

CHAPTER 15

Functions

CONTENTS

1	Introduction	1
2	Tables and graphs	1
3	Exercises	1
3.1	Exercise 1	2
3.2	Exercise 2	5
3.3	Exercise 3	8
4	Answers to Exercises	10
4.1	Exercise 1	10
4.2	Exercise 2	12
4.3	Exercise 3	15

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1 INTRODUCTION

Some background information

- A function is a relationship where each input has a single output.
- A relationship exists between two quantities where the one quantity, the input value (independent variable), is substituted into a formula to give another value, the output value (dependent variable).
- A function can be represented in the following different, but equivalent ways:
 - in a flow diagram: input value $\longrightarrow \times 3 \longrightarrow +5 \longrightarrow$ output value
 - verbally: for example, multiply the input number by 3 and then add 5 to get the corresponding output number - $y = 3x + 5$
 - a table of values that shows the input and corresponding output values as pairs of numbers
 - a graph on which the ordered pairs of numbers are plotted as points.
- We can switch from one representation to another.
- The scales on one or both of the axes of a graph can change, which will change the appearance of the graph, but not the information in the graph.
- The graphs of relationships show clearly what the patterns are that the output values show. Some patterns show:
 - Increases in the output values as the input values increase, for example in $y = 10 + x$
 - decreases in the output values as the input values increase, for example in $y = 10 - x$
 - both decreases and increases in the output values as the input values increase, for example $y = x^2$

2 TABLES AND GRAPHS

Misconceptions: When starting work on graph paper to plot points, it is easy to forget that it is an ordered pair of which the first number is plotted along the horizontal axis and the second number along the vertical axis. They easily switch the numbers in their efforts to plot them. Or they plot the one number on the $x - axis$ and the other on the $y - axis$, so that they have two points, one on each axis.

3 EXERCISES

3.1 Exercise 1

1. Each formula indicates a relationship between two sets of numbers that may be called the input numbers and the output numbers. For each formula, calculate the output numbers that correspond to the input numbers 0, 1, 2 and 10

(a) $y = 3x + 5$

(b) $y = 3(x + 5)$

(c) $y = 3x + 5x$

(d) $y = 3x^2 + 5$

(e) $y = 3x^2 + 5x$

(f) $y = 3x(x + 5)$

2. The information provided in the formula $y = 5x^2 - 5x$ can also be represented in words, for example: To get the output number, you have to subtract three times the input number from five times the square of the input number. Represent each formula in the question in words:

(a) $y = 3x + 5$

(b) $y = 3(x + 5)$

(c) $y = 3x + 5x$

(d) $y = 3x^2 + 5$

(e) $y = 3x^2 + 5x$

(f) $y = 3x(x + 5)$

3. For each instruction, write a formula that provides the same information.

(a) Multiply the input number by 10, then subtract 3 to get the output number.

(b) Subtract 3 from the square of the input number, then multiply by 10 to get the output number.

(c) Multiply the square of the input number by 10, then add 5 times the input number to get the output number.

(d) Subtract 7 times the square of the input number from 100, then multiply by 3 to get the output number.

(e) Add 4 to the input number, then subtract the answer from 50 to get the output number.

(f) Multiply the input number by 3, then subtract the answer from 15 to get the output number.

4. Complete the following table for the input values of 1, 5 and 10.

(a)

Input number	1	5	10
multiply the input number by 10, then subtract 3 to get the output number			

(b)

Input number	1	5	10
$y = 10x - 3$			

(c)

Input number	1	5	10
subtract 3 from the square of the input number, then multiply by 10 to get the output number			

(d)

Input number	1	5	10
$y = 10(x^2 - 3)$			

(e)

Input number	1	5	10
multiply the square of the input number by 10, then add 5 times the input number to get the output number			

(f)

Input number	1	5	10
$y = 10x^2 + 5x$			

(g)

Input number	1	5	10
subtract 7 times the square of the input number from 100, then multiply by 3 to get the output number			

(h)

Input number	1	5	10
$y = 3(100 - 7x^2)$			

(i)

Input number	1	5	10
add 4 to the input number, then subtract the answer from 50 to get the output number			

(j)

