CHAPTER 8

Algebraic Expressions

MathU Teaching Emporium

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An algebraic expression is a description of a set of operations that are to be done in a certain order. In this chapter, you will learn to specify a different set of operations that will produce the same results as a given set of operations. Two different expressions that produce the same results are called equivalent expressions.

1 ALGEBRAIC LANGUAGE

1.1 Words, diagrams and expressions

Note

An **algebraic expression** indicates **a sequence of operations** that can also be described in words. In some cases they can be described with flow diagrams.

Expressions in brackets should always be calculated first. If there are no brackets in an algebraic expression, it means that multiplication and division must be done first, and addition and subtraction afterwards.

For example, if $x = 5$ the expression $12 + 3x$ means "multiply 5 by 3, then add 12". It does **not** mean "add 12 and 3, then multiply by 5".

If you wish to say "add 12 and 3, then multiply by 5", the numerical expression should be $5 \times (12 + 3)$ or $(12 + 3) \times 5$.

1.2 Some words we use in algebra

An expression with one term only, like $3x^2$, is a **monomial**.

An expression which is a sum of two terms, like $5x + 4$, is called a **binomial**.

An expression which is a sum of three terms, like $3x^3 + 2x + 9$, is called a **trinomial**.

The symbol x is often used to represent the **variable** in an algebraic expression, but other letter symbols may also be used.

In the monomial $3x^2$, the 3 is called the **coefficient** of x^2.

In the binomial $5x + 4$, and the trinomial $3x^2 + 2x + 9$ the numbers 4 and 9 are called **constants**.

1.3 Equivalent algebraic expressions

Note

Equivalent expressions are algebraic expressions that have different sequences of operations, but have the same numerical value for any given value of x.

Note

It is often convenient not to work with a given expression, but to **replace** it with an equivalent expression.

1.4 Conventions for writing algebraic expressions

Here are some things that mathematicians have agreed upon, and it makes mathematical work much easier if all people stick to these agreements.

Note

A **convention** is something that people have agreed to do in the same way.

Note

The multiplication sign is often omitted in algebraic expressions: We normally write $4x$ instead of $4 \times x$ and $4(x–5)$ instead of $4 \times (x–5)$. It is a convention to write a known number first in a product, i.e. we write 3Ã— x rather than x Ã— 3, and we write 3x **but not x3**.

Note

People all over the world have agreed that, in expressions that do not contain brackets, addition and subtraction should be performed as they appear from left to right in the expression.

According to this convention, $x–y + z$ means that you first have to subtract y from x, then add z. For example if $x = 10$, $y = 5$ and $z = 3$, $x–y + z$ is $10–5 + 3$ and it means $10–5 = 5$, then $5 + 3 = 8$. It does not mean $5 + 3 = 8$, then $10–8 = 2$.

Note

People have also agreed that, in expressions that do not contain brackets, we should do **multiplication** (and division) **before addition and subtraction**.

Hence $5 + 3 \times 4$ should be understood as "multiply 4 by 3, then add the answer to 5" and not as "add 5 and 3 then multiply the answer by 4".

Also, $3 \times 4 + 5$ should be understood to mean "multiply 4 by 3, then add 5 to the answer", and not as "add 4 and 5 then multiply the answer by 3".

If we want to specify the calculations in 7(a) and 7(c) without using words we face challenges.

We cannot write 20 - 4 + 5 for "*add 4 and 5 then subtract the answer from 20*", because that would mean "*subtract 4 from 20 then add 5*". We need a way to indicate, without using words, that we want the addition to be performed before the subtraction in this case.

Similarly we cannot write 4 + 5 $\times$ 3 for "*add 4 and 5 then multiply the answer by 3*", because that would mean "*multiply 3 by 5 and then add the answer to 4*". We need a way to indicate, without using words, that we want the addition to be performed before the multiplication in this case.

Mathematicians have agreed to use brackets to address the above challenges. The following convention is used all over the world:

Note

Whenever there are brackets in an expression, the calculations within the brackets should be performed first.

Hence $20 – (4 + 5)$ means *add 4 and 5 then subtract the answer from 20, but 20 - 4 + 5 means subtract 4 from 20 then add 5*.

$(4 + 5) \times 3$ or $3 \times (4 + 5)$ means *add 4 and 5 then multiply the answer by 3*, but $4 + 5 \times 3$ means *multiply 3 by 5 then add the answer to 4*.

