CHAPTER 2

Integers

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1 INTRODUCTION

Integers are used in everyday life. The highest or lowest places above or below sea level is written as integers. The temperature water freezes at is an integer.

These integers teach us how to calculate these real-life concepts using mathematics. It can help you calculate the points gained or lost in a sport. There are many more examples of integers in life. Can you think of any?

1.1 Why people decided to have negative numbers

Numbers such as -7 and -500, the additive inverses of whole numbers, are included with all the whole numbers and called **integers**.

Note

Fractions can be negative too, e.g.- $\frac{3}{4}$ and $-3,46$.

Natural numbers are used for counting and fractions (rational numbers) are used for measuring. Why do we also have negative numbers?

1.2 Properties of integers

Basic Rules

- $a + (-a) = 0$
- $a \times \frac{1}{a} = 1$ or $a \div a = 1$
- $a + 0 = a$
- $a \times 1 = a$

When a larger number is subtracted from a smaller number, the answer may be a negative number: $5-12 = -7$. This number is called **negative 7**.

One of the most important reasons for inventing negative numbers was to provide solutions for equations like the following:

Equation	Solution	Required property of negative numbers
$17+x=10$	$x=-7$ because $17+(-7)=17-7=10$	Adding a negative number is just like subtracting the corresponding positive number
$5-x=9$	$x=-4$ because $5-(-4)=5+4=9$	Subtracting a negative number is just like adding the corresponding positive number
$20+3x=5$	$x=-5$ because $3\times(-5)=-15$	The product of a positive number and a negative number is a negative number.

2 ADDING AND SUBTRACTING WITH INTEGERS

2.1 Addition and subtraction of negative numbers

Examples: $(–5)+(–3)$ and $(–20)–(–7)$

This is done in the same way as the addition and subtraction of positive numbers.

$_5)+(-3)=-8$ and $-20-(-7)=-13$

This is just like $5+3=8$ and $20-7=13$, or R$5+$R$3=$R8, and R$20-$R$7=$R13

2.2 Subtraction of a larger number from a smaller number

Examples: $5–9$ and $29–51$

Let us first consider the following:

$5+(-5)=0 \quad 10+(-10)=0$ and $20+(-20)=0$

If we subtract 5 from 5, we get 0, but then we still have to subtract 4:

$$\begin{aligned}5-9&=5-5-4\\&=0-4\\&=-4\end{aligned}$$

We know that $-9=(-4)+(-5)$

Suppose the numbers are larger, for example $29-51$:

$$29-51=29-29-22$$

How much will be left of the 51, after you have subtracted 29 from 29 to get 0?

How can we find out? Is it $51–29$?

2.3 Addition of a positive and a negative number

Examples: $7 + (-5); 37 + (-45)$ and $(-13) + 45$

The following statement is true if the unknown number is 5:

$$20 - (a\ certain\ number) = 15$$

We also need numbers that will make sentences like the following true:

$$20 + (a\ certain\ number) = 15$$

But to go from 20 to 15 you have to subtract 5.
The number we need to make the sentence $20 + (a\ certain\ number) = 15$ true, must have the following strange property:
If you **add** this number, it should have the **same effect** as **subtracting 5**.

So mathematicians agreed that the number called negative 5 will have the property that if you add it to another number, the effect will be the same as subtracting the natural number 5. negative 5
This means that mathematicians agreed that $20 + (-5)$ is equal to $20 - 5$.
In other words, instead of adding *negative* 5 to a number, you may subtract 5.

Note

Adding a negative number has the same effect as subtracting a corresponding natural number.
For example: $20 + (-15) = 20 - 15 = 5$.

2.4 Subtraction of a negative number

We have dealt with cases like $-20 - (-7)$ previously.
The following statement is true if the number is 5:

$$25 + (a\ certain\ number) = 30$$

We also need a number to make this statement true:

$$25 - (a\ certain\ number) = 30$$

If you subtract this number, it should have the same effect as adding 5.
It was agreed that $25 - (-5)$ is *equal* to $25 + 5$.

Instead of subtracting the negative number, you add the corresponding positive number (the additive inverse):

$$\begin{aligned} 8-(-3) &= 8+3 \\ &= 11 \\ -5-(-12) &= -5+12 \\ &= 7 \end{aligned}$$

Note

We may say that for each "positive" number there is a **corresponding** or **opposite** negative number. Two positive and negative numbers that correspond, for example 3 and (-3), are called **additive inverses**.