Input number	1	5	10
$y = 50 - (x + 4)$			

(k)

Input number	1	5	10
multiply the input number by 3, then subtract the answer from 15 to get the output number			

(l)

Input number	1	5	10
$y = 15 - 3x$			

5. In certain cases, flow diagrams can be used to provide instructions on how output numbers can be calculated. For each flow diagram, represent the information in a formula and also in words:

- (a) $\boxed{\times 3} \rightarrow \boxed{+17} \rightarrow$
- (b) $\boxed{+5} \rightarrow \boxed{\times 3} \rightarrow \boxed{+2} \rightarrow$
- (c) $\boxed{-2} \rightarrow \boxed{\times 3} \rightarrow \boxed{+23} \rightarrow$
- (d) $\boxed{\times 2} \rightarrow \boxed{+3} \rightarrow \boxed{\times 5} \rightarrow \boxed{+4} \rightarrow$
- (e) $\boxed{+3} \rightarrow \boxed{\times 2} \rightarrow \boxed{+4} \rightarrow \boxed{\times 5} \rightarrow$
- (f) $\boxed{\times 10} \rightarrow \boxed{+19} \rightarrow$
- (g) $\boxed{+5} \rightarrow \boxed{\times 10}$

6. Complete the following table:

(a)

x	0	1	2	3
$3x + 17$				

(b)

x	0	1	2	3
$3(x + 5) + 2$				

(c)

x	0	1	2	3
$3(x - 2) + 23$				

7. Complete the following table:

(a)

x	-3	-2	-1	0
$5(2x + 3) + 4$				

(b)

x	-3	-2	-1	0
$5[2(x + 3) + 4]$				

(c)

x	-3	-2	-1	0
$10x + 19$				

(d)

x	-3	-2	-1	0
$10(x + 5)$				

8. Explain why the output numbers of the following formulas are the same.

Formula 1 : $y = 3x + 17$

Formula 2 : $3(x + 5) + 2$

Formula 3 : $3(x - 2) + 23$

3.2 Exercise 2

1. Complete the table to show some of the input and output numbers of the relationship described by the formula $y = 2x - 3$.

Input numbers	-5	0	2	4	6	8
Output numbers						

2. The vertical blue line on this graph represents the input number 6. The heavy horizontal red line represents the output number 9. The black point where the blue and red lines intersect indicates that the input number 6 is associated with the output number 9. We also say the black point represents the **ordered number pair (6;9)**.

(a) Which ordered number pair does the red point on the graph represent?

(b) Which ordered number pair does the blue point on the graph represent?

3. Do the two graphs show the same relationship, or different relationships between two variables?

4. How do the two graphs differ?

5. Use one of the graphs to find out what the output number is for an input number of 12.

6. Do the tables below represent the same relationship? Explain your answer.

Values of the independent variable	3	4	5	6	7	8
Values of the dependent variable	12	14	16	18	20	22

Values of the independent variable	0	5	10	15	20	25
Values of the dependent variable	8	18	28	38	48	58

7. Complete the following table for the relationship described by $y = x^2$:

x	-2	-1	0	1	2
y					

8. Which graph represents the ordered number pairs in the table below?

x	-2	-1	0	1	2
y	4	1	0	1	4

9. Complete the table for the relationship $y = 15 + x$.

x	-10	-5	0	5	10
$15 + x$					

10. Which graph represents the ordered number pairs in the table?

x	-10	-5	0	5	10
$15 + x$	5	10	15	20	25

11. Complete the table for the relationship $y = 15 - x$.

x	-10	-5	0	5	10
$15 - x$					

12. Which graph represents the ordered number pairs in the table?

x	-10	-5	0	5	10
$15 - x$	25	20	15	10	5

13. The output values for $y = x^2$ and $y = 15 + x$ show patterns. Describe, in words, how the patterns differ. Use the words increase and decrease in your description.

14. Describe, in words, how the graphs of $y = x^2$ in red and $y = 15 + x$ in black differ besides the colour.

15. Describe, in words, how the patterns in the output values for $y = 15 + x$ and $y = 15 - x$ differ. Use the words increase and decrease in your description.

