



# CHAPTER 10

*Construction Of Geometric Figures*

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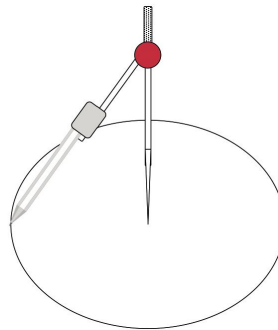
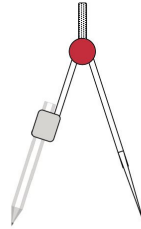
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October 19, 2021

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# 1 CONSTRUCTION OF GEOMETRIC FIGURES

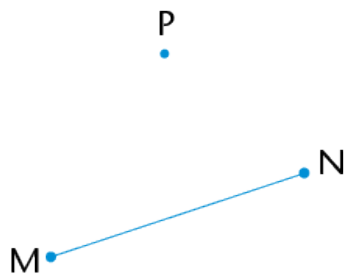


## 1.1 Using circles to draw perpendicular lines

### Case 1: A perpendicular through a point that is not on the line segment

Copy the steps below:

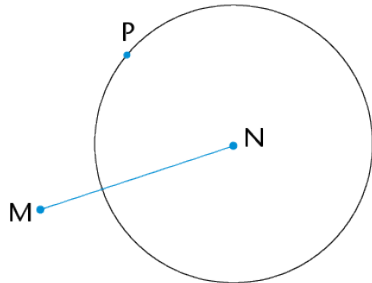
You are given line segment  $MN$  with point  $P$  at a distance from it. You must construct a line that is perpendicular to  $MN$ , so that the perpendicular passes through point  $P$ .



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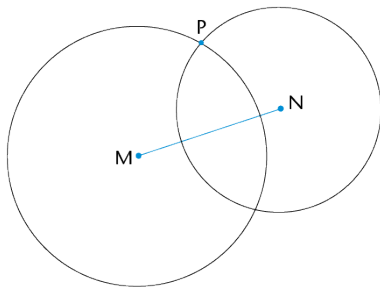
**step 1**

Use your compass to draw a circle whose centre is the one end point of the line segment (N) and passes through the point (P).



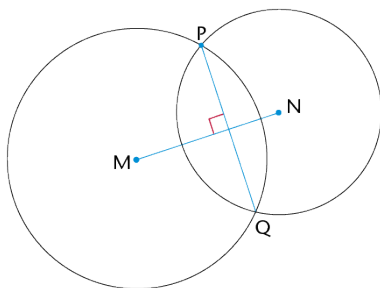
**step 2**

Repeat step 1, but make the centre of your circle the other end point of the line segment (M).



**step 3**

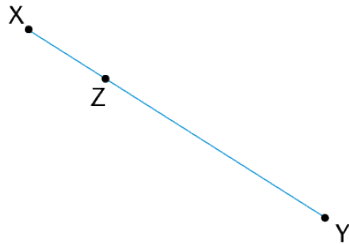
Join the points where the circles intersect:  $PQ \perp MN$ .



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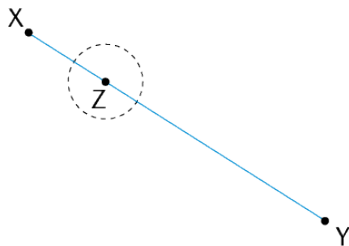
### Case 2: A perpendicular at a point on the line segment

You are given line segment  $XY$  with point  $Z$  on it. You must construct a perpendicular line passing through  $Z$ .



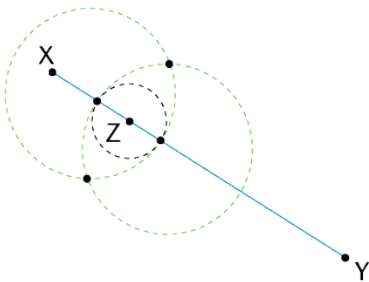
#### step 1

Use your compass to draw a circle whose centre is  $Z$ . Make its radius smaller than  $ZX$ . Note the two points where the circle intersects  $XY$ .



#### step 2

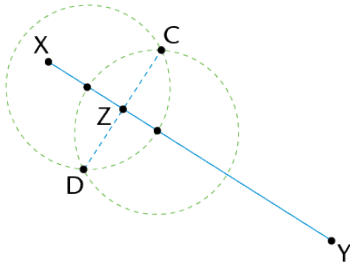
Set your compass wider than it was for the circle with centre  $Z$ . Draw two circles of the same size whose centres are at the two points where the first (black) circle intersects  $XY$ . The two circles (green) will overlap.



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**step 3**

Join the intersection points of the two overlapping circles. Mark these points C and D:  $CD \perp XY$  and pass through point Z.