2.5 Subtraction of a positive number from a negative number

For example: $-7-4$ actually means $(-7)-4$.
Instead of subtracting a positive number, you add the corresponding negative number.
For example $-7-4$ can be seen as $(-7)+(-4)=-11$

3 MULTIPLYING AND DIVIDING WITH INTEGERS

3.1 Multiplication with integers

Commutative Property: A commutative property is a property where the order in which you complete the calculations does not matter. Addition and multiplication is commutative.

$$a+b=b+a, \qquad a\times b=b\times a$$

Please note: Subtraction and division is not commutative.

EXAMPLE

Multiplication of integers is commutative:

$(-20)\times 5=5\times(-20)$

Distributive Property: Numbers (or variables) can be added first and then multiplied, or you can first multiply the numbers (or variables) and then add together.

$$a(b+c)=ab+ac$$

- The product of two positive numbers is a positive number, for example $5 \times 6 = 30$.
- The product of a positive number and a negative number is a negative number, for example $5 \times (-6) = -30$.
- The product of a negative number and a positive number is a negative number, for example $(-5) \times 6 = -30$.

Note

To make sure that multiplication distributes over addition and subtraction in the system of integers, we have to agree that
(a negative number) $\times$ **(a negative number) is a positive number**,
for example $(-10) \times (-3) = 30$.

Summary

- When a number is added to its additive inverse, the result is 0. For example, $(+12) + (-12) = 0$.
- Adding an integer has the same effect as subtracting its additive inverse. For example, $3 + (-10)$ can be calculated by doing $3 - 10$, and the answer is -7.
- Subtracting an integer has the same effect as adding its additive inverse. For example, $3 - (-10)$ can be calculated by calculating $3 + 10$ is 13.

- The product of a positive and a negative integer is negative. For example, $(-15) \times 6 = -90$.
- The product of a negative and a negative integer is positive. For example $(-15) \times (-6) = 90$.

Division with integers

- The quotient of a positive number and a negative number is a negative number.
- The quotient of two negative numbers is a positive number.

3.2 Powers, roots and word problems

Note

The symbol $\sqrt{}$ means that you must take the **positive square root** of the number.

3^2 is 9 and $(-3)^2$ is also 9.

3^3 is 27 and $(-5)^3$ is 125.

Both (-3) and 3 are **square roots** of 9.

3 may be called the **positive square root** of 9, and (-3) may be called the **negative square root** of 9.

3 is called the **cube root** of 27, because $3^3 = 27$.

-5 is called the cube root of -125 because $(-5)^3 = -125$.

10^2 is 100 and $(-10)^2$ is also 100. Both 10 and (-10) are called **square roots** of 100.

4 EXERCISES

4.1 Exercise 1

1. Solve for x:

   1.1 $20 - x = 50$

   1.2 $50 + x = 20$

   1.3 $20 - 3x = 50$

   1.4 $50 + 3x = 20$

2. Calculate each of the following:

   2.1 $-7 + 18$

   2.2 $24 - 30 - 7$

   2.3 $-15 + (-14) - 9$

   2.4 $35 - (-20)$

   2.5 $30 - 47$

   2.6 $(-12) - (-17)$

4.2 Exercise 2

1. Calculate each of the following:

   1.1 $-7 + -7 + -7 + -7 + -7 + -7 + -7 + -7 + -7 + -7$

   1.2 $-10 + -10 + -10 + -10 + -10 + -10 + -10$

   1.3 $10 \times (-7)$

   1.4 $7 \times (-10)$

2. State whether the following statements true or false.

   2.1 $10 \times (-7) = 70$

   2.2 $9 \times (-5) = (-9) \times 5$

   2.3 $(-7) \times 10 = 7 \times (-10)$

   2.4 $5 \times (-9) = 45$

   2.5 $-(-9) \times (-9) = -(-(-9)) \times 9$

3. Calculate the following:

   3.1 $(-35) \div 5$

   3.2 $(-35) \div (-7)$

   3.3 $(-60) \div 20$

   3.4 $(-60) \div (-3)$

   3.5 $60 \div (-20)$

   3.6 $60 \div (-3)$

   3.7 $50 \div (-5)$

   3.8 $50 \div (-10)$

4.3 Exercise 3

1. Calculate each of the following.

   1.1 $20 + (-5)$

   1.2 $4 \times (20 + (-5))$

   1.3 $4 \times 20 + 4 \times (-5)$

   1.4 $4 \times ((-5) + (-20))$

   1.5 $4 \times (-5) + 4 \times (-20)$

2. Evaluate the following:

   2.1 $20 + (-15)$

   2.2 $10 + (-5)$

   2.3 What properties of integers is demonstrated in your previous two answers?

3. Evaluate the following:

   3.1 $4 \times 20 + 4 \times (-15)$

   3.2 $4 \times (-15) + 4 \times (-20)$

   3.3 $(-4) \times 10 + ((-4) \times (-5))$

4. Without doing the calculations, determine which two of the following expressions you expect to have the same solution.