16. Describe, in words, how the graphs of $y = 15 + x$ in black and $y = 15 - x$ in red differ besides the colour.

17. Complete the following table by extending the pattern in the output numbers.

Input numbers	0	5	10	15	20	25	30
Output numbers	0	4	8	12			

18. Which graph represents the ordered number pairs in the table?

Input numbers	0	5	10	15	20	25	30
Output numbers	0	4	8	12	16	20	24

19. Complete the following table by extending the pattern in the output numbers.

Input numbers	0	5	10	15	20	25	30
Output numbers	0	2	4	6			

20. Which graph represents the ordered number pairs in the table.

Input numbers	0	5	10	15	20	25	30
Output numbers	0	2	4	6	8	10	12

21. How do the patterns in two tables differ?

Input numbers	0	5	10	15	20	25	30
Output numbers	0	4	8	12	16	20	24

Input numbers	0	5	10	15	20	25	30
Output numbers	0	2	4	6	8	10	12

22. Complete the table by extending the patterns in the output values:

(a)

x	0	1	2	3	4	5	6	7
y	2	5	8					

(b)

x	0	1	2	3	4	5	6	7
y	3	1	-1	-3				

(c)

x	0	1	2	3	4	5	6	7
y	-10	-5	0	5				

(d)

x	0	1	2	3	4	5	6	7
y	-5	-3	-1	1				

(e)

x	0	1	2	3	4	5	6	7
y	6	3	0	-3				

(f)

x	0	1	2	3	4	5	6	7
y	3	2	1	0				

(g)

x	0	1	2	3	4	5	6	7
y	3	5	7	9				

3.3 Exercise 3

1. Complete the tables for $y = x^2$, $z = x^2 + 1^2$, $w = x^2 + 2^2$ and $s = x^2 + 3^2$

(a)

x	1	2	3	4	5
y					

(b)

x	1	2	3	4	5
z					

(c)

x	1	2	3	4	5
w					

(d)

x	1	2	3	4	5
s					

2. Complete the tables for $y = x^2$, $p = (x + 1)^2$, $q = (x + 2)^2$ and $r = (x + 3)^2$:

(a)

x	1	2	3	4	5
p					

(b)

x	1	2	3	4	5
y					

(c)

x	1	2	3	4	5
$p - y$					

(d)

x	1	2	3	4	5
q					

(e)

x	1	2	3	4	5
$q - y$					

(f)

x	1	2	3	4	5
r					

(g)

x	1	2	3	4	5
$r - y$					

3. Complete the tables, for $y = x^2$, $p = (x + 1)^2$, $q = (x + 2)^2$ and $r = (x + 3)^2$:

(a)

x	1	2	3	4	5
$p - y$					

(b)

x	1	2	3	4	5
$q - y$					

(c)

x	1	2	3	4	5
$r - y$					

4. Complete the following table:

(a)

x	1	2	3	4	5
$2x + 1$					

(b)

x	1	2	3	4	5
$4x + 4$					

(c)

x	1	2	3	4	5
$6x + 9$					

5. What are the constant differences in the sequences of values of $2x + 1$, $4x + x$ and $6x + 9$ for $x = 1, 2, 3, 4 \dots$?

6. Do you have an idea whether or not the corresponding sequence for $12x + 36$ will also have a constant difference and what the constant difference may be?

7. There are certain patterns in the coefficients and constant terms in the expressions presented below. Continue the pattern and find the next two expressions. $2x + 1, 4x + x, 6x + 9, \dots$

8. Complete the following table:

(a)

x	1	2	3	4	5
$8x + 16$					

(b)

x	1	2	3	4	5
$10x + 25$					

9. Briefly explain why the values in the two tables are exactly the same.

x	1	2	3	4	5
$(x + 1)^2 - x^2$	3	5	7	9	11
$(x + 2)^2 - x^2$	8	12	16	20	24
$(x + 3)^2 - x^2$	15	21	27	33	39

x	1	2	3	4	5
$2x + 1$	3	5	7	9	11
$4x + 4$	8	12	16	20	24
$6x + 9$	15	21	27	33	39

10. Show that the following expressions can be represented by binomials, for example: $2x + 1$, $(x + 4)^2 - x^2$ and $(x + 5)^2 - x^2$