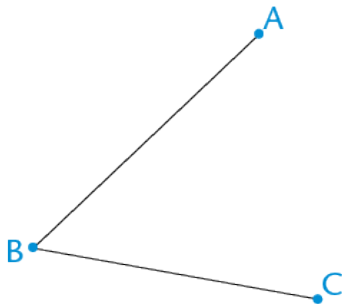
## 1.2 Bisecting angles

### Using circles to bisect angles

**Bisect** an angle. Do the steps yourself in your exercise book. The word **bisect** means 'to cut in half'.

#### Example

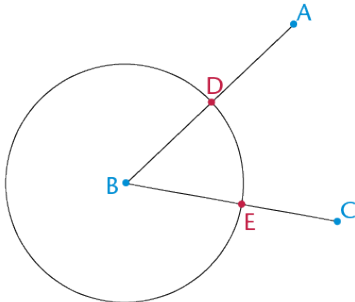
You are given  $\hat{A}BC$ . You must bisect the angle.



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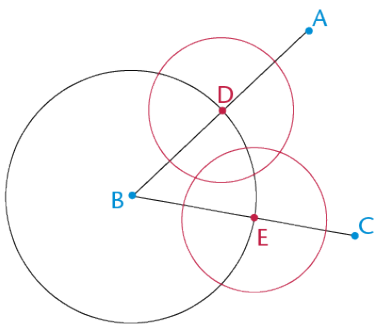
**step 1**

Draw a circle with centre B to mark off an equal length on both arms of the angle. Label the points of intersection D and E:  $DB = BE$ .

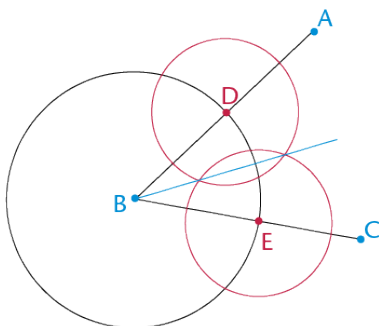


**step 2**

Draw two equal circles with centres at D and at E. Make sure the circles overlap.



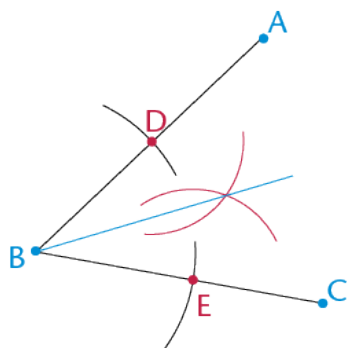
**step 3**



Same construction as in step 3 above

**Note**

Can you explain why the method above works to bisect an angle?



### Note

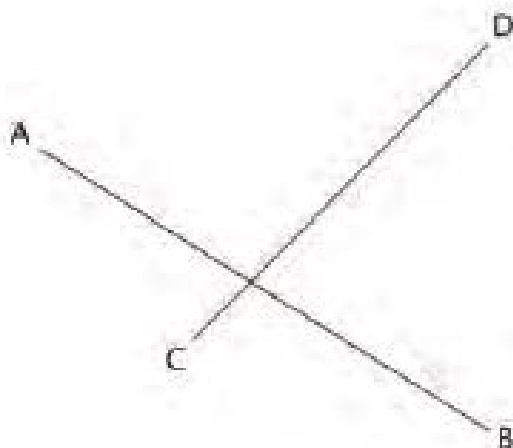
Can you also see that we need not draw full circles, but can merely use parts of circles (arcs) to do the above construction?

## 2 EXERCISES

### 2.1 Exercise 1

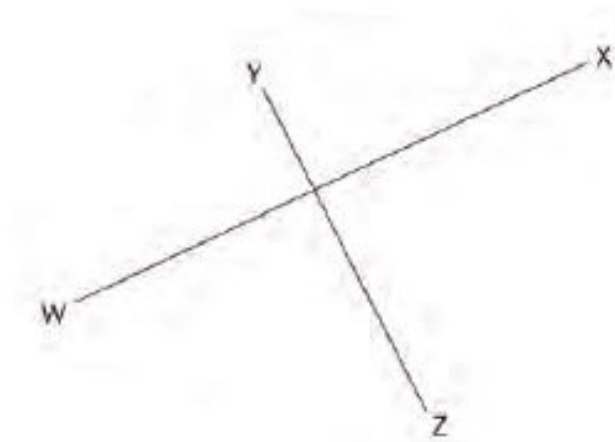
1. What does it mean if we say that two lines are perpendicular?
2. Use your protractor to measure the angles between the following pairs of lines. Then state whether they are perpendicular or not.

2.1 .



2.2 .





