CHAPTER 12

Geometry Of Straight Lines

CONTENTS

October 19, 2021

1 ANGLE RELATIONSHIPS

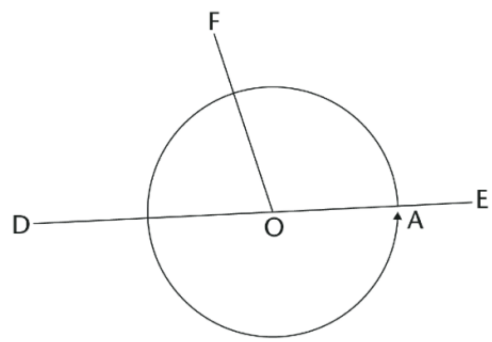
Note:

Remember that $360°$ is one full revolution. If you look at something and then turn all the way around so that you are looking at it again, you have turned through an angle of $360°$. If you turn only halfway around so that you look at something that was right behind your back, you have turned through an angle of $180°$.

1.1 Exercise 1

Complete in App

1. Answer the questions about the figure below:



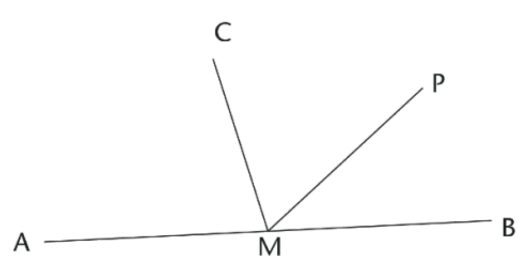
1.1 Is angle FOD in the figure bigger or smaller than a right angle?

1.2 Is angle FOE in the above figure smaller or bigger than a right angle?

Note:

The sum of the angles on a straight line is $180°$. When the sum of the angles is $180°$, the angles are called supplementary.

2. $C\hat{M}A$ in the figure below is $75°$.



2.1 How big is $C\hat{M}A$

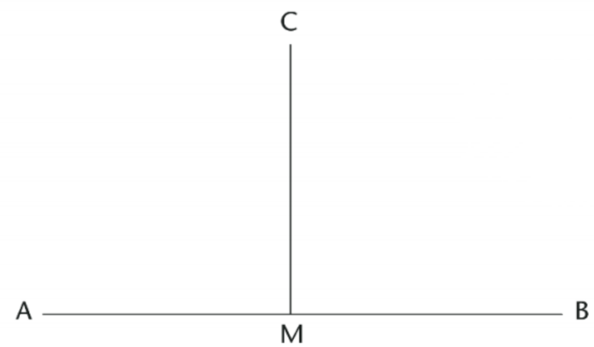
2.2 Why do you say so?

3. $P\hat{M}B$ in the figure in question 2 is $40°$

   (a) How big is $C\hat{M}P$?

   (b) Explain your reasoning.

4. In the figure below, AMB is a straight line and $A\hat{M}C$ and $B\hat{M}C$ are equal angles.

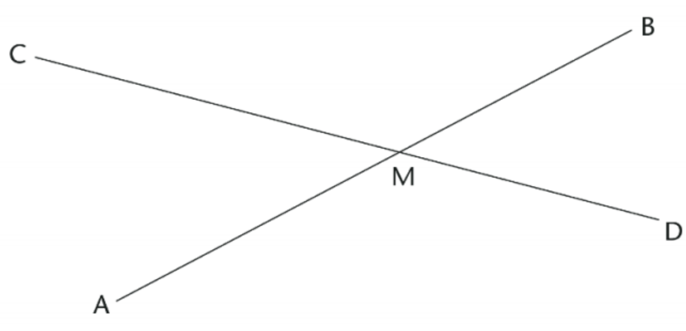


   (a) How big are these angles?

   (b) How do you know this?

> **Note:**
>
> When one line forms two equal angles where it meets another line, the two lines are said to be perpendicular. Because the two equal angles are angles on a straight line, their sum is $180°$, hence each angle is $90°$.

5. In the figure below, lines AB and CD intersect at point M.



5.1 Does it look as if $C\hat{M}A$ and $B\hat{M}D$ are equal?

5.2 Can you explain why they are equal?

5.3 What does $C\hat{M}A + D\hat{M}A$ equal? Why do you say so?

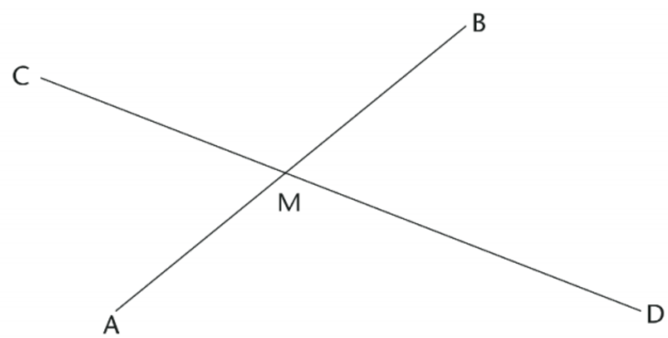
5.4 What is $C\hat{M}A + C\hat{M}B$? Why do you say so?

