CHAPTER 6

Patterns

MathU Teaching Emporium

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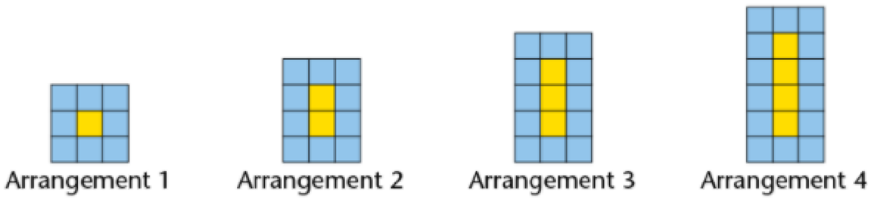
October 19, 2021

1 PATTERNS

1.1 Exercise 1

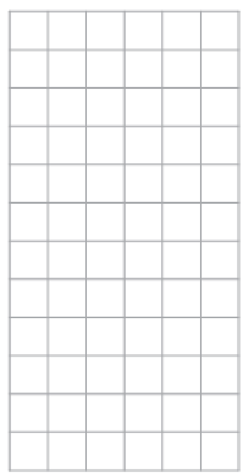


1. Blue and yellow square tiles are combined to form the above arrangements.

   1.1 How many yellow tiles are there in each arrangement?

   1.2 How many blue tiles are there in each arrangement?

   1.3 If more arrangements are made in the same way, how many blue tiles and how many yellow tiles will there be in arrangement 5? Check your answer by drawing the arrangement on the grid on the below.



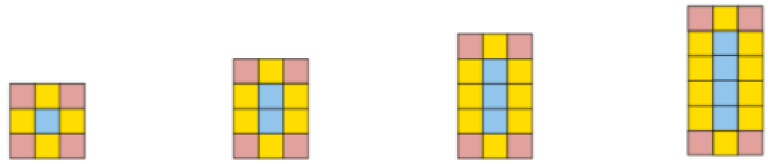
   1.4 Complete this table.

Number of yellow tiles	1	2	3	4	5	8
Number of blue tiles						

   1.5 How many blue tiles will there be in a similar arrangement with 26 yellow tiles?

   1.6 How many blue tiles will there be in a similar arrangement with 100 yellow tiles?

2. 2.1 In these arrangements there are red tiles too. Complete this table.



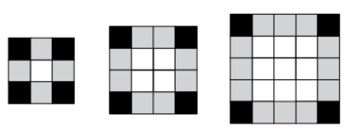
Arrangement 1 Arrangement 2 Arrangement 3 Arrangement 4

Number of blue tiles	1	2	3	4	5	6	7
Number of yellow tiles	A	B	8	10	C	14	16
Number of red tiles	4	4	4	4	4	4	D

Note

The number of red tiles in arrangements like those above is **constant**. It is always 4, no matter how many blue and yellow tiles there are. The number of blue tiles is different for different arrangements. We can say the number of blue tiles **varies**. We can also say the number of blue tiles is a **variable**.

3. Look at these three arrangements. They consist of black squares, grey squares and white squares.



3.1 Sketch the next consecutive term in the pattern.

3.2 Describe what is constant in these arrangements.

3.3 What are the variables in these arrangements?

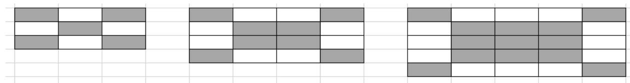
3.4 Complete the table based upon the pattern above.

Arrangement number	1	2	3	4	5	6	7	10	20
Number of black squares									
Number of grey squares									
Number of white squares									

3.5 How many grey squares do you think there will be in arrangement 15?

3.6 How many black squares do you think there will be in arrangement 15?

4.

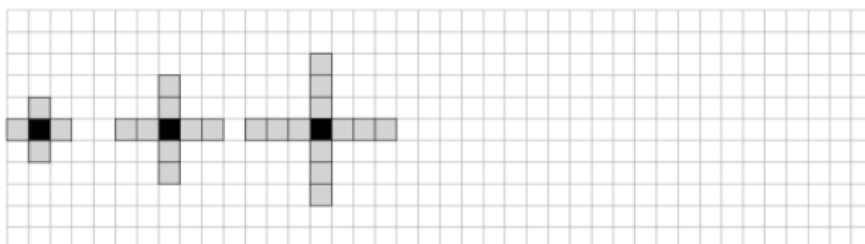


4.1 Draw the next arrangement that follows the same pattern.

4.2 How many grey squares will there be in term 12?

1.2 Exercise 2

1. 1.1 Make two more arrangements of black and grey squares so that a pattern is formed.