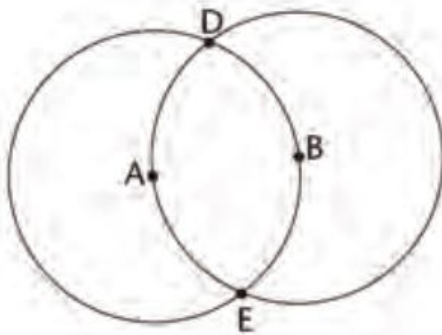
3. Do the following:

3.1 Use a compass to draw two overlapping circles of different sizes. Draw a line through the points where the circles intersect (overlap). Draw a line to join the centres of the circles. Use your protractor to measure the angles between the intersecting lines.

3.2 What can you say about the intersecting lines?

4. What do you know about the sides and angles in an equilateral triangle?

5. Refer to the two circles with properties: The circles are the same size, each circle passes through the other circle's centre, the centres of the circles are labelled as  $A$  and  $B$ , and the points of intersection of the circle are labelled  $D$  and  $E$ . What can you say about the lengths of  $AB$ ,  $AD$  and  $DB$ ?



6. What kind of triangle is  $ABD$ ?

7. Therefore, what do you know about  $\hat{A}$ ,  $\hat{B}$  and  $\hat{D}$ ?

8. Copy and complete the table below. The first one has been done for you.

Angle	Multiples below $360^\circ$	Angle	Multiples below $360^\circ$
$30^\circ$	$30^\circ; 60^\circ; 90^\circ; 120^\circ; 150^\circ; 180^\circ;$ $210^\circ; 240^\circ; 270^\circ; 300^\circ; 330^\circ$	$45^\circ$	
$60^\circ$		$90^\circ$	

9. Construct the following angles without using a protractor. You will need to do more than one construction to create each angle.

9.1  $120^\circ$

9.2  $135^\circ$

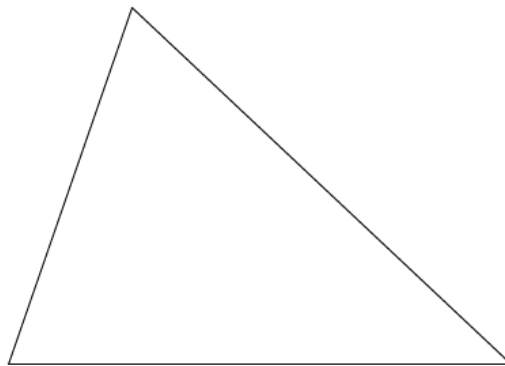
9.3  $270^\circ$

9.4  $240^\circ$

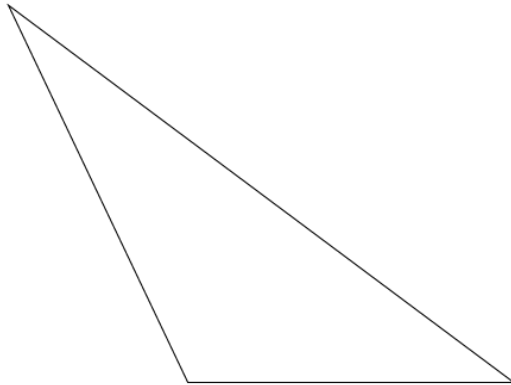
9.5  $150^\circ$

## 2.2 Exercise 2

1. Copy the acute triangle below. Bisect each of the angles of the acute triangle. Extend each of the bisectors to the opposite side of the triangle. What do you notice?

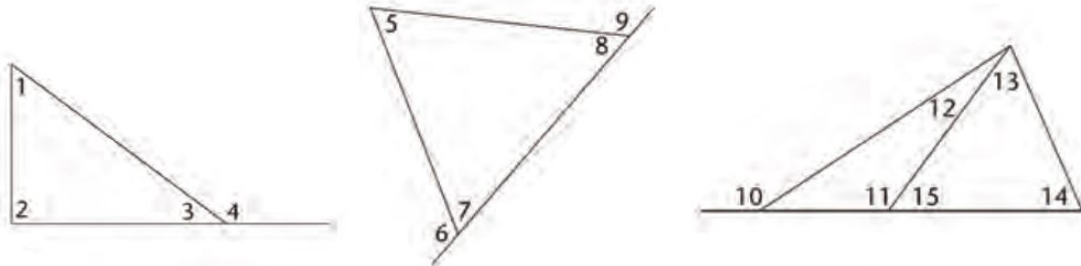


2. Copy the obtuse angle below, Do the same with the obtuse triangle. What do you notice?

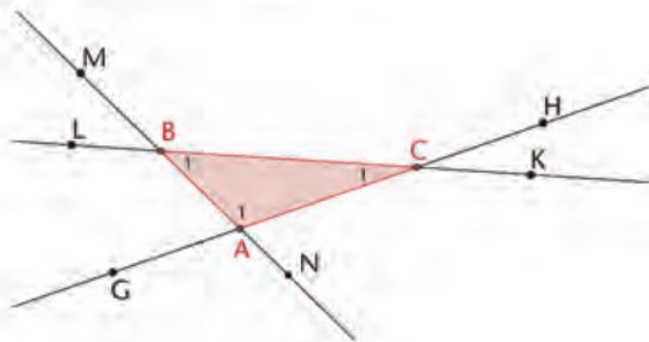


3. Copy the following table and name each exterior angle and its two interior opposite angles below.

Ext. $\angle$					
Int. opp. $\angle$ s					



4.  $\triangle ABC$  below has each side extended in both directions to create six exterior angles.