5.5 Is it true that $C\hat{M}A + D\hat{M}A = C\hat{M}A + C\hat{M}B$?

5.6 Which angle occurs on both sides of the equation is question 5.5?

6. Look carefully at your answers to question 5.3 to 5.5. Now try to explain your observation in question 5.1.

7. In the figure below, AB and CD intersect at M. Four angles are formed. Angle CMA and angle BMD are also vertically opposite.



Hint:

When two straight lines intersect, the vertically opposite angles are equal.

(a) If angle $BMC = 125°$, how big is angle AMD?

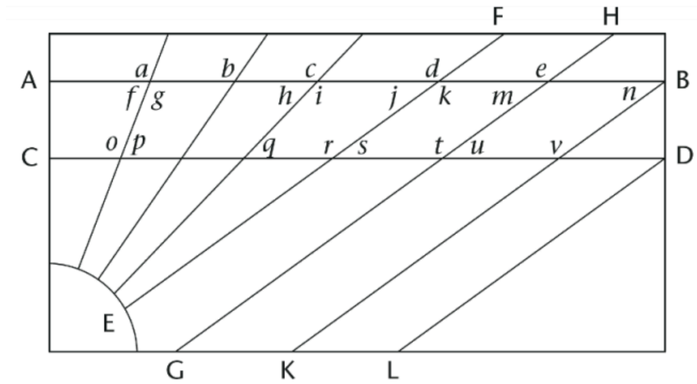
(b) Why do you say so?

1.1.1 Lines and angles

Note:

A line that intersects other lines is called a transversal.

8. In the figure below, AB is parallel to CD and $EF \parallel GH \parallel KB \parallel$.



8.1 Angles a, b, c, d and e are corresponding angles. Do the corresponding angles appear to be equal?

8.2 Investigate whether or not the corresponding angles are equal by using tracing paper. Trace the angle you want to compare to other angles and place it on top of the other angle to find out if they are equal. What do you notice?

8.3 Angles f, h, j, m and n are also corresponding angles. Identify all the other groups of corresponding angles in the pattern.

8.4 Describe the position of corresponding angles that are formed when a transversal intersects other lines.

8.5 The following are pairs of alternate angles: g and o ; j and s ; and k and r. Do these angles appear to be equal?

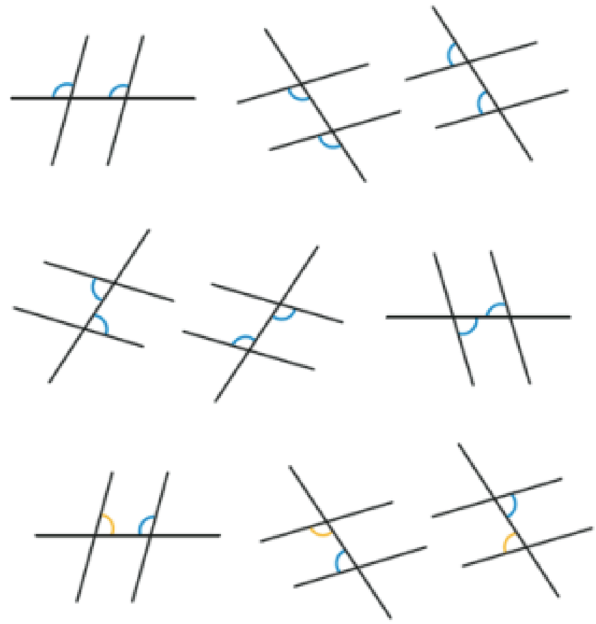
8.6 Investigate whether or not the alternate angles are equal by using tracing paper Trace the angle you want to compare and place it on top of the other angle to find out if they are equal. What do you notice?

8.7 Identify two more pairs of alternate angles.

8.8 Clearly describe the relative position of alternate angles that are formed when a transversal intersects other lines.

8.9 Did you notice anything about some of the pairs of corresponding angles when you did the investigation in question 6? Describe your finding.

8.10 Angles f and o, i and q and k and s are all pairs of co-interior angles. Identify three more pairs of co-interier angles in the pattern.



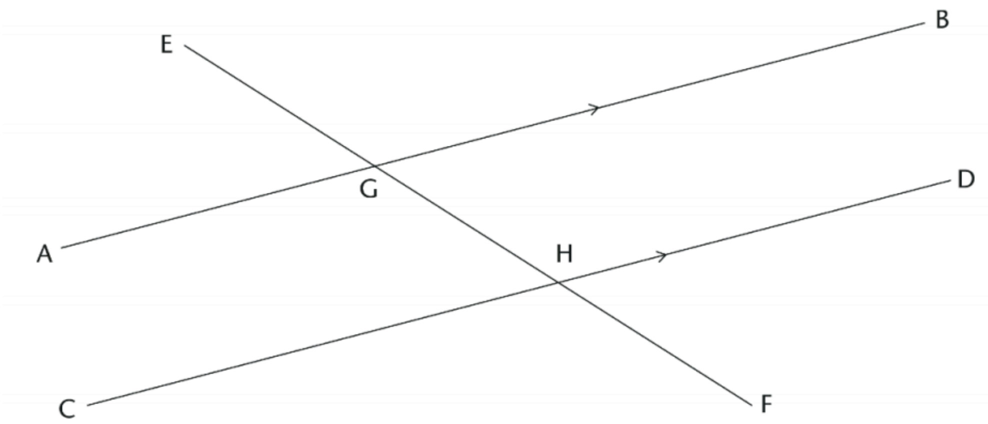
Note:

The angles in the same relative position at each intersection where a straight line crosses two others are called corresponding angles.Angles on different sides of a transversal and between two other lines are called alternate angles. Angles on the same side of the transversal and between two other lines are called co-interior angles.

1.1.2 Angles formed by parallel lines

Corresponding angles:

The lines AB and CD shown in the figure below never meet. Lines that never meet and are at a fixed distance from one another are called parallel lines. We write $AB \parallel CD$. Parallel lines have the same direction, i.e. they form equal corresponding angles with any line that intersects them.



9. In the figure above, the line EF cuts AB and CD at H. EF is a transversal that cuts parallel lines AB and CD.
    9.1 Look carefully at the angles EGA and EHC in the above figure. they are called corresponding angles. Do they appear to be equal?
    9.2 Measure the two angles to check if they are equal. What do you notice?
10. Suppose $E\hat{G}A$ and $E\hat{H}C$ are really equal. Would $E\hat{G}B$ and $E\hat{H}D$ then also be equal? Give reasons for your answer.

Note:

When two parallel lines are cut by a transversal, the corresponding angles are equal.

Alternate angles:

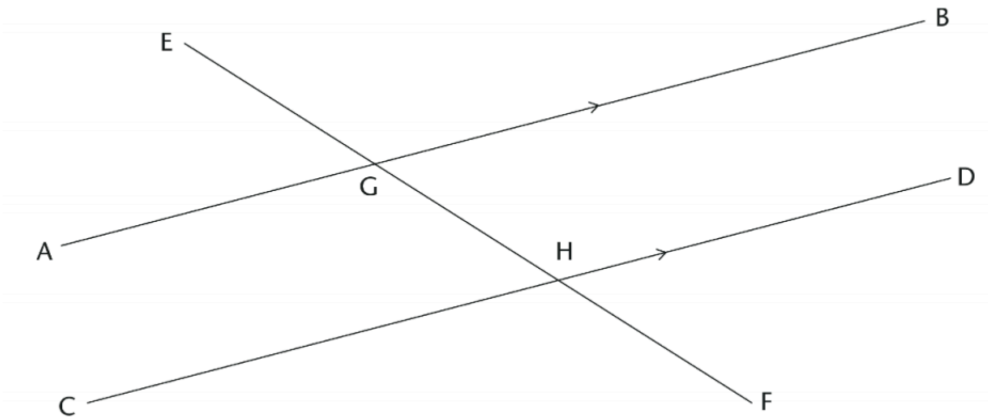
The angles $B\hat{G}F$ and $C\hat{H}E$ in the figure above are called alternate angles. They are on opposite sides of the transversal.

11. Do you think angles AGF and DHE should also be called alternate angles?
12. Do you think alternate angles are equal? Investigate by using the tracing paper like you did previously, or measure the angles accurately with your protractor. What do you notice?

Note:

When parallel lines are cut by a transversal, the alternate angles are equal.

13. Are angles $B\hat{G}H$ and $D\hat{H}F$ corresponding angles? what do you know about corresponding angles?
14. Refer to the figure below and complete the following:



14.1 What can you say about $B\hat{G}H + A\hat{G}H$? give a reason.

14.2 What can you say about $D\hat{H}G + C\hat{H}G$? Give a reason.

14.3 Is it true that $B\hat{G}H + A\hat{G}H = D\hat{H}G + C\hat{H}G$? Explain.

14.4 Will the equation in question 14.3 still be true if you replace angle $B\hat{G}H$ on the left-hand side with angle $C\hat{H}G$

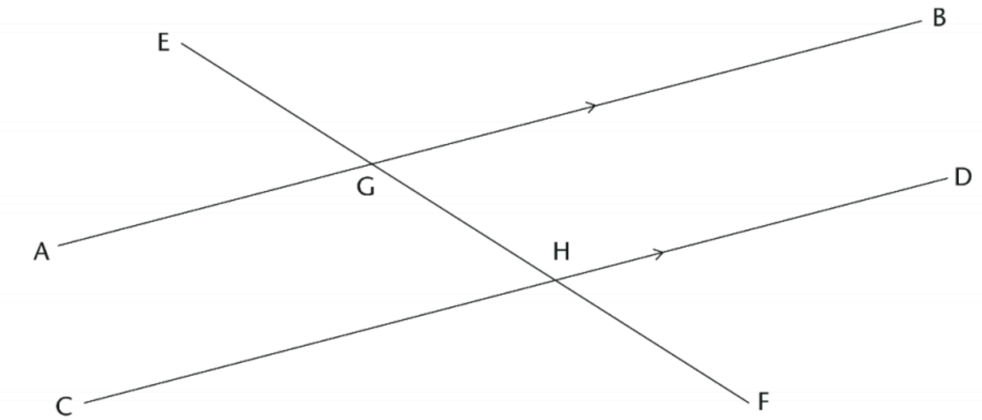
15. Look carefully at your work in question 14 and write an explanation why alternate angles are equal, when two parallel lines are cut by a transversal.