$10 + 2(5 + 9)$ means *add 5 and 9, multiply the answer by 2, then add this answer to 10:*

$5 + 9 = 14$
$14 \times 2 = 28$
$28 + 10 = 38$

In algebra, we normally write $3(x + 2y)$ instead of $(x + 2y) \times 3$, and we write $3(x – 2y)$ instead of $(x – 2y) \times 3$. Don't let this conventional way of writing in algebra confuse you. The expression $3(x + 2y)$ does not mean that multiplication by 3 is the first thing you should do when you evaluate the expression for certain values of x and y. The first thing you should do is to add the values of x and y. That is what the brackets tell you!

However, performing the instructions 3(x + 2y) is not the only way in which you can find out how much 3(x + 2y) is for any given values of x and y. Instead of working out 3(x + 2y), you may work out 3x + 6y. In this case you will multiply each term before you add them together.

2 PROPERTIES OF OPERATIONS

You should have noticed that for each row the results are the same. This is because operations with numbers have certain properties, namely the **distributive**, **commutative** and **associative** properties.

The **distributive** property is used each time you multiply a number in parts. For example:

The number thirty-four is actually $30 + 4$. You may calculate 5×34 by calculating 5×30 and 5×4, and then adding the two answers:

$5 \times 34 = 5 \times 30 + 5 \times 4$

Note

The word "distribute" means to spread out. The distributive property may be described as follows:

$a(b + c) = ab + ac$

where a, b and c can be any numbers.

We may say: "multiplication distributes over addition"

For any values of x and y,

- $x + y$ and $y + x$ give the same answers, and
- xy and yx give the same answers.

This is called the **commutative property** of addition, and multiplication.

The **associative property** allows you to arrange three or more numbers in any sequence when adding or multiplying. For any values of x, y and z, the following expressions all have the same answer:

$x + y + z$
$y + x + z$
$z + y + x$

The associative property of multiplication allows you to simplify something like the following.

$abc + bca + cba$

Because the order of multiplication does not change the result we can rewrite this expression as: $abc+abc+abc$.

This then can be simplified by adding like terms to be $3abc$. You will be able to use these properties throughout this chapter and when you do algebraic manipulations.

Note

When you form an expression that is equivalent to a given expression you say that you manipulate the expression.

3 COMBINING LIKE TERMS IN ALGEBRAIC EXPRESSIONS

3.1 Rearrange terms, then combine like terms

To check whether two expressions are possibly equivalent, you can evaluate both expressions for several different values of the variable.

Some expressions can be simplified by rearranging the terms and combining "like terms".

In the expression $5x^2 + 13x + 7 + 2x^2 – 8x – 12$, the terms $5x^2$ and $2x^2$ are like terms.

Note

Two or more like terms can be combined to form a single term.

$5x^2 + 2x^2$ can be replaced by $7x^2$ because for any value of x, for example $x = 2$ or $x = 10$, calculating $5x^2 + 2x^2$ and $7x^2$ will produce the same output value (try it!).

4 MULTIPLICATION OF ALGEBRAIC EXPRESSIONS

4.1 Multiply polynomials by monomials

The fact that if you work correctly, you get the same answer in questions 1(a) and 1(b), is a demonstration of the **distributive property**.

> **Note**
>
> The distributive property may be described as follows:
>
> $a(b + c) = ab + ac$ and
>
> $a(b–c) = ab–ac$,
>
> where a, b and c can be any numbers.

What you saw in question 1 was that $3 \times 100 = 3 \times 38 + 3 \times 62$.

This can also be expressed by writing $3(38 + 62) = 3 \times 38 + 3 \times 62$.

Performing the instructions $5(x + y)$ is not the only way in which you can find out how much $5(x + y)$ is for any given values of x and y. Instead of doing $5(x + y)$ you may do $5x + 5y$. In this case you will multiply first, and again, before you add.

> **Note**
>
> What you do in this question is sometimes called ”multiplication of a polynomial by a monomial”.
>
> One may also say that in each case you **expand** the expression, or you write an equivalent expression in **expanded form**.

4.2 Squares and cubes and roots of monomials

The square root of $16x^2$ is $4x$, because $(4x)^2 = 16x^2$.