4.1 Write down the names of the interior angles of the triangle.

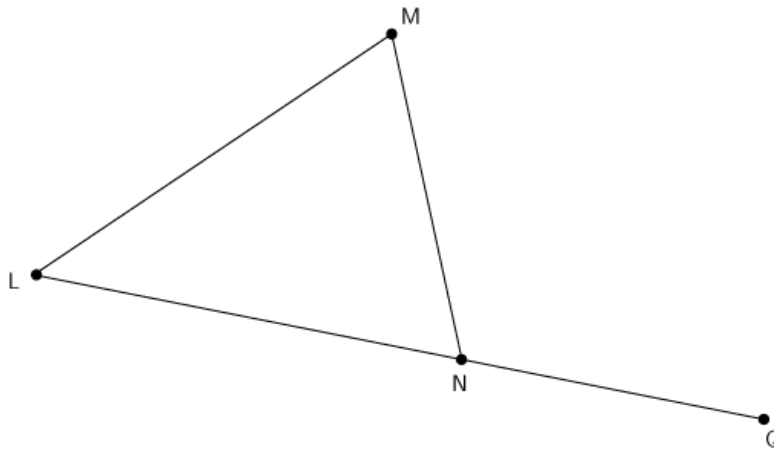
4.2 Since a triangle has three sides that can be extended in both directions, there are two exterior angles at each vertex. Write down the names of all the exterior angles of the triangle.

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4.3 Explain why  $M\hat{B}L$  is not an exterior angle of  $\triangle ABC$

4.4 Write down two other angles that are neither interior nor exterior.

5. Consider  $\triangle LMN$ .



5.1 Write down the name of the exterior angle.

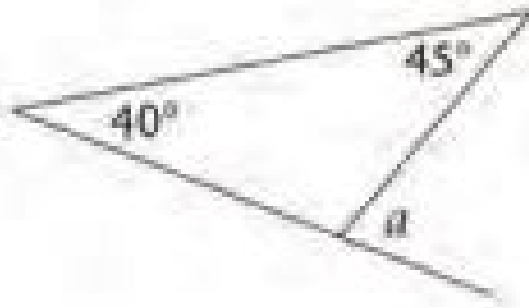
5.2 Use a protractor to measure the interior angles and the exterior angle. Copy the drawing and write the measurements on the drawing. Use your findings in question 2 to complete the following sum:

$$L\hat{M}N + M\hat{L}N = \dots$$

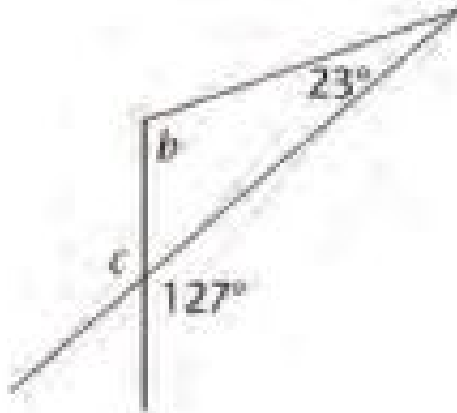
6. What is the relationship between the exterior angle of a triangle and the sum of the interior opposite angles?

7. Work out the sizes of angles  $a$  to  $f$  below, without using a protractor. Give reasons for the statements you make as you work out the answers.

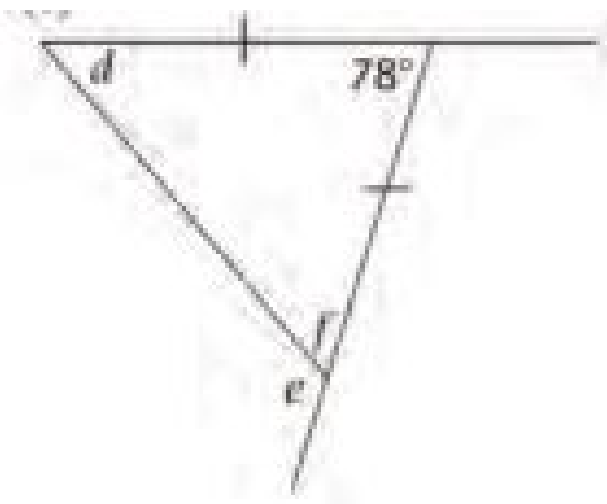
7.1 .