Co-interior angles

The angles $A\hat{G}H$ and $C\hat{H}G$ in the figure on the following page are called co-interior angles. They are on the same side of the transversal. The prefix "co-" means together. The word "co-interior" means on the same side.

16. Are angles BGH and $D\hat{H}F$ in the figure corresponding angles? What do you know about corresponding angles?

17. Refer to the figure below and answer the questions that follow.



17.1 What do you know about $C\hat{H}G + D\hat{H}G$? Explain.

17.2 What do you know about $B\hat{G}H + A\hat{G}H$? Explain.

17.3 What do you know about $B\hat{G}H + C\hat{H}G$? Explain.

17.4 What conclusion can you draw about $A\hat{G}H + C\hat{H}G$? Give detailed reasons for your conclusion.

Note:

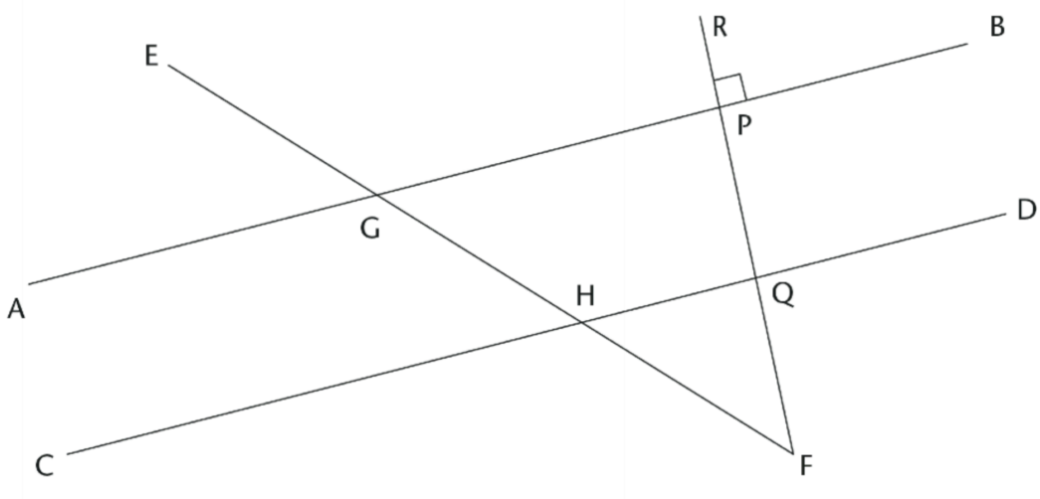
When two parallel lines are cut by a transversal, the sum of the two co-interior angles is $180°$. Another way of saying this is to say that the two co-interior angles are supplementary.

2 IDENTIFY AND NAME ANGLES

2.1 Exercise 2

Complete in App

1. In the figure below, the line RF is perpendicular to AB.



1.1 Is RF also perpendicular to CD? Justify your answer.

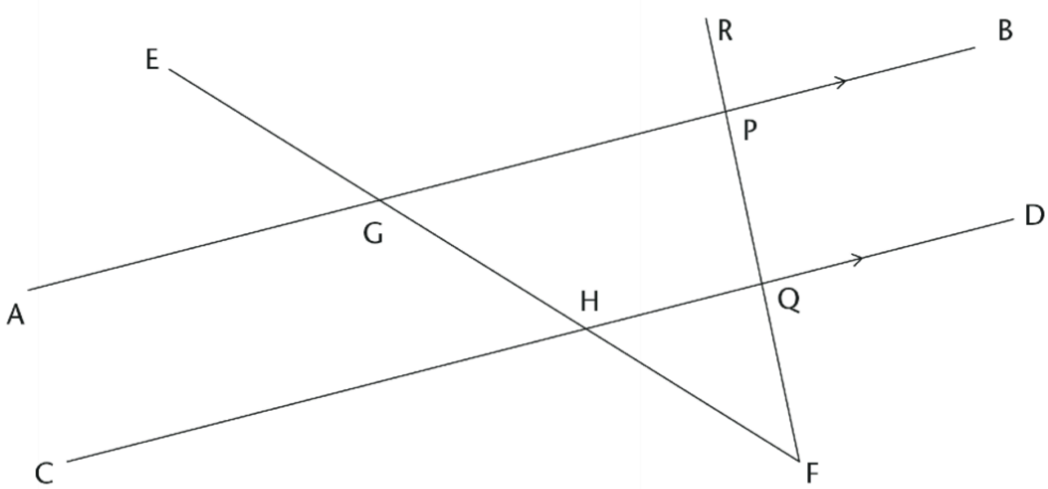
1.2 Name four pairs of supplementary angles in the figure. In each case, say how you know the angles are supplementary.