1.2 Is there a constant in your pattern? If yes, what is its value?

1.3 Is there a variable in your pattern? If yes, give the values of the variable.

2. 2.1 Make three more arrangements with dots to form the sequence $1; 3; 6; 10; 15...$

2.2 How many dots will there be in the sixth and seventh arrangements? Explain how you got your answer.

2.3 How many dots are there in arrangements 1 and 2 together?

2.4 How many dots are there in arrangements 2 and 3 together?

2.5 How many dots are there in arrangements 3 and 4 together?

2.6 How many dots are there in arrangements 4 and 5 together?

2.7 Describe the pattern in your answers for 2.3, 2.4, 2.5 and 2.6.

3. 3.1 Sketch the next two consecutive terms of the pattern.

3.2 What are the variables in your pattern?

3.3 The number of black squares is a variable in these arrangements. The value of this variable is 4 in the first arrangement and 8 in the second arrangement. What is the value of this variable in the third arrangement?

3.4 What are the values of each of the variables in the fifth arrangement in your pattern? Explain your answers.

1.3 Exercise 3

1. 1.1 Write the next three numbers in each of the sequences below.

Sequence A: $5 \;\; 9 \;\; 13 \;\; 17 \;\; 21$

Sequence B: $5 \;\; 10 \;\; 20 \;\; 40 \;\; 80$

Sequence C: $5 \;\; 10 \;\; 17 \;\; 26 \;\; 37$

Note

The numbers in a sequence are also called the **terms** of the sequence.

Note

A sequence can be formed by repeatedly adding or subtracting the same number. In this case the **difference** between consecutive terms in a sequence is **constant**. A sequence can be formed by repeatedly multiplying or dividing. In this case the **ratio** between consecutive terms is **constant**. A sequence can also be formed in such a way that neither the difference nor the ratio between consecutive terms is constant.

Note

To write more terms of sequence A, you **added** 4 **repeatedly**.

Note

To write more terms of sequence B, you **multiplied** by 2 **repeatedly**.

Note

To write more terms of sequence C you did not add the same number each time, nor did you multiply by the same number.

2. In each case, follow the instruction to make a sequence with eight terms.
   2.1 Start with 1 and multiply by 2 repeatedly.
   2.2 Start with 256 and divide by 2 repeatedly.
   2.3 Start with 2 and raise it to the n^{th} power.
3. Complete the following tables:

3.1

Term number	1	2	3	4	5	6
Term value	A	B	80	160	320	C

3.2

Term number	3	4	5	6	7
Term value	A	B	3	8	13

3.3

Term number	1	2	3	4	5
Term value	A	57	B	51	C

A description of the **relationship between consecutive terms**. In other words the calculations that you do to a term to produce the next term, as in questions 3.2 and 3.3. The first (or another) term must be given. This kind of formula has two parts, the first term, and the relationship between terms.

A description of the **relationship between the value of the term and its position in the sequence**. This relationship describes the calculations that can be done **on the term number** to produce the **term value**, as in question 3.1.

1.4 Exercise 4

1. Consider the two incomplete sequences in the table below.

Term number	1	2	3	4	5	6	7	8
Sequence 1	5	10	A	B	C	30	35	40
Sequence 2	3	8	D	18	23	E	33	F

1.1 Determine the values of A, B and C.

1.2 Determine the values of $D,\ E$ and F.

1.3 What is similar between the sequences A and B?

1.4 What is the formula for the sequence A?

1.5 What is the formula for the sequence B?

2. Determine the formula for the following sequences:

2.1

Term number	1	2	3	4	5	6	7	8
Term value	3	6	9	12	15	18	21	24

2.2

Term number	1	2	3	4	5	6	7	8
Term value	1	4	9	16	25	36	49	64

2.3

Term number	1	2	3	4	5	6	7	8
Term value	-1	-8	-27	-64	-125	-216	-343	-512

3. Consider the sequence defined by the following formula:

$T(n) = \frac{1}{\sqrt{5}}[(\frac{1+\sqrt{5}}{2})^n - (\frac{1+\sqrt{5}}{2})^n$

3.1 Draw a table for the first 5 terms of the sequence.

3.2 What is the name of this famous sequence?

2 LINEAR NUMBER PATTERNS

2.1 Examples:

Question 1

Consider the following sequence:

$-3; -7; -11; -15; ...$

1.1 Determine the formula of the following sequence in the form $T_n = ...$

1.2 What term will have a value of -39?

1.3 Determine the value of T_{13}.

Memo

Term: $1;\ 2;\ 3;\ 4$

Value; $-3; -7; -11; -15$

First difference: -4

1.1 $T_n = an + b$

$-3 = -4(1) + b$

$-3 = -4 + b$

$\therefore b = 1$

$T_n = -4n + 1$

1.2 $T_n = -4n + 1$

$-39 = -4n + 1$

$-40 = -4n$

$\therefore n = 10$

$T_{10} = -39$

1.3 $T_n = -4n + 1$

$T_{13} = -4(13) + 1$

$T_{13} = -51$

Question 2

Determine the general term of the following sequences.