7.2 .



7.3 .



## 2.3 Exercise 3

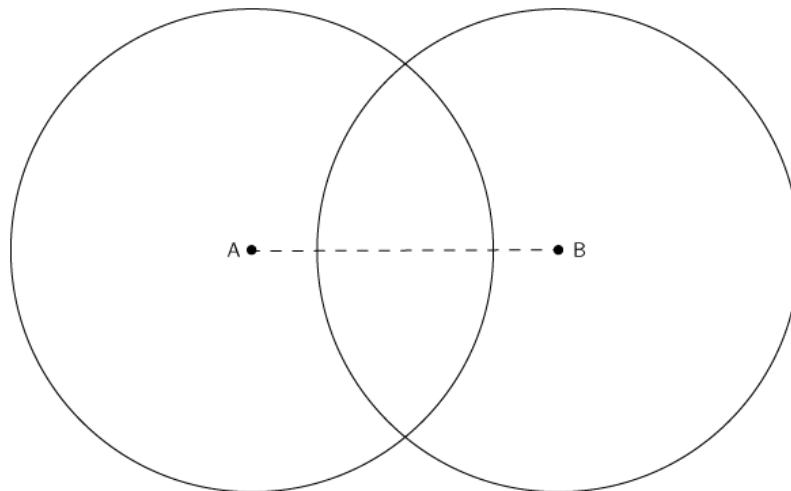
1. Copy and complete the table. Write down whether or not we can construct a congruent triangle when the following conditions are given.

Conditions	Congruent?
Three sides (SSS)	
Two sides (SS)	
Three angles (AAA)	
Two angles and a sides (AAS)	
Two sides and an angle not between the sides (SSA)	
Two sides and an angle between the sides (SAS)	
Right-angled with the hypotenuse and a side (RHS)	

2. How many sides does a quadrilateral have?  
3. How many angles does a quadrilateral have?  
4. How many diagonals does a quadrilateral have?

## 2.4 Exercise 4

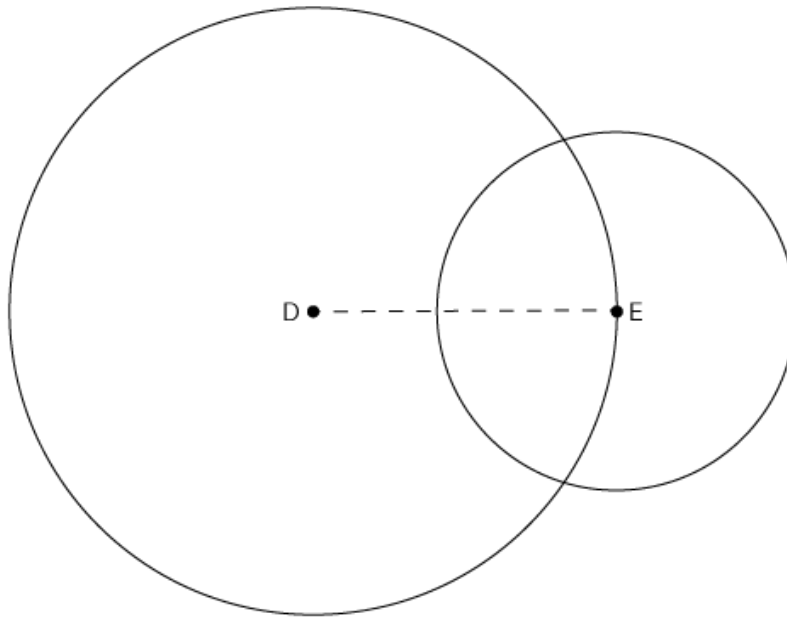
1. Below are two overlapping circles with centres  $A$  and  $B$ . The circles are the same size.



- 1.1 Construct a rhombus inside the circles by joining the centre of each circle with the intersection points of the circles. Join  $AB$ . Copy the circles and construct the perpendicular bisector of  $AB$ . What do you find?  
1.2 Do the diagonals bisect each other?

1.3 Complete the sentence: The diagonals of a thombus will always . . .

2. Below are two overlapping circles with centres  $D$  and  $E$ . The circles are different sizes.



Copy the circles and construct a kite by joining the centre points of the circles to the intersection points of the circles. Draw in the diagonals of the kite. Mark all lines that are of the same length.