1.3 Name four pairs of co-interior angles in the figure.

1.4 Name four pairs of corresponding angles in the figure.

1.5 Name four pairs of alternate angles in the figure.

2. Now you are given that AB and CD in the figure below are parallel.



2.1 If it also given that RF is perpendicular to AB, will RF also be perpendicular to CD? Justify your answer.

2.2 Name all pairs of supplementary angles in the figure. In each case, say how know that the angles are supplementary.

2.3 Suppose $E\hat{G}A = x$. Give the size of as many angles in the figure as you can, in terms of x. Each time give a reason for your answer.

3 SOLVING PROBLEMS

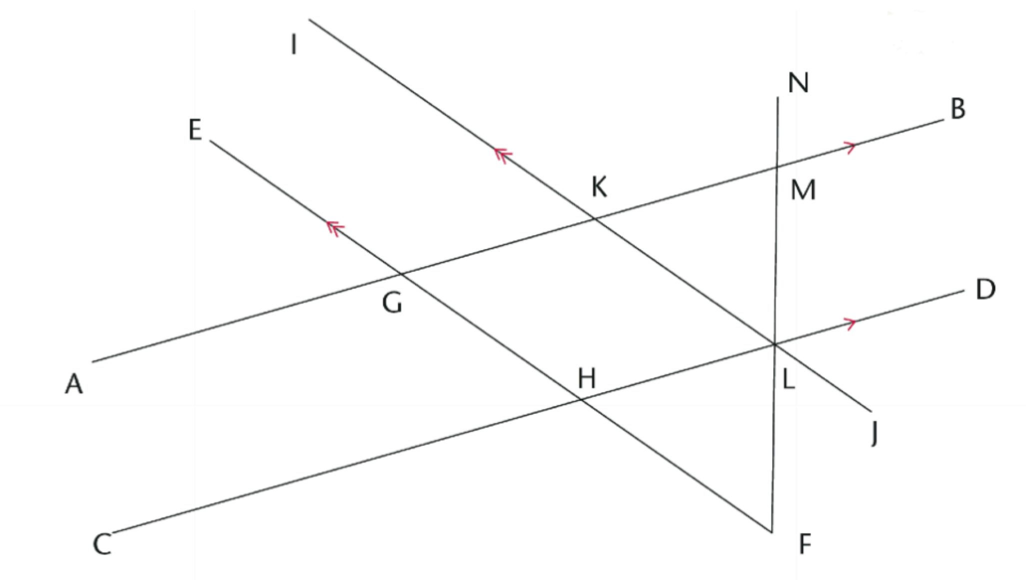
Note:

When you solve problems in geometry you can use a shorthand way to write your reasons. For example, if two angles are equal because they are corresponding angles, then you can write (corr $\angle$s, AB$\|$CD) as the reason.

3.1 Exercise 3

Complete in App

1. Line segments AB and CD in the figure below are parallel. EF and IJ are also parallel. Use the figure to answer the questions that follow.

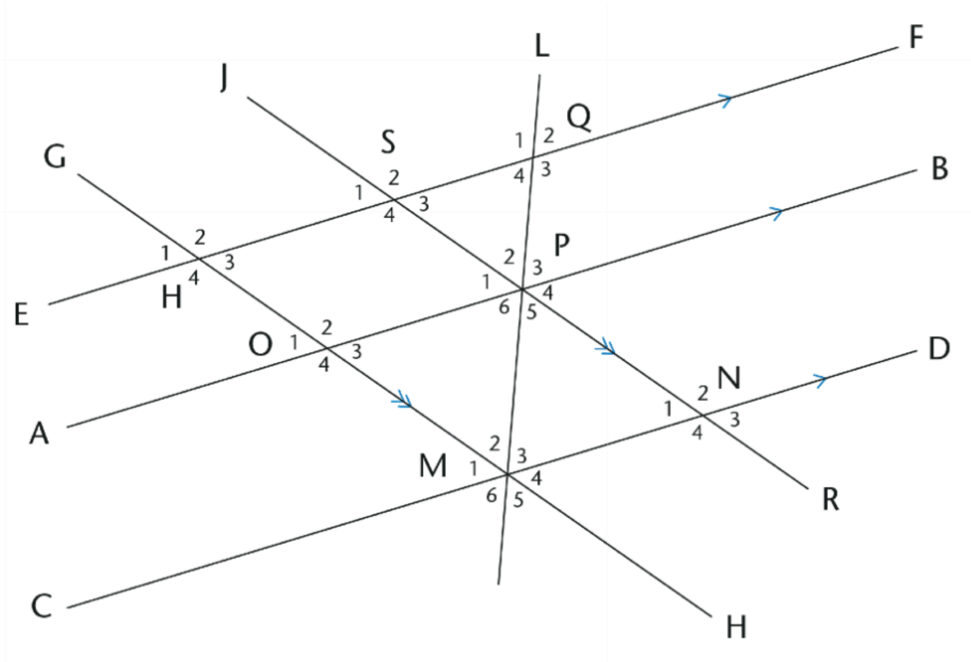


   (a) Name five angles in the figure that are equal to $G\hat{H}D$. Give a reason for each of your answers

   (b) Name all the angles in the figure that are equal to $A\hat{G}H$. Give a reason for each of your answers.