4 ANSWERS TO EXERCISES

4.1 Exercise 1

- 5; 8; 11; 35
 - 15; 18; 21; 45
 - 0; 8; 16; 80
 - 5; 8; 17; 305
 - 0; 8; 22; 350
 - 0; 18; 42; 450
- Multiply the input number by 3 and add 5 to get the output number.
 - Add 5 to the input number and multiply the answer by 3.
 - Multiply the input number by 3, multiply the input number by 5 and add the two answers.
 - Square the input number, multiply the answer by 3 and add 5 to that ' answer.
 - Square the input number and multiply the answer by 3. Also, multiply the input number by 5 and add to the previous answer.
 - Add 5 to the input number and multiply the answer by the input number. Multiply this answer by 3.

3. (a) $y = 10x - 3$
 (b) $y = 10(x^2 - 3)$
 (c) $y = 10x^2 + 5x$
 (d) $3(100 - 7x^2)$
 (e) $y = 50 - (x + 4)$
 (f) $y = 15 - 3x$

4. (a)

	1	5	1
verbal description	7	47	97
formula	7	47	97

(b)

	1	5	1
verbal description	-20	220	970
formula	-20	220	970

(c)

	1	5	1
verbal description	15	275	1 050
formula	15	275	1 050

(d)

	1	5	1
verbal description	279	-225	-1 800
formula	279	-225	-1 800

(e)

	1	5	1
verbal description	45	41	36
formula	45	41	36

(f)

	1	5	1
verbal description	12	0	-15
formula	12	0	-15

5. (a) Formula: $y = 3x + 17$

To get the output number, multiply the input number by 3 and add 17.

(b) Formula: $y = 3(x + 5) + 2$

To get the output number, add 5 to the input number, multiply by 3 and then add 2.

(c) Formula: $y = 3(x - 2) + 23$

To get output number, subtract 2 from the input number, multiply by 3 and add 23.

(d) Formula: $y = 5(2x + 3) + 4$

To get the output number, multiply the input number by 2, then add 3, then multiply that answer by 5 and add 4.

(e) Formula: $y = 5[2(x + 3) + 4]$

To get the output number, add 3 to the input number, then multiply by 2, then add 4 to that answer and multiply the last answer by 5.

(f) Formula: $y = 10x + 19$

To get the output number, multiply the input number by 10 and add 19.

(g) Formula: $y = 10(x + 5)$

To get the output number, add 5 to the input number and then multiply by 10.

6. (a)

x	0	1	2	3
y according to your formula for 5(a)	17	20	23	26
y according to your formula for 5(b)	17	20	23	26
y according to your formula for 5(c)	17	20	23	26

(b) Learner's own work

7. (a)

x	-3	-2	-1	0
y according to your formula for 5(d)	-11	-1	9	19
y according to your formula for 5(e)	20	30	40	50
y according to your formula for 5(f)	-11	-1	9	19
y according to your formula for 5(g)	20	30	40	50

8. The formula for (b) is $y = 3(x + 5) + 2 = 3x + 15 + 2 = 3x + 17$, as in (a).

The formula for (c) is $y = 3(x - 2) + 23 = 3x - 6 + 23 = 3x + 17$, as in (a).

4.2 Exercise 2

1.

Input numbers	-5	0	2	4	6	8
Output number	-13	-3	1	5	9	13

2. (a) (8; 13)

(b) (2; 1)

3. The same data, because the output numbers associated with each input number are the same. For example, the output number associated with 3 is 12 on both graphs. This is true of all the points on the graphs.