2.1 Are the diagonals of the kite perpendicular?

2.2 Do the diagonals of the kite bisect each other?

2.3 What is the difference between the diagonals of a rhombus and those of a kite?

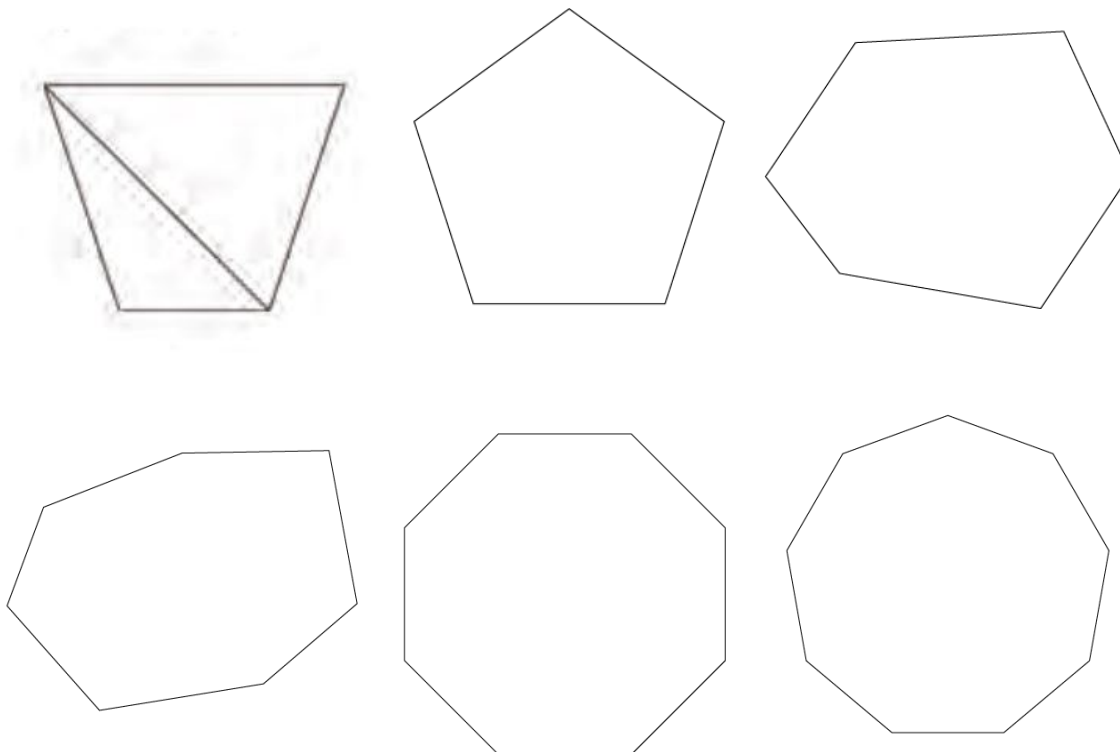
3. Draw a parallelogram, rectangle and square onto grid paper. Draw in the diagonals of the quadrilaterals. Indicate on each shape all the lengths of the diagonals that are equal. (Use a ruler.) Use the information you have found to copy and complete the table below. Fill in "yes" or "no".

Quadrilateral	Diagonals equal	Diagonals bisect	Diagonals meet at $90^\circ$
Parallelogram			
Rectangle			
Square			

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4. We can divide a quadrilateral into two triangles by drawing in one diagonal.

Copy the polygons below and draw in diagonals to divide each of the polygons into as few triangles as possible. Write down the number of triangles in each polygon.



## 3 ANSWERS FOR EXERCISES

### 3.1 Exercise 1

1. Two lines are perpendicular when the angle of intersection is a right angle.

2.1 The angles are  $103^\circ$ ,  $77^\circ$ ,  $103^\circ$  and  $77^\circ$ . The lines are not perpendicular.

2.2 The angles are all equal to  $90^\circ$ , so the lines are perpendicular.

3.1  $90^\circ$

3.2 If all the angles at the point of intersection are equal, then they are all right angles, and the lines are perpendicular to each other.



4.1 The sides are equal and all angles are equal to  $60^\circ$ .

5.1 All are equal, since they are radii of circles that are the same size.

6.1 Equilateral

7.1 They are all equal to  $60^\circ$ .

	Angle	Multiples below $360^\circ$	Angle	Multiples below $360^\circ$
8.1	$30^\circ$	$30^\circ; 60^\circ; 90^\circ; 120^\circ; 150^\circ; 180^\circ;$ $210^\circ; 240^\circ; 270^\circ; 300^\circ; 330^\circ$	$45^\circ$	$45^\circ; 90^\circ; 135^\circ; 180^\circ; 225^\circ; 270^\circ;$ $315^\circ$
	$60^\circ$	$60^\circ; 120^\circ; 180^\circ; 240^\circ; 300^\circ$	$90^\circ$	$90^\circ; 180^\circ; 270^\circ$

9.1 Draw a circle and draw its diameter. At the centre, construct an angle of  $60^\circ$  using another circle with the same radius. The supplementary angle of the  $60^\circ$  angle is  $120^\circ$ .