2. AB and CD in the figure above are parallel. $N\hat{M}B = 80°$ and $J\hat{L}F = 40°$. Find the sizes of as many angles in the figure as you can, giving reasons.

3. In the figure below, $AB \parallel CD$; $EF \parallel AB$; $JR \parallel GH$. You are also given that $P\hat{M}N = 60°$, $R\hat{N}D = 50°$.



3.1 Find the sizes of as many angles in the figure as you can, giving reasons.

3.2 Are EF and CD parallel? Give reasons for your answers.

4 ANSWERS TO EXERCISES

4.1 Exercise 1

1.1 smaller

1.2 bigger

2.1 $105°$

2.2 $C\hat{M}A + C\hat{M}B = A\hat{M}B = 180°$

3.1 $105° - 40° = 65°$

3.2 $C\hat{M}P + P\hat{M}B = 105° = C\hat{M}B$

4.1 $90°$

4.2 $A\hat{M}C + B\hat{M}C$ form straight line $A\hat{M}B$. If they are equal, each angle must be $90°$. $(90° + 90° = 180°)$

5.1 Yes

5.2 Yes, by using the angle definition of a straight line. $C\hat{M}A + C\hat{M}B = 180°$ and $B\hat{M}D + C\hat{M}B = 180°$. If $C\hat{M}B$ is subtracted on both sides, it follows that $C\hat{M}A = B\hat{M}D$.

5.3 $180°$. They are angles on straight line CD.

5.4 $180°$. They are angles on straight line AB.

5.5 Yes

5.6 $C\hat{M}A$

6. Each side of the equal equation has one angle that is the same size, so the other two angles have to be equal to each other.

7.1 $125°$

7.2 It is vertically opposite $B\hat{M}C$.

8.1 No, some of the angles look bigger or smaller than others.

8.2 Only $d = e$

8.3 (g,i,k); (o,r,t,v); (u,s,q,p)

8.4 Angles on the same side of the transversal and either all angles left of the other lines, or angles on the right-hand side of the other lines.

8.5 Yes

8.6 Yes, they are equal and they are angles on lines AB and CD, which are parallel to each other.

8.7 m and u; h and q; f and p

8.8 They are on alternate sides of the transversal and between other lines.

8.9 Corresponding angles are also equal when AB is parallel to CD.

8.10 g and p; j and r; m and t; v and n

9.1 Yes

9.2 They are equal.

10. Yes. These angles are corresponding angles and equal because $AB \parallel CD$.

11. Yes, they are on different sides of transversal EF and between lines AB and CD.

12. They are equal if $AB \parallel CD$

13. $\angle GHD =$ corresponding $\angle EGB$ but $\angle EGB =$ vertically opposite $\angle AGH$, so $\angle AGH =$ alternate $\angle GHD$. In general, an angle is vertically opposite to the alternate angle of its equal corresponding angle.

14. Yes. They are in the same positions if the lines are crossed by a transversal and they are equal if the lines are parallel.

15.1 They are supplementary angles (their sum is $180°$), because AB is a straight line.

15.2 They are supplementary angles (their sum is $180°$), because CD is a straight line.

15.3 Yes, because the sums on either side of the equal sign are both equal to $180°$.

15.4 Yes, because $C\hat{H}G$ is vertically opposite and equal to $D\hat{H}F$, which is in turn corresponding to $B\hat{G}H$, because AB and CD are parallel.

16. Corresponding angles are equal if parallel lines are intersected by a transversal. Alternate angles will also be equal, because their vertically opposite anges are equal.

17.1 They are supplementary (their sum is $180°$) because CD is a straight line.

17.2 They are supplementary (their sum is $180°$) because AB is a straight line.

17.3 They are alternate angles and they are equal, because $AB \parallel CD$.

17.4 They are also supplementary (their sum = $180°$). This is because we have already shown that:
$C\hat{H}G$ is equal to $B\hat{G}H$ ($\angle$'s on a str line).

