



CHAPTER 7

Algebraic Equations 1

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April 20, 2021

Yes

1 SETTING UP EQUATIONS

NOTE

An **equation** is a mathematical sentence that is true for some numbers but false for other numbers. The following are examples of equations:

$$x + 3 = 11 \text{ and } 2^x = 8$$

$x + 3 = 11$ is true if $x = 8$, but false if $x = 3$.

When we look for a number or numbers that make an equation true we say that we are **solving the equation**. For example, $x = 4$ is the **solution** of $2x + 8$ because it makes $2x + 8$ true. (Check: $2 \times 4 = 8$)

2 SOLVING EQUATIONS BY INSPECTION

2.1 The answer is in plain sight

NOTE

Two or more equations can have the same solution. For example, $5x = 10$ and $x + 2 = 4$ have the same solution; $x = 2$ is the solution for both equations.

NOTE

Two equations are called **equivalent** if they have the same solution.

3 MORE EXAMPLES

3.1 Looking for and checking solutions

NOTE

When a certain number is the solution of an equation we say that the number **satisfies** the equation. For example, $x = 4$ satisfies the equation $3x = 12$ because $3 \times 4 = 12$.

4 EXERCISES

4.1 Exercise 1: Looking for numbers to make statements true

1. Are the following statements true or false? Justify your answer.

1.1 $x - 3 = 0$ if $x = -3$

1.2 $x^3 = 8$ if $x = -2$

1.3 $3x = -6$ if $x = -3$

1.4 $3x = 1$ if $x = 1$

1.5 $6x + 5 = 47$ if $x = 7$

2. Find the original number. Show your reasoning.

2.1 A number multiplied by 10 is 80

2.2 Add 83 to a number and the answer is 100

2.3 Divide a number by 5 and the answer is 4

2.4 Multiply a number by 4 and the answer is 20

2.5 Twice the number is 100

2.6 A number raised to the power 5 is 32

2.7 A number raised to the power 4 is -81

2.8 Fifteen times a number is 90

2.9 93 added to a number is -3

2.10 Half a number is 15

3. Write the equations below in words using "a number" in place of the letter symbol x . Then write what you think "the number" is in each case.

For example: $4 + x = 23$. Four plus a number equals twenty-three. The number is 19

3.1 $8x = 72$

3.2 $\frac{2x}{5} = 2$

3.3 $2x + 5 = 21$

3.4 $12 + 9x = 30$

3.5 $30 - 2x = 40$

3.6 $5x + 4 = 3x + 10$

4.2 Exercise 2: The answer is in plain sight

NOTE

You can read the solutions of an equation from a table.

1. Seven equations are given in the following table. Use the table to find out for which of the given values of x it will be the that the left-hand side of the equation is equal to the right-hand side.

x	-3	-2	-1	0	1	2	3	4
$2x + 3$	-3	-1	1	3	5	7	9	11
$x + 4$	1	2	3	4	5	6	7	8
$9 - x$	12	11	10	9	8	7	6	5
$3x - 2$	-11	-8	-5	-2	1	4	7	10
$10x - 7$	-37	-27	-17	-7	3	13	23	33
$5x + 3$	-12	-7	-2	3	8	13	18	23
$10 - 3x$	19	16	13	10	7	4	1	-2

1.1 $2x + 3 = 5x + 3$

1.2 $5x + 3 = 9 - x$

1.3 $2x + 3 = x + 4$

1.4 $10x - 7 = 5x + 3$

1.5 $3x - 2 = x + 4$

1.6 $9 - x = 2x + 3$

1.7 $10 - 3x = 3x - 2$

2. A previous questions gave seven equations in a table:

x	-3	-2	-1	0	1	2	3	4
$2x + 3$	-3	-1	1	3	5	7	9	11
$x + 4$	1	2	3	4	5	6	7	8
$9 - x$	12	11	10	9	8	7	6	5
$3x - 2$	-11	-8	-5	-2	1	4	7	10
$10x - 7$	-37	-27	-17	-7	3	13	23	33
$5x + 3$	-12	-7	-2	3	8	13	18	23
$10 - 3x$	19	16	13	10	7	4	1	-2

Various equations are equal to each other, which can be verified by inspection. There are solutions for x where the following equations are equal:

a) $2x + 3 = 5x + 3$

b) $5x + 3 = 9 - x$

c) $2x + 3 = x + 4$

d) $10x - 7 = 5x + 3$

e) $3x - 2 = x + 4$

f) $9 - x = 2x + 3$

g) $10 - 3x = 3x - 2$

Which of these equations a) to g) have the same solutions?

Explain.