9.2 Draw a line and mark a point on it. Construct a right angle and bisect it to get  $45^\circ$ . The supplementary angle is  $135^\circ$ .

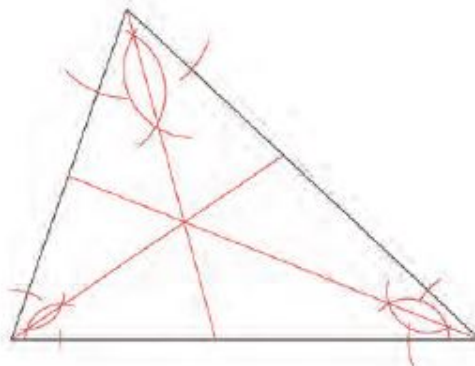
9.3 Construct an angle of  $90^\circ$ . The outer angle is the  $270^\circ$  angle.

9.4 Construct two  $60^\circ$  angles adjacent to each other making a  $120^\circ$  angle. The outer angle is the  $240^\circ$  angle.

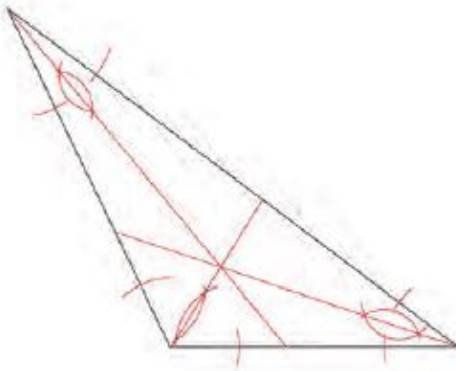
9.5 Draw a line and mark a point on it. Construct an angle of  $60^\circ$ , bisect it to get  $30^\circ$ . The supplementary angle to  $30^\circ$  is the  $150^\circ$  angle.

## 3.2 Exercise 2

1. The bisectors all intersect at the same point.



2. The bisectors all intersect at the same point.



Ext. $\angle$	4	9	6	10	11
3. Int. opp. $\angle$ s	1 and 2	5 and 7	5 and 8	11 and 12 12 & 13 and 14	13 and 14

4.1  $\angle ABC, \angle BCA$  and  $\angle CAB$

4.2  $\angle MBC, \angle LBA, \angle BAG, \angle NAC, \angle ACK, \angle HCB$

4.3 It does not comply with the definition: an exterior is an angle between a side of a triangle and another side that is extended.

4.4  $\angle GAN$  and  $\angle HCK$

5.1  $M\hat{N}Q$

5.2  $67^\circ + 45^\circ = 112^\circ$

6. They are equal.

7.1  $a = 85^\circ$  (ext  $\angle$  of  $\Delta$ )

7.2  $b = 104^\circ$  (ext  $\angle$  of  $\Delta$ )

$c = 127^\circ$  (vertically opposite angles are equal)

$d + f = 180^\circ - 78^\circ = 102^\circ$  (sum of  $\angle$ s of  $\Delta$ )

7.3  $d = f$  (angles opposite equal sides)

Therefore  $d = 51^\circ = f$

$e = 78^\circ + 51^\circ = 129^\circ$  (ext.  $\angle$  of  $\Delta$ )

### 3.3 Exercise 3

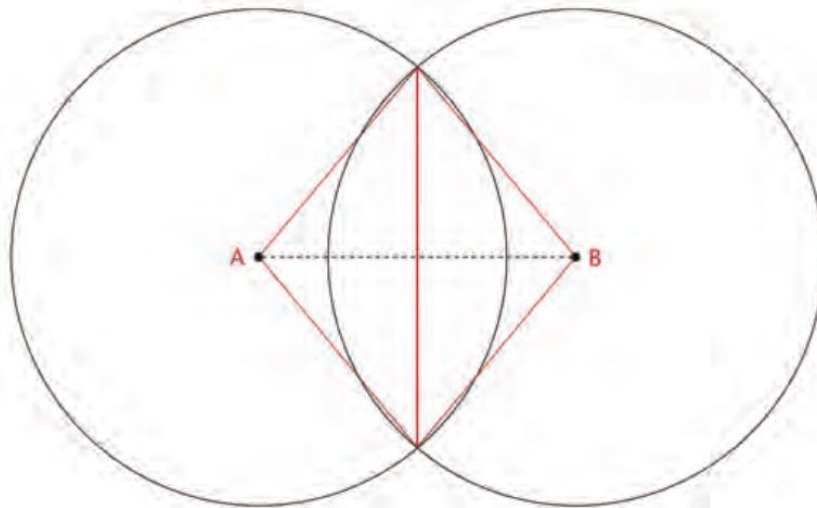
Conditions	Congruent?
Three sides (SSS)	Yes
Two sides (SS)	No
Three angles (AAA)	No
1. Two angles and a sides (AAS)	Yes
Two sides and an angle not between the sides (SSA)	No
Two sides and an angle between the sides (SAS)	Yes
Right-angled with the hypotenuse and a side (RHS)	Yes