4.2 Exercise 2

1.1 Not necessarily, as we don't know whether $AB \parallel CD$.

1.2 There are many possibilities: $R\hat{P}B$ and $B\hat{P}Q$ (on str. line RF); angles $B\hat{P}Q$ and $G\hat{P}Q$ (on str. line AB); $A\hat{G}E$ and $E\hat{G}P$ (on str line AB); angles $A\hat{G}E$ and $A\hat{G}H$ (on str. line EF).
Co-interior angle pairs $A\hat{G}H$ and $C\hat{H}G$, $P\hat{G}H$ and $G\hat{H}Q$, $G\hat{P}Q$ and $P\hat{Q}H$, $B\hat{P}Q$ and $P\hat{Q}D$ will be supplementary only if $AB \parallel CD$. Remember that supplementary angles are angles that add up to $180°$, they do not have to be adjacent angles.

1.3 Co-interior angles $A\hat{G}H$ and $C\hat{H}G$, $P\hat{G}H$ and $G\hat{H}Q$, $G\hat{P}Q$ and $P\hat{Q}H$, $B\hat{P}Q$ and $P\hat{Q}D$

1.4 $E\hat{G}P$ and $G\hat{H}Q$; $P\hat{G}H$ and $Q\hat{H}F$; $B\hat{P}Q$ and $D\hat{Q}F$; $R\hat{P}B$ and $P\hat{Q}D$ (there are four other possibilities as well)

1.5 $A\hat{G}H$ and $Q\hat{H}G$; $P\hat{G}H$ and $C\hat{H}G$; $G\hat{P}Q$ and $P\hat{Q}D$; $B\hat{P}Q$ and $H\hat{Q}P$. (Can also list the alternate exterior angles in the figure.)

2.1 Yes, $AB \parallel CD$, therefore RF will be perpendicular to CD because $H\hat{Q}P$ and $G\hat{P}R$ are corresponding angles and thus $H\hat{Q}P$ will also be $90°$.

2.2 Because it is stated that $AB \parallel CD$, the co-interior angles will also be supplementary: $P\hat{G}H$ and $G\hat{H}Q$; $G\hat{P}Q$ and $H\hat{Q}P$; $B\hat{P}Q$ and $D\hat{Q}P$; $A\hat{G}H$ and $C\hat{H}G$.
Other angles that are supplementary are adjacent angles on a straight line:
$C\hat{H}F$ and $F\hat{H}Q$ on line CD; $H\hat{Q}P$ and $H\hat{Q}F$ on line RF; ect.

2.3 $G\hat{H}C = E\hat{G}A = x$ (Corr. $\angle$'s; $AB \parallel CD$);
$P\hat{G}H = E\hat{G}A = x$ (vert. opp. $\angle$'s);
$Q\hat{H}F = G\hat{H}C = x$ (vert. opp. $\angle$'s);
$E\hat{G}P = 180° - E\hat{G}A = 180° - x$ ($\angle$'s on a str. line)
$A\hat{G}H = E\hat{G}P = 180° - x$ (vert. opp. $\angle$'s);
$G\hat{H}Q = E\hat{G}P = 180° - x$ (corr. $\angle$'s; $AB \parallel CD$);
$C\hat{H}F = G\hat{H}Q = 180° - x$ (vert. opp. $\angle$'s);
$Q\hat{F}H = 90° - x$ ($H\hat{Q}F = 90°$; sum of $\angle$'s of a Δ).

4.3 Exercise 3

1.1 $C\hat{H}F = G\hat{H}D$ (vert. opp. $\angle$'s);
$D\hat{L}K = G\hat{H}D$ (corr. $\angle$'s; $EF \parallel IJ$);
$A\hat{G}H = G\hat{H}D$ (Alt. $\angle$'s; $AB \parallel CD$);
$E\hat{G}K = G\hat{H}D$ (corr. $\angle$'s; $AB \parallel CD$);
$H\hat{L}J = G\hat{H}D$ (Alt. $\angle$'s; $EF \parallel IJ$)

1.2 $A\hat{K}J = A\hat{G}H$ (corr. $\angle$'s; $EF \parallel IJ$);
$E\hat{G}K = A\hat{G}H$ (vert. opp $\angle$'s);
$I\hat{K}M = E\hat{G}K$ (corr. $\angle$'s; $EF \parallel IJ$) and $E\hat{G}K = A\hat{G}H$ (proven above);
$C\hat{H}F = A\hat{G}H$ (corr. $\angle$'s; $AB \parallel CD$);
$G\hat{H}L = A\hat{G}H$ (alt. $\angle$'s; $AB \parallel CD$)

2. $K\hat{M}L = 80°$ (vert. opp. $\angle$)
$N\hat{M}K = 100°$ and $B\hat{M}L = 100°$ ($\angle$'s on str. lines AB and NF)
$M\hat{L}D = 80°$ (corr. $\angle$'s equal; $AB \parallel CD$)
$M\hat{L}H = 100°$ ($\angle$'s on str. line CD)
$H\hat{L}F = 80°$ (vert. opp. to $M\hat{L}D$)
$M\hat{L}K = 40°$ (vert. opp. to $J\hat{L}F$)
$I\hat{K}G = 60°$ (corr. $\angle$ to $H\hat{L}K$)
$G\hat{K}L = 120°$ ($\angle$'s on a str. line IJ)
$E\hat{G}K = 120°$ (corr. $\angle$ to IKB)
$E\hat{G}A = 60°$ ($\angle$'s on str. line AB)
$G\hat{H}L = 120°$ (corr. $\angle$ with $E\hat{G}K$)
$C\hat{H}F = 120°$ (vert. opp. $G\hat{H}L$)