4. The scales are different on the axes that represent the input numbers.

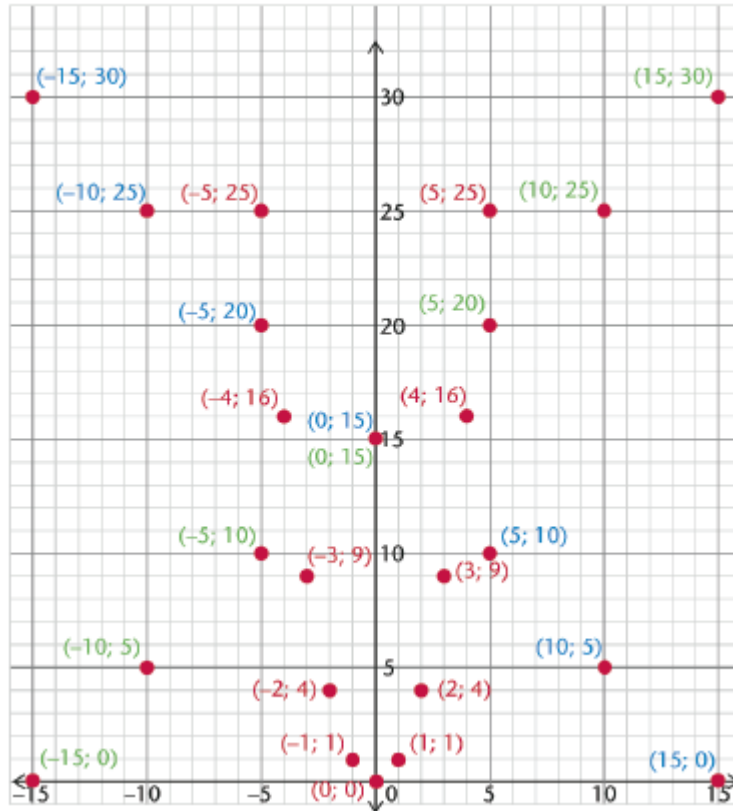
5. 30

6. No – there is a different relationship between input and output numbers, because the output numbers associated with the input number 5 are different.

7. (a)

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	25	16	9	4	1	0	1	4	9	16	25

(b) Use different colours for the answers to questions 7, 8 and 9.



8.

x	-15	-10	-5	0	5	10	15
$15 + x$	0	5	10	15	20	25	30

9.

x	-15	-10	-5	0	5	10	15
$15 - x$	30	25	20	15	10	5	0

10. (a) For $y = x^2$ the rate at which the output values increase and decrease is not constant, but for $y = 15 + x$, the output values increase at a constant rate.

(b) The graph of $y = x^2$ is a curve and the graph of $y = 15 + x$ is a straight line.

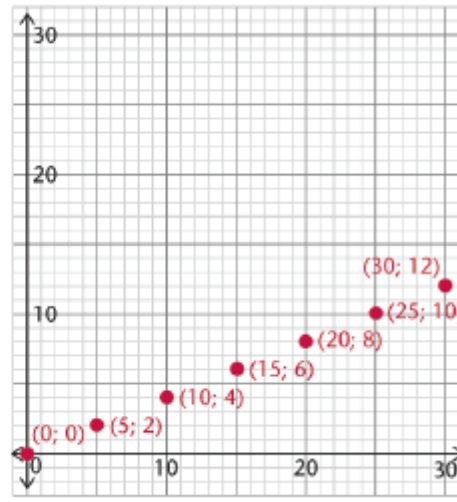
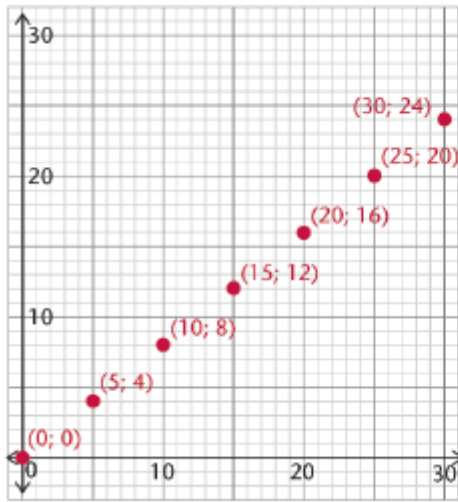
11. (a) For $y = 15 + x$ the output values increase by 5, for $y = 15 - x$ they decrease by 5. In both cases the input values increase by 5.
- (b) Both are straight lines but their directions differ: $y = 15 + x$ goes upwards from left to right as the input values increase, and $y = 15 - x$ goes downwards from left to right as the input values increase.