2.1 $16; 25; 36; 49$

2.2 $3; 10; 29; 66$

Memo

2.1 $T_n = (n+3)^2$

2.2 $T_n = n^3 + 2$

Question 3

Consider the following sequence:

$\frac{3}{15}; \frac{6}{13}; \frac{9}{11}; \frac{12}{9}; \ldots$

Determine the value of T_{30}.

Memo

Term: $1;\ 2;\ 3;\ 4;\ n$

Value: $\frac{3}{15}; \frac{6}{13}; \frac{9}{11}; \frac{12}{9}; \frac{n \times 3}{-2n+17}$

$T_{30} = \frac{30 \times 3}{-2(30)+17} = -\frac{90}{43}$

Question 4

Consider the following sequence:

$2; 6; 10; 14$

4.1 What will the value of the x-term be?

4.2 Determine the term with the value of 34.

4.3 Determine the value of T_{50}.

Memo

4.1 $T_n = an + b$
$= 4n + b$
$2 = 4(1) + b$
$2 = 4 + b$
$\therefore b = -2$
$T_x = 4x - 2$

4.2 $T_n = 4n - 2$
$34 = 4n - 2$
$36 = 4n$
$n = 9$
$\therefore T_9 = 34$

4.3 $T_n = 4n - 2$
$T_{50} = 4(50) - 2$
$= 200 - 2$
$= 198$

2.2 Exercise 5

1. Consider the following sequence:
   $-5; -11; -17; -23; ...$
   1.1 Determine the formulae of the following sequence in the form $T_n = ...$
   1.2 What term will have a value of -76?
   1.3 Determine the value of T_{10}.
2. Determine the formulae of the following sequences.
   2.1 $8; 27; 64; 125$
   2.2 $-1; 6; 25; 62$

3. Consider the following sequence:
   $\frac{2}{3}; \frac{5}{7}; \frac{8}{11}; \frac{11}{15}; \ldots$
   Determine the value of T_{25}.

4. Consider the following sequence
   $3; 10; 17; 24$

   4.1 What will the value of the x-term be?

   4.2 Determine the term with the value of 80.

   4.3 Determine the value of T_{30}.

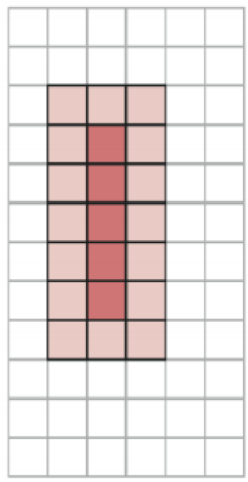
3 ANSWERS TO EXERCISES

3.1 Exercise 1

1.1 There are $1, 2, 3$ and 4 yellow tiles in arrangements $1, 2, 3$ and 4 respectively.

1.2 There are $8, 10, 12$ and 14 blue tiles in arrangements $1, 2, 3$ and 4 respectively.

1.3 Five yellow tiles and 16 blue tiles



1.4

Number of yellow tiles	1	2	3	4	5	8
Number of blue tiles	8	10	12	14	16	22

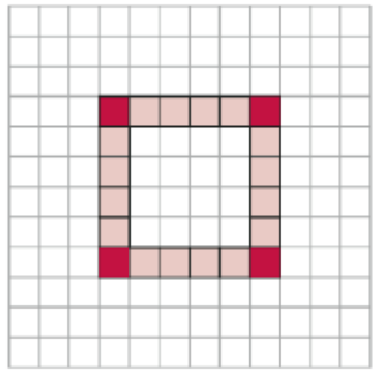
1.5 58 blue tiles

1.6 206

2.

Number of blue tiles	1	2	3	4	5	6	7
Number of yellow tiles	4	6	8	10	12	14	16
Number of red tiles	4	4	4	4	4	4	4

3.1



3.2 The number of black squares

3.3 The number of grey squares and the number of white squares are both variables

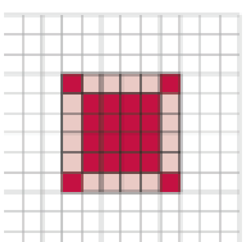
3.4

Arrangement number	1	2	3	4	5	6	7	10	20
Number of black squares	4	4	4	4	4	4	4	4	4
Number of grey squares	4	8	12	16	20	24	28	40	80
Number of white squares	1	4	9	16	25	36	49	100	400

3.5 $15^2 = 2254$ Or, by adding increasing odd number steps, for example: $100 + 21 = 121, 121 + 23 = 144$, etc. (Note: the last answer is quite sophisticated.)