$\therefore H\hat{F}L = 180° - H\hat{L}F - F\hat{H}L$ (sum of $\angle$'s of a triangle)
$\therefore H\hat{F}L = 180° - 80° - 60° = 40°$
$D\hat{L}K = 60°$ (vert. opp . to $H\hat{L}K$)
$D\hat{L}F = 100°$ (vert. opp. to $M\hat{L}H$)
$H\hat{L}K = 100° - 40° = 60°$
$L\hat{K}M = 60°$ (vert. opp. to $I\hat{K}G$)
$I\hat{K}B = 120°$ (vert. opp. $G\hat{K}L$)
$A\hat{G}H = 120°$ (vert. opp. $E\hat{G}K$)
$K\hat{G}H = 60°$ ($\angle$'s on str. line AB)
$G\hat{H}C = 60°$ ($\angle$'s on str. line CD)
$F\hat{H}L = 60° = G\hat{H}C$ (vert. opp. $\angle$'s)

3.1 $\hat{N}_1 = 50°$ (vert. opp. $R\hat{N}D$);
$\hat{N}_2 = 130°$ ($\angle$'s on str. line);
$\hat{N}_4 = 130°$ ($\angle$'s on str. line);
$\hat{P}_1 = 50°$ (corr. to $\hat{N}_1$; $AB \parallel CD$);
$S\hat{P}B = 130°$ ($\angle$'s on str. line);
$\hat{P}_4 = 50°$ (vert. opp. $\hat{P}_1$);
$O\hat{P}N = 130°$ (vert. opp. $S\hat{P}B$);
$\hat{S}_1 = 50°$ (corr. to $\hat{P}_1$; $EF \parallel AB$);
$\hat{S}_2 = 130°$ ($\angle$'s on a str. line);
$\hat{S}_3 = 50°$ (vert. opp. $\hat{S}_1$);
$\hat{S}_4 = 130°$ ($\angle$'s on a str. line);
$\hat{H}_2 = 130°$ (co-int $\angle$ with $\hat{S}_1$; $GH \parallel JR$);
$\hat{H}_1 = 50°$ ($\angle$'s on a str. line);
$\hat{H}_4 = 130°$ (vert. opp. $G\hat{H}S$);
$\hat{H}_3 = 50°$ (vert. opp. $\hat{H}_1$);
$\hat{O}_2 = 130°$ (corr. to $\hat{H}_2$; $EF \parallel AB$);
$\hat{O}_1 = 50°$ ($\angle$'s on str. line);
$\hat{O}_4 = 130°$ (vert. opp. $\hat{O}_2$);
$\hat{O}_3 = 50°$ ($\angle$'s on a str. line);
$\hat{M}_1 = 50°$ (co-int. with $\hat{O}_4$; $AB \parallel CD$);
$\hat{M}_4 = 50°$ (vert. opp. $\hat{M}_1$);
$N\hat{M}O = 130°$ ($\angle$'s on a str. line);
$C\hat{M}H = 130°$ (vert. opp. $N\hat{M}O$);
$\hat{Q}_2 = \hat{P}_3 = P\hat{M}N = 60°$ (corr. $\angle$'s; $EF \parallel AB$ and $AB \parallel CD$);
$\hat{Q}_1 = 120°$ ($\angle$'s on str. line);
$Q\hat{P}O = P\hat{M}C = \hat{Q}_1 = 120°$ (corr. $\angle$'s; $EF \parallel AB$ and $AB \parallel CD$);
$\hat{Q}_4 = 60°$ (vert. opp. $\hat{Q}_2$);
$\hat{Q}_3 = 120°$ (vert. opp. $\hat{Q}_1$);

$\hat{P}_6 = 60°$ (corr. to $\hat{Q}_4$; $SQ \parallel PO$);
$B\hat{P}M = 120°$ ($\angle$'s on str. line);
$\hat{P}_2 = Q\hat{P}O - \hat{P}_1 = 120° - 50° = 70°$;
$\hat{M}_2 = \hat{P}_2 = 70°$ (corr. $\angle$'s; $JR \parallel GH$);
$Q\hat{P}N = 110°$ ($\angle$'s on str. line);
$\hat{M}_5 = \hat{M}_2 = 70°$ (vert. opp $\angle$'s);
$\hat{M}_6 = \hat{M}_3 = 60°$ (vert. opp $\angle$'s);
$\hat{P}_6 = 60°$ (vert. opp. $\hat{P}_3$);
$\hat{P}_5 = 70°$ (vert. opp. $\hat{P}_2$)

3.2 Yes, corresponding angles $J\hat{S}H$ and $M\hat{N}P$ are equal. Also, if $AB \parallel CD$ and $AB \parallel EF$, then it follows that $EF \parallel CD$