3. Copy and complete the following table. Then answer the questions that follow:

x	0	5	10	15	20	25	30	35	40
$2x + 3$									
$3x - 10$									

3.1 Can you find solution for $2x + 3 = 3x - 10$ in the table?

3.2 What happens to the values of $2x + 3$ and $3x - 10$ as x increases? Do they become bigger or smaller?

3.3 Is there a point where the value of $3x - 10$ becomes bigger or smaller than the value of $2x + 3$ as the value of x increases? If so, between which x values does this happen?

3.4 What is the value for x where $2x + 3 = 3x - 10$?

4.3 Exercise 3: Looking for and checking solutions

NOTE

You can also do a search by narrowing down the possible solution to an equation.

NOTE

The point where the two expressions are equal is called the **break-even point**.

1. Solve the following equations:

1.1 $x - 3 = 4$

1.2 $x + 2 = 9$

1.3 $3x = 21$

1.4 $3x + 1 = 22$

2. Solve the following equations. Show your working.

2.1 $12x = 84$

2.2 $\frac{84}{x} = 12$

2.3 $48 = 8k + 8$

2.4 $19 - 8m = 3$

2.5 $6y - 4 = 20$

2.6 $x^3 = -64$

2.7 $5^x = 125$

2.8 $2^x = 8$

2.9 $x^2 = 9$

3. What makes the following equations true? Check your answers.

3.1 $m + 8 = 100$

3.2 $x + 60 = 80$

3.3 $26 - k = 0$

3.4 $105 \times y = 0$

3.5 $k \times 10 = 10$

3.6 $5x = 100$

3.7 $\frac{15}{t} = 5$

3.8 $\frac{t}{5} = 3$

4. Solve the following equations by inspection. Check your answers.

4.1 $12x + 14 = 50$

4.2 $15m + 25 = 100$

4.3 $\frac{100}{x} = 20$

4.4 $7m + 5 = 40$

4.5 $2x + 8 = 10$

4.6 $3x + 10 = 31$

4.7 $-1 + 2x = -11$

4.8 $2 + \frac{x}{7} = 5$

4.9 $64 + 9x = 100$

4.10 $\frac{2x}{6} = 4$

NOTE

"Searching" for the solution of an equation by using tables or by narrowing down to the possible solution is called **solution by inspection**.

5 ANSWERS FOR EXERCISES

5.1 Exercise 1

1.1 False. $-3 - 3 = -6$

1.2 False. $(-2)(-2)(-2) = -8$

1.3 False. $3(-3) = 9$

1.4 False. $3(1) = 3$

1.5 True. $6(7) + 5 = 47$

2.1 8

2.2 17

2.3 20

2.4 5

2.5 50

2.6 2

2.7 Does not exist

2.8 6

2.9 -96

2.10 30

3.1 Eight times a number is seventy-two. The number is 9

3.2 Two fifths of a number equals two. The number is 5.

3.3 Two times a number plus five equals twenty-one. The number is 8.

3.4 Twelve plus nine times a number equals thirty. The number is 2.

3.5 Thirty minus two times a number equals forty. The number is -5 .

3.6 Five times a number plus four equals three times that same number plus ten. The number is 3.

5.2 Exercise 2

1.1 $x = 0$

1.2 $x = 1$

1.3 $x = 1$

1.4 $x = 2$

1.5 $x = 3$

1.6 $x = 2$

1.7 $x = 2$

2. b) and c) have the same solutions. Furthermore, d), f), and g) have the same solutions.

3.1 No. There is no solution in the table.

3.2 Both expressions become bigger

3.3 Yes. Between $x = 10$ and $x = 15$.

3.4 $x = 13$

5.3 Exercise 3

1.1 $x = 7$

1.2 $x = 7$

1.3 $x = 7$

1.4 $x = 7$

2.1 $x = 7$

2.2 $x = 7$

2.3 $k = 5$

2.4 $m = 2$

2.5 $y = 4$

2.6 $x = -4$

2.7 $x = 3$

-
- 2.8 $x = 3$
- 2.9 $x = 3$
- 3.1 $m = 92$
- 3.2 $x = 20$
- 3.3 $k = 26$
- 3.4 $y = 0$
- 3.5 $k = 1$
- 3.6 $x = 20$
- 3.7 $t = 3$
- 3.8 $t = 15$
- 4.1 $x = 3$
- 4.2 $m = 5$
- 4.3 $x = 5$
- 4.4 $m = 5$
- 4.5 $x = 1$
- 4.6 $x = 7$
- 4.7 $x = -5$
- 4.8 $x = 21$
- 4.9 $x = 4$
- 4.10 $x = 12$