3.6 The number of black tiles on the corners remains the same: 4.

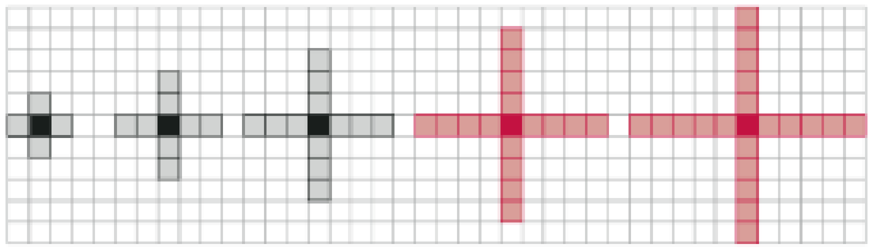
4.1



4.2 $12^{12} + 4 = 148$

3.2 Exercise 2

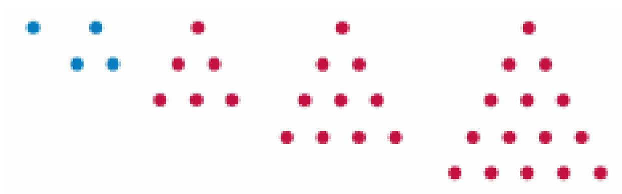
1.1



1.2 Yes, the number of black squares is constant. It is 1 in all the arrangements.

1.3 Yes, the number of grey squares is a variable: $4, 8, 12, 16, 20$.

2.1



2.2 21 in the sixth arrangement, 28 in the seventh.

The pattern is $1, 3, 6, 10, 15, 21, 28...$

Or: The number added increases by 1 each time.

2.3 4

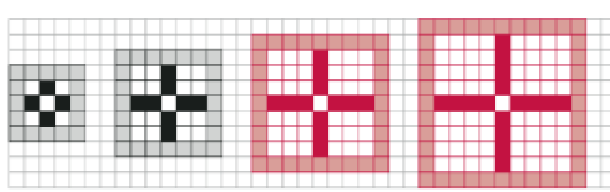
2.4 9

2.5 16

2.6 25

2.7 $4, 9, 16, 25$. These are the squares of the whole numbers from 2.

3.1



3.2 The numbers of white squares, grey squares and black squares.

3.3 12

3.4 20 black squares, because the pattern is $4, 8, 12, 16, 20$; add 4 to consecutive terms, starting with 4.
101 white squares, because the pattern is $5, 17, 37, 65, 101$ which is: $4+1; 16+1; 36+1; 64+1$, and this can be written as
$12 \times 4 + 1; 22 \times 4 + 1; 32 \times 4 + 1; 42 \times 4 + 1$
48 grey squares, because the pattern is $16, 24, 32, 40, 48$; add 8 to consecutive terms starting with 16.

3.3 Exercise 3

1.1 Sequence A: $5 \quad 9 \quad 13 \quad 17 \quad 21 \quad 25 \quad 29 \quad 33$

Sequence B: $5 \quad 10 \quad 20 \quad 40 \quad 160 \quad 320 \quad 640$

Sequence C: $5 \quad 10 \quad 17 \quad 26 \quad 37 \quad 50 \quad 65 \quad 82$

2.1 $1 \quad 2 \quad 4 \quad 8 \quad 16$

2.2 $256 \quad 128 \quad 64 \quad 32 \quad 16$

2.3 $2 \quad 4 \quad 8 \quad 16 \quad 32$

3.1

Term number	1	2	3	4	5	6
Term value	20	40	80	160	320	640

3.2

Term number	3	4	5	6	7
Term value	-7	-2	3	8	13

3.3

Term number	1	2	3	4	5
Term value	60	57	54	51	48

3.4 Exercise 4

1.1 $A = 15$
$B = 20$
$C = 25$

1.2 $D = 13$
$E = 28$
$F = 38$

1.3 The difference between each consecutive term is 5

1.4 $T(n) = 5n$

1.5 $T(n) = 5n - 2$

2.1 $T(n) = n + 2$

2.2 $T(n) = n^2$

2.3 $T(n) = -n^3$

3.1

Term number	1	2	3	4	5
Term value	1	1	2	3	5

3.2 The fibonacci sequence

3.5 Exercise 5

1.1 $T_n = -6n + 2$

1.2 $T_{13} = -39$

1.3 $T_{10} = -58$

2.1 $T_n = (n + 1)^3$

2.2 $T_n = n^3 - 2$

3. $\frac{74}{99}$

4.1 $T_x = 7x - 4$

4.2 $T_{12} = 80$

4.3 $T_{30} = 206$