 3 and so on.

1.6 $1\ 458; 4\ 374; 13\ 122$ - Multiply by 3 each time.

1.7 $37; 41; 45 -$ Add 4 each time.

1.8 $128; 256; 512$ - Multiply by 2 each time.

6.2 Exercise 2

1. Sequence D and sequence G. A constant number is added each time.

2. 2.1 $22; 29; 36; 43; 50$ - add 7

   2.2 $10\ 020$; **10 010** ; **10 000** ; **9 990** ; $9\ 980$; $9\ 970$; **9 960** ; **9 950** ; $9\ 940$; $9\ 930$ - subtract 10

   2.3 $1,5$; $3,0$; $4,5$; **6, 0** ; **7, 5** ; **9, 0** ; **10, 5** ; **12, 0** - add $1,5$

   2.4 $2,2$; $4,0$; $5,8$; **7, 6** ; **9, 4** ; **11, 2** ; **13, 0** ; **14, 8** $-$ add $1,8$

   2.5 $45\frac{3}{4}$; $46\frac{1}{2}$; $47\frac{1}{4}$; 48 ; $\mathbf{48\frac{3}{4}}$; $\mathbf{49\frac{1}{2}}$; $\mathbf{50\frac{1}{4}}$; **51** ; $\mathbf{51\frac{3}{4}}$ $-$ add $\frac{3}{4}$

   2.6 **100, 60** ; $100,49$; $100,38$; $100,27$; **100, 16** ; **100, 05** ; $99,94$; $99,83$; $99,72$ $-$ subtract $0,11$

3. $a = 9$, $b = 10, c = 12$
   $d = 8$, $e = 19$, $f = 23$,
   $g = n + 7$

6.3 Exercise 3

1. Yes, it works; you multiply each number by 2 to find the next number.

2. 2.1 Multiply each number by 10 to calculate the next number.
   $10\ 000$; $100\ 000$; $1\ 000\ 000$; $10\ 000\ 000$; $100\ 000\ 000$

   2.2 Divide each number by 2 and calculate the next number.
   1 ; $0,5$; $0,25$; $0,125$; $0,0625$

   2.3 Multiply by -3 and calculate the next number.
   567 ; $-1\ 701$; $5\ 103$; $-15\ 309$; $45\ 927$

   2.4 Multiply each number by 4 and calculate the next number.
   192 ; 768 ; $3\ 072$; $12\ 288$; $49\ 152$

   2.5 Divide each number by -3 and calculate the next number.
   27 ; -9 ; 3 ; -1 ; $\frac{1}{3}$

3. $a = 12,\ b = 18,\ c = 30,$
   $d = 6,\ e = 72,\ f = 6x$

6.4 Exercise 4

1.1 Sequence E. Different numbers are added to each term.

1.2 A different calculation is done to find each new term (neither adding the same number nor multiplying by the same number).

2. $65 \,;82 \,;101 \,;122 \,;145$

3. $50 + 15 = 65$
   $65 + 17 = 82$
   $82 + 19 = 101$
   $101 + 21 = 122$
   $122 + 23 = 145$

4.1 Yes. Test: The difference between 17 and 10 is 7. Increase the difference by 2 to get 9. To get the next term, add 9 to 17 to get 26. Continue this way of reasoning to get the other terms.

4.2 No. Test: The difference between 17 and 10 is 7 . Increase the difference by 1 and you get 8 . To get the next term add 8 to 17 . It gives 25 which is not the next term in the sequence.

4.3 Yes. Test: The difference between 17 and 10 is 7 . Add two more and you get 9 . To get the next term, add 9 to 17 to get 26 . Continue this way of reasoning to get the other terms

5.1 $36; 49; 64; 81; 100$
The term value is the square of the term number.

5.2 $77; 98; 121; 146; 173-$
Add consecutive odd numbers: $11; 13; 15; \ldots$

5.3 $104; 139; 179; 224; 274-$
Add consecutive multiples of $5; 10; 15; 20; \ldots$

5.4 $26; 33; 41; 50; 60-$
Add consecutive natural numbers: $1; 2; 3; \ldots$

6.5 Exercise 5

1. Sequence D and sequence G. A constant number is added each time.

2. 2.1 The term number or the position of the number in the sequence.
   2.2 It is the constant difference between the terms.

3. 3.1 $7 \times 3 + 2 = 21 + 2 = 23$
   3.2 $10 \times 3 + 2 = 30 + 2 = 32$

3.3 hundredth term $= 100 \times 3 + 2 = 300 + 2 = 302$

4.1 Position of the number $\times 2 + 1$ (or $2n + 1$). $x = 2$ and $a = 1$

4.2 twenty-eighth term $= 28 \times 2 + 1 = 56 + 1 = 57$

6.6 Exercise 6

1.1 $a = 31$
$b = 163$

1.2 $a = 39$
$b = 79$

1.3 $a = 39$
$b = 75$
$c = 351$

2. $a = 6$
$b = 12$
$c = 20$
$d = 30$
$e = 42$

6.7 Exercise 7

1. 1.1 25

   1.2 36

   1.3 49

   1.4 There will be 144 windowpanes. To find the number of windowpanes you square the type number. So for type 12 there are $12 \times 12 = 144$ windowpanes.

2. $a = 225$ and $b = 400$

6.8 Exercise 8

1. 1.1 5

   1.2 4

   1.3 3

   1.4 2

   1.5 1

1.6 15 circles: $5 + 4 + 3 + 2 + 1 = 15$ (add the number of circles in each row)

1.7 $7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$ circles

1.8 $8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36$ circles

2. $a = 15$
   $b = 21$
   $c = 78$
   $d = 120$

3. 3.1 Picture 3 is 4 circles long and **3** circles wide.

   3.2 Picture 1 is **2** circles long and **1** circle wide.

   3.3 Picture 4 is **5** circles long and **4** circles wide.

   3.4 Picture 5 is **6** circles long and **5** circles wide.

4. 4.1 90

   4.2 42

   4.3 30

   4.4 380

5. 5.1 210

   5.2 630

6.9 Exercise 9

1. 1.1 13

   1.2 43

   1.3 $a = 13$
   $b = 16$
   $c = 58$

2. 2.1 Both A and B. Both plans work if you try them out.

   2.2 Yes. Test: An input of 1 produces 1, an input of 2 produces 4 and so on.

6.10 Exercise 10

1.1 53 tiles

1.2 71 tiles

1.3 152 tiles

1.4 The operator for (a) must be the multiplication of 3. The operator for (b) must be the addition of 2.

(a) $= \times 3$

(b) $= +2$

6.11 Exercise 11

1.1 $58 ; 74 ; 92 ; 112$

Add consecutive even numbers: 2 then 4 then 6 and so on.

1.2 $486 ; 1458 ; 4374 ; 13\,122$

Multiply each number by 3 to find the next term.

1.3 $31 ; 37 ; 43 ; 49$

Add 6 to find the next term.

2.1 $a = 24$

$b = 31$

$c = 45$

$d = 66$

2.2 Multiply the position number by 7 and subtract 4.

3.1 125

3.2 $a = 64$

$b = 125$

$c = 216$

$d = 1000$

3.3 Cube the stack number, i.e. (stack number) ^{3}.