2. 4

3. 4

4. 2

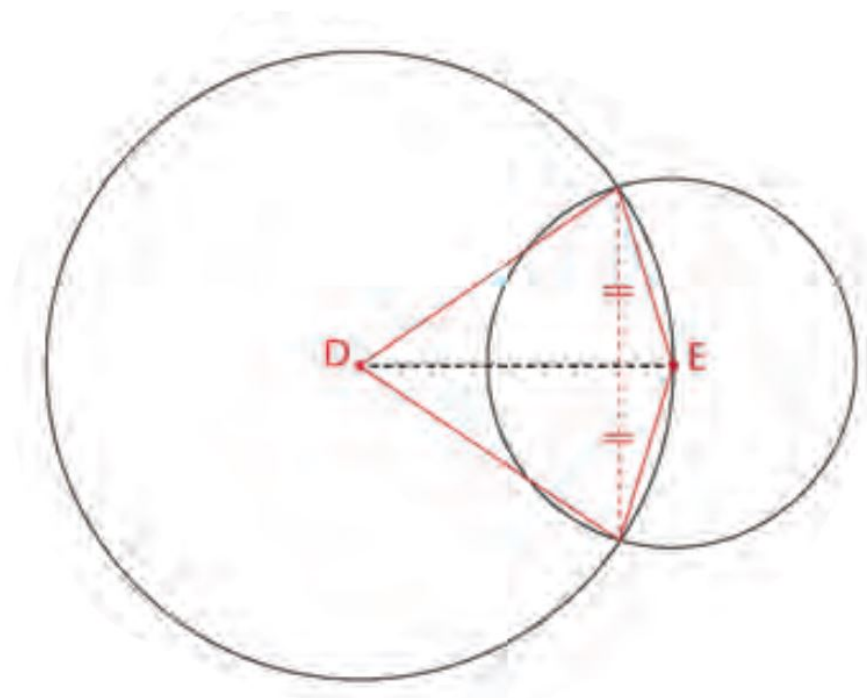
### 3.4 Exercise 4



1.1 The lines are the diagonals of the rhombus.

1.2 Yes

1.3 ... intersect at  $90^\circ$  and will bisect each other.



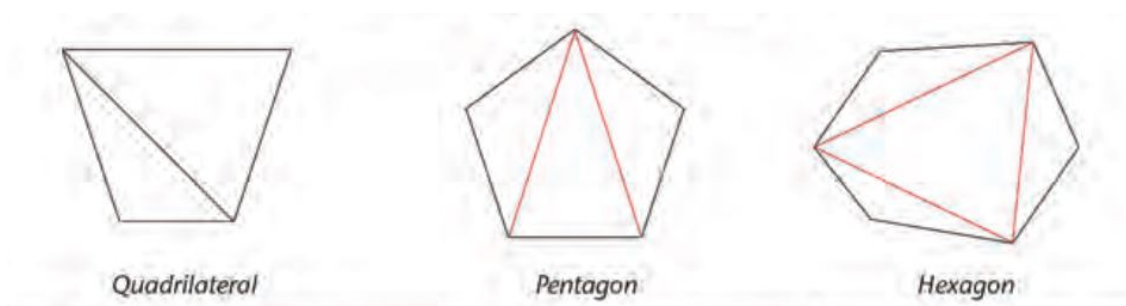
2.1 Yes

2.2 No, only one of the diagonals is bisected.

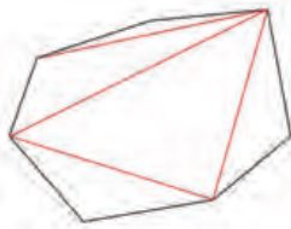
2.3 The diagonals of a rhombus both bisect each other.

Quadrilateral	Diagonals equal	Diagonals bisect	Diagonals meet at $90^\circ$
3.1 Parallelogram	No	Yes	No
Rectangle	Yes	Yes	No
Square	Yes	Yes	Yes

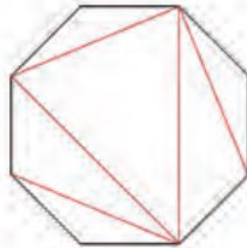
4.1



No. of $\Delta$ s	2	3	4
Sum of $\angle$ s	$2 \times 180^\circ = 360^\circ$	$3 \times 180^\circ = 540^\circ$	$4 \times 180^\circ = 720^\circ$



Heptagon



Octagon



Nonagon

No. of $\Delta$ s	5	6	7
Sum of $\angle$ s	$900^\circ$	$1080^\circ$	$1260^\circ$