   A. $16 \times (53 + 68)$

   B. $53 \times (16 + 68)$

   C. $16 \times 53 + 16 \times 68$

   D. $16 \times 53 + 68$

5. Determine $((-10) \times (5 + (-3))$

6. Calculate the following:

   6.1 $20(-50 + 7)$

   6.2 $20 \times (-50) + 20 \times 7$

   6.3 $20(-50 + -7)$

   6.4 $20 \times (-50) + 20 \times -7$

   6.5 $-20(-50 + -7)$

   6.6 $-20 \times -50 + -20 \times -7$

7. Calculate the following:

   7.1 $40 \times (-12 + 8) - 10 \times (2 + (-8)) - 3 \times (-3 - 8)$

   7.2 $(9 + 10 - 9) \times 40 + (25 - 30 - 5) \times 7$

   7.3 $-50(40 - 25 + 20) + 30(-10 + 7 + 13) - 40(-16 + 15 - 2)$

   7.4 $-4 \times (30 - 50) + 7 \times (40 - 70) - 10 \times (60 - 100)$

   7.5 $-3 \times (-14 - 6 + 5) \times (-13 - 7 + 10) \times (20 - 10 - 15)$

4.4 Exercise 4

1. Calculate the values of A, B, C, D, E and F in the following table:

x	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12
x^2	1	4	9	16	25	36	C	64	81	100	121	E
x^3	1	8	A	64	125	B	343	D	729	1000	1331	F

2. Calculate the following:

   2.1 $\sqrt{4} - \sqrt{9}$

   2.2 $\sqrt[3]{27} + (-\sqrt[3]{64})$

   2.3 $-(3^2)$

   2.4 $(-3)^2$

   2.5 $4^2 - 6^2 + 1^2$

   2.6 $3^3 - 4^3 - 2^3 - 1^3$

   2.7 $\sqrt{81} - \sqrt{4} \times \sqrt[3]{125}$

   2.8 $-(4^2)(-1)^2$

   2.9 $\frac{(-5)^2}{\sqrt{37-12}}$

   2.10 $\frac{-\sqrt{36}}{-1^3-2^3}$ (Round off to two decimal places)

3. Determine each of the following:

   3.1 The overnight temperature in Polokwane drops from $11°$C to $-2°$C. By how many degrees has the temperature dropped (in °C)?

   3.2 The temperature in Escourt drops from $2°$C to $-1°$C in one hour, and then another two degrees in the next hour. How many degrees in total did the temperature drop over the two hours?

   3.3 A submarine is 75 m below the surface of the sea. It then rises by 21 m. How far (in meters) below the surface is it now?

   3.4 A submarine is 37 m below the surface of the sea. It then sinks a further 15 m. How far (in meters) below the surface is it now?

5 ANSWERS FOR EXERCISES

5.1 Exercise 1

1.1 $x = -30$

1.2 $x = -30$

1.3 $x = -10$

1.4 $x = -10$

2.1 11

2.2 -13

2.3 -38

2.4 55

2.5 -17

2.6 5

5.2 Exercise 2

1.1 -70

1.2 -70

1.3 -70

1.4 -70

2.1 False

2.2 True

2.3 True

2.4 False

2.5 True

3.1 -7

3.2 5

3.3 -3

3.4 20

3.5 -3

3.6 -20

3.7 -10

3.8 -5

5.3 Exercise 3

1.1 15

1.2 60

1.3 60

1.4 -100

1.5 -100

2.1 5

2.2 5

2.3 Adding an integer is the same as subtracting its additive inverse.

3.1 20

3.2 -140

3.3 -20

4. A and C

5. -20

6.1 -860

6.2 -860

6.3 $-1\,140$

6.4 $-1\,140$

6.5 $1\,140$

6.6 $1\ 140$

7.1 -67

7.2 330

7.3 $-1\ 330$

7.4 270

7.5 $2\ 250$

5.4 Exercise 4

1. A $= -27$
   B $= -216$
   C $= 49$
   D $= -512$
   E $= 144$
   F $= -1\ 728$

2.1 -1

2.2 -1

2.3 -9

2.4 9

2.5 -19

2.6 -46

2.7 -1

2.8 -16

2.9 5

2.10 $\frac{2}{3}$

3.1 $13°$C

3.2 $5°$C

3.3 54 m

3.4 52 m