12. (a)

Input numbers	0	5	10	15	20	25	30
Output numbers	0	4	8	12	16	20	24

(b)

Input numbers	0	5	10	15	20	25	30
Output numbers	0	2	4	6	8	10	12



13. The output numbers in (a) increase by double the amount in (b). The graph of (a) is steeper than the one in (b).

14. (a) A.

x	0	1	2	3	4	5	6	7
y	2	5	8	11	14	17	20	23

B.

x	0	1	2	3	4	5	6	7
y	3	1	-1	-3	-5	-7	-9	-11

C.

x	0	1	2	3	4	5	6	7
x	-10	-5	0	5	10	15	20	25

D.

x	0	1	2	3	4	5	6	7
y	-5	-3	-1	1	3	5	7	9

E.

x	0	1	2	3	4	5	6	7
y	6	3	0	-3	-6	-9	-12	-15

F.

x	0	1	2	3	4	5	6	7
y	3	2	1	0	-1	-2	-3	-4

G.

x	0	1	2	3	4	5	6	7
y	3	5	7	9	11	13	15	17

(b) Sample answers are given for the descriptions to produce more output values.

A: $y = 3x + 2$; add three to each consecutive answer, or multiply the input number by three and add two

B: $y = -2x + 3$; subtract two from each consecutive answer, or multiply the input number by minus two and add three

C: $y = 5x - 10$; add five to each consecutive answer, or multiply the input number by five and subtract ten

D: $y = 2x - 5$; add two to each consecutive answer, or multiply the input number by two and subtract five

E: $y = -3x + 6$; subtract three from each consecutive answer, or multiply the input number by minus three and add six

F: $y = -x + 3$; subtract one from each consecutive answer, or multiply the input number by minus one and add three

G: $y = 2x + 3$; add two to each consecutive answer, or multiply the input number by two and add three

4.3 Exercise 3

1.

x	1	2	3	4	5	6	7	8	9	10
y	1	4	9	16	25	36	49	64	81	100
z	2	5	10	17	26	37	50	65	82	101
w	5	8	13	20	29	40	53	68	85	104
s	10	13	18	25	34	45	58	73	90	109

2. (a)

x	1	2	3	4	5	6	7	8	9	10
p	4	9	16	25	36	49	64	81	100	121
y	1	4	9	16	25	36	49	64	81	100
$p - y$	3	5	7	9	11	13	15	17	19	21

(b)

x	1	2	3	4	5	6	7	8	9	10
q	9	16	25	36	49	64	81	100	121	144
y	1	4	9	16	25	36	49	64	81	100
$q - y$	8	12	16	20	24	28	32	36	40	44

(c)

x	1	2	3	4	5	6	7	8	9	10
p	16	25	36	49	64	81	100	121	144	169
r	1	4	9	16	25	36	49	64	81	100
$r - y$	15	21	27	33	39	45	51	57	63	69

3. The output values for the relationships $z; w$ and s have already been calculated in question 1, and the values for p, q and r have been done in question 2.

4. (a)

x	1	2	3	4	5	6	7	8	9	10
$p - y$	3	5	7	9	11	13	15	17	19	21
$q - y$	8	12	16	20	24	28	32	36	40	44
$r - y$	15	21	27	33	39	45	51	57	63	69

(b)

x	10	11	12	13	14	15	16	17
$p - y$	21	23	25	27	29	31	33	35
$q - y$	44	48	52	56	60	64	68	72
$r - y$	69	75	81	87	93	99	105	111

5. (a)

x	1	2	3	4	5	6	7	8	9	10
$2x + 1$	3	5	7	9	11	13	15	17	19	21
$4x + 4$	8	12	16	20	24	28	32	36	40	44
$6x + 9$	15	21	27	33	39	45	51	57	63	69

(b) 2, 4 and 6 respectively

(c) Learners should be allowed to make their own conjectures.

(d)

x	1	2	3	4	5	6	7	8	9	10
$8x + 16$	24	32	40	48	56	64	72	80	88	96
$10x + 25$	35	45	55	65	75	85	95	105	115	125

6. (a) The expression $2x + 1$ has the same values as $(x + 1)^2 - x^2$ for all values of x .

This can be explained by expanding the right-hand side:

$$(x + 1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1 \text{ and similarly for the other cases.}$$

(b) $8x + 16$ and $10x + 25$