The cube root of $64x^3$ is $4x$, because $(4x)^3 = 64x^3$

5 DIVIDING POLYNOMIALS BY INTEGERS AND MONOMIALS

> **Note**
>
> Division is **right-distributive** over addition and subtraction, for example, $(2 + 3) \div 5 = (2 \div 5) + (3 \div 5)$. The division symbol is to the right of the brackets. But it is not left-distributive, for example, $10\tilde{A} \div (2 + 4) \neq (10\tilde{A} \div 2) + (10\tilde{A} \div 4)$.

For example
$(200+40) \div 20 = (200 \div 20) + (40 \div 20) = 10 + 2 = 12$, and

$(500 + 200 – 300) \div 50 = (500 \div 50) + (200 \div 50) – (300 \div 50)$

The distributive property of division can be expressed like this:

Note

$(x+y) \div z = (x \div z) + (y \div z)$

$(x – y) \div z = (x \div z) – (y \div z)$

The instruction $72 \div 6$ may also be written as $\frac{72}{6}$.

This notation, which looks just like the common fraction notation, is often used to indicate division.

Hence, instead of $(10x^2 + 20x – 15) \div 5$ we may write $\frac{10x^2+20x–15}{5}$.

Since $(10x^2 + 20x – 15) \div 5$ is equivalent to
$(10x^2 \div 5) + (20x \div 5) – (15 \div 5)$,

$\frac{10x^2+20x–15}{5}$ is equivalent to $\frac{10x^2}{5} + \frac{20x}{5} – \frac{15}{5}$.

6 PRODUCTS AND SQUARES OF BINOMIALS

How can we obtain the expanded form of $(x+2)(x+3)$?

In order to expand $(x+2)(x+3)$, you can first keep $(x+2)$ it is, and apply the distributive property:

$(x+2)(x+3)$
$= (x+2)x + (x+2)3$
$= x^2 + 2x + 3x + 6$
$= x^2 + 5x + 6$

To expand (x - y)(x + 3y) it can be written as (x - y)x + (x - y)3y and the two parts can then be expanded.

$(x – y)(x+3y)$
$= (x – y)x + (x – y)3y$
$= x^2 – xy + 3xy – 3y^2$

$= x^2 + 2xy – 3y^2$

Note

All the expressions in questions 4 and 5 are **squares of binomials**, for example $(ax + b)^2$ and $(ax – b)^2$

6.1 Manipulating Expressions

The process of writing a polynomial as a product is called **factorisation**. This is the inverse of expansion.

$$\xrightarrow{factorisation}$$

$$x^2 + 5x + 6 = (x + 2)(x + 3).$$

$$\xleftarrow{expansion}$$

TIP

A numerical or algebraic expression that requires multiplication as a last step, is called **product**. For example, $12(37 + 63), 2x(x - 5)$ and xyz are called products. A product is a monomial

Each part of a product is called a **factor** of the expression. If $c = ab$, then a and b are factors of c. $(x + 2)$ and $(x + 3)$ are the factors of $(x + 2)(x + 3)$. Since :

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

$x + 2$ and $x + 3$ are the factors of $x^2 + 5x + 6$.

TIP

Important : A statement like :

$$2(x + 3) = 2x + 6,$$

which is true for all values of x you can think of, is called an **identity**.

In the following sections you will learn how to factorise certain types of expressions.
The following identities are useful for the purposes of factorisation:

- $a(b + c) = ab + ac$
- $(x + a)(x + b) = x^2 + (a + b)x + ab$
- $(a + b)(a - b) = a^2 - b^2$

7 FACTORS OF EXPRESSIONS OF THE FORM $ab + ac$

7.1 The greatest common factor

Suppose you have to factorise $4x^3 + 2x^2 - 6x$:
It is clear that $2x$ is a factor of every term, hence it is a factor of $4x^3 + 2x^2 - 6x$.

By division we get $\frac{4x^3+2x^2-6}{2x} = 2x^2 + x - 3$. Hence $4x^3 + 2x^2 - 6x = 2x(2x^2 + x - 3)$

It is always a good idea to check factorisation by expanding the answer and making sure that the result is equal to the original expression.

TIP

Note that:

$$b - a = -a + b = -(a - b)$$

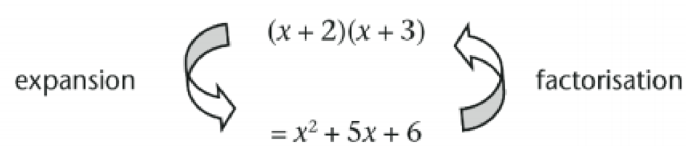
8 FACTORS OF EXPRESSIONS OF THE FORM $x^2 + (b + c)x + bc$

The expanded form of a product of two linear binomials, such as $(x + 3)(x + 8)$, or $(x + 3)(x - 8)$, is a **quadratic trinomial**, such as $x^2 + 11x + 24$ or $x^2 - 5x - 24$ with:

- A term in x^2
- A term in x that is called the **middle term**, which is:
  11 in $x^2 + 11x + 24$ and
  $-5x$ in $x^2 - 5x - 24$
- A constant term also called the **last term**, which is :
  $+24$ in $x^2 + 11x + 24$, and
  -24 in $x^2 - 5x - 24$.

To factorise an expression, such as $x^2 + 5x + 6$, means to reverse the process of expansion.
This means that we have to find out which binomials will produce the trinomial when the product of the binomials is expanded, for example:

$$x^2 + 5x + 6 = (\mathbf{?} + \mathbf{?})(\mathbf{?} + \mathbf{?})$$



The product of the first terms of the factors must be equal to the x^2 term of the trinomial.

Meaning: $x \times x = x^2$

Meaning: $2 \times 3 = 6$

The product of the last terms of the factors must be equal to the last term (the constant term) of the trinomial.
The sum of the inner and outer products must be equal to the term in x (the middle term) of the trinomial.

Meaning: $2x + 3x$

$$= (2+3)x$$

$$= 5x$$

WORKED EXAMPLE 1: ALTERNATIVE METHOD

Question

Factorise $ac + bc + bd + ad$

Solution

Step 1: Order and group with common factors

$$ab + bc + bd + ad = (ac + bc) + (bd + ad)$$

Step 2: Take out the common factor

$$= c(a+b) + d(b+a)$$

Step 3: Write the expression as a product

$$= (a+b)(c+d)$$

$$\therefore ac + bc + bd + ad = (a+b)(c+d)$$

WORKED EXAMPLE 2: ANOTHER METHOD

Question

Factorise $x^2 + 4x + 3$

Solution

Step 1: Rewrite the middle terms as the sum of two terms

$$= x^2 + x + 3x + 3$$

Step 2: Group

$$= (x^2 + x) + (3x + 3)$$

WORKED EXAMPLE 2: ANOTHER METHOD continued...

Step 3: Take out the greatest common factor

$$= x(x\mathbf{+1}) + 3(x\mathbf{+1})$$

Step 4: Write it as a product

$$= (x\mathbf{+1})(x + 3)$$

$$x^2 + 4x + 3 = (x\mathbf{+1})(x + 3)$$

WORKED EXAMPLE 3: ANOTHER METHOD

Question

Factorise $x^2 + 3x - 4$

Solution

Step 1: Rewrite the middle terms as the sum of two terms

$$= x^2 - x + 4x - 4$$

Step 2: Group

$$= (x^2 - x) + (4x - 4)$$

Step 3: Take out the greatest common factor

$$= x(x\mathbf{-1}) + 4(x\mathbf{-1})$$

Step 4: Write it as a product

$$=(x\mathbf{-1})(x + 4)$$

$$\therefore x^2 + 3x - 4 =(x\mathbf{-1})(x + 4)$$

The challenge is to re-write the middle term as the sum of two terms in a way that you are able to take out the common factor.

9 FACTORS OF EXPRESSIONS OF THE FORM $a^2 - b^2$

9.1 Factorising difference between two squares expressions

An expression of the form $a^2 - b^2$ is called the **diffrence between two squares**.
To factorise a difference between two squares , we use the identity:

$$a^2 - b^2 = (a + b)(a - b)$$

where a and b represent numbers or algebraic expressions.

TIP

Important: If p and q are perfect squares, also "algebraic squares", then :

$$p - q = (\sqrt{p} + \sqrt{q})(\sqrt{p} - \sqrt{q})$$

$$9x^4 - 4y^2 = (\sqrt{9x^4} + \sqrt{4y^2})(\sqrt{9x^4} - \sqrt{4y^2})$$

$$= (3x^2 + 2y)(3x^2 - 2y)$$

(Note the operations within the brackets differ.)

Remember:

- Always factorise completely
- Always take out the greatest common factor if there is one.
- One is a perfect square: $1 = 1^2$ and $1^m = 1$.
- The exponential law: $a^m \cdot a^n = a^{m+n}$

10 SIMPLIFICATION OF ALGEBRAIC FRACTIONS

10.1 Working with algebraic fractions

Important: It is useful to manipulate quotient expressions, such as :

$$\frac{x^2+5x+6}{x+2},$$

into simpler but equivalent sum expressions like $x+3$ in this case. It Makes substitution and the solving of equations easier.

WORKED EXAMPLE 4: SIMPLIFICATION OF ALGEBRAIC FRACTIONS

Question

Determine the value of $\frac{x^2-2x-3}{x-3}$ for $x=4,6$

Solution

Step 1: Factorise the numerator

$$\frac{x^2-2x-3}{x-3}=\frac{(x-3)(x+1)}{x-3}$$

Step 2: Simplify the expression

$$\frac{(x-3)(x+1)}{x-3}=x+1$$

Step 3: Substitute $x=4,6$

$$=4,6+1$$
$$=5,6$$

10.2 Dividing by zero cannot be done

Division by 0 is not possible.

TIP

The algebraic fraction:

$$\frac{x+2}{x-2},$$

cannot have a value if the denominator ($x-2$) is equal to 0.

We say the expression $\frac{x+2}{x-2}$ is **undefined** for $x-2=0$, ie. for $x=2$.

We also say $x=2$ is an **excluded value** of x for $\frac{x+2}{x-2}$

10.3 Simplifying algebraic frations

To simplify an algebraic fraction that contains a polynomial as a numerator or denominator, the polynomial should be factorised first.

> **TIP**
>
> Important: To prevent division by zero, the excluded values must be stated.

11 EXERCISES

11.1 Exercise 1

1. Complete the table in each question using the table below as an example.

Words	Flow Diagram	Expression
Multiply a number by 5 and then subtract by 3 from the answer	$\boxed{\times 5} \longrightarrow \boxed{-3} \longrightarrow$	$5x - 3$

1.1	Add 5 to a number and then multiply the answer by 3	$\boxed{+5} \longrightarrow \boxed{\times 3} \longrightarrow$	
1.2		$\boxed{-3} \longrightarrow \boxed{\times 5} \longrightarrow$	$(x-3) \times 5$ or $5(x-3)$
1.3	Multiply a number by 2, add 3 and multiply the answer by 3	$\square \longrightarrow \square \longrightarrow \square \longrightarrow$	$3(2x+3)$

2. Describe each of these sequences of calculations with an algebraic expression:

   2.1 Multiply a number by 10, subtract 5 from the answer, and multiply the answer by 3.

   2.2 Subtract 5 from a number, multiply the answer by 10 and multiply this answer by 3.

3. Evaluate each of these expressions for $x = 10$:

   3.1 $200 - 5x$

   3.2 $(200 - 5)x$

   3.3 $5x + 40$

3.4 $5(x + 40)$

3.5 $40 + 5x$

3.6 $5x + 5 \times 40$

11.2 Exercise 2

1. Consider the polynomial pattern starting with $7x^5 + 5x^4 + 3x^3 + x^2 + \dots$

   1.1 What is the coefficient of the fourth term?

   1.2 What is the exponent value of the fifth term?

   1.3 Do you think the sixth term will be a constant? Why?

2. Copy and complete the table, using the completed first row as an example:

Expression	Type of expression	Symbol	Constant	Coefficient of ...
$x^2 + 6x + 10$	Trinomial	x	10	the second term is: 6

2.1

Expression	Type of expression	Symbol	Constant	Coefficient of ...
$6s^3 + s^2 + 5$				s^2 is:

2.2

Expression	Type of expression	Symbol	Constant	Coefficient of ...
$\frac{k}{3} + 12$				the first term is:

2.3

Expression	Type of expression	Symbol	Constant	Coefficient of ...
$4p^10$	Monomial			p^10 is:

11.3 Exercise 3

1. Copy and complete the table by doing the necessary calculations. Calculate the numerical value of the expressions for the various values of x.

1.1

x	-2	-1	0	1	2
$3x + 2$					

1.2

x	-2	-1	0	1	2
$2x - 3$					

1.3

x	-2	-1	0	1	2
$3x + 2 + 2x - 3$					

1.4

x	-2	-1	0	1	2
$2x - 3 + 3x + 2$					

1.5

x	-2	-1	0	1	2
$5x-1$					

1.6

x	-2	-1	0	1	2
$(3x+2)(2x-3)$					

1.7

x	-2	-1	0	1	2
$3x(2x-3)+2(2x-3)$					

1.8

x	-2	-1	0	1	2
$6x^2-5x-6$					

1.9

x	-2	-1	0	1	2
$\frac{(3x+2)(2x-3)}{3x+2}$					

1.10

x	-2	-1	0	1	2
$\frac{6x^2-5x-6}{3x+2}$					

2. Consider the tables with the following expressions. Although they may look different, make a list of all the algebraic expressions which have the same numerical value for the same value of x.

x	-2	-1	0	1	2
$3x+2$	-4	-1	2	5	8
$2x-3$	-7	-5	-3	-1	1
$3x+2+2x-3$	-11	-6	-1	4	9
$2x-3+3x+2$	-11	-6	-1	4	9
$5x-1$	-11	-6	-1	4	9
$(3x+2)(2x-3)$	28	5	-6	-5	8
$3x(2x-3)+2(2x-3)$	28	5	-6	-5	8
$6x^2-5x-6$	28	5	-6	-5	8
$\frac{(3x+2)(2x-3)}{3x+2}$	-7	-5	-3	-1	1
$\frac{6x^2-5x-6}{3x+2}$	-7	-5	-3	-1	1

3. Copy and complete the following table:

3.1

x	2	3	5	10	-5	-10
$12x-7+3x+10-5x$						

3.2

x	2	3	5	10	-5	-10
$10x+3$						

4. Consider the expressions in the table. Is $10x + 3$ equivalent to $12x - 7 + 3x + 10 - 5x$? Explain your answer.

5. Suppose you need to know how much $12x - 7 + 3x + 10 - 5x$ is for $x = 37$ and $x = -43$ What do you think is the easiest way to find out?

11.4 Exercise 4

1. The multiplication sign is often omitted in algebraic expressions: We normally write $4x$ instead of $4 \times x$ and $4(x - 5)$ instead of $4 \times (x - 5)$. Rewrite each of the following in the normal way of writing algebraic expressions:

    1.1 $x \times 4 + x \times y - y \times 3$

    1.2 $7 \times (10 - x) + (5 \times x + 3)10$

2. According to this convention, $x - y + z$ means that you first have to subtract y from x then add z. For example if $x = 10$, $y = 5$, and $z = 3$, $x - y + z$ is $10 - 5 + 3$ and it means $10 - 5 = 5$ then $5 + 3 = 8$. It does not mean $5 + 3 = 8$, then $10 - 8 = 2$.
   Using this information calculate the following:

    2.1 $50 - 20 + 30$

    2.2 $50 + 30 - 20$

    2.3 $50 - 30 + 20$

3. Evaluate each of the following expressions for $x = 10, y = 5$ and $z = 2$

    3.1 $x + y - z$

    3.2 $x - z + y$

    3.3 $10y - 3x + 5z - 4y$

    3.4 $10y - 3x - 5z + 4y + 3x$

4. Do each of the following calculations:

    4.1 Multiply 4 by 3, then add 5 to the answer.

    4.2 Add 4 and 5, then multiply the answer by 3.

    4.3 Multiply 4 by 3, then add the answer to 5.

    4.4 Add 5 and 3, then multiply the answer by 4 .

5. Rewrite the instruction in each question without using words.

    5.1 Multiply 4 by 3 , then add 5 to the answer.

    5.2 Multiply 4 by 3 , then add the answer to 5 .

6. Calculate each of the following:

   6.1 $10 \times 5 + 30$

   6.2 $30 + 10 \times 5$

   6.3 $10 \times 5 - 30$

   6.4 $30 - 10 \times 5$

7. Do each of the following calculations:

   7.1 Add 4 and 5 , then subtract the answer from 20.

   7.2 Subtract 4 from 20 and then add 5 .

   7.3 Add 4 and 5 then multiply the answer by 3.

   7.4 Multiply 3 by 5 and then add the answer to 4.

8. Whenever there are brackets in an expression, the calculations within the brackets should be performed first. Calculate each of the following:

   8.1 $100 + 50 - 30$

   8.2 $100 + (50 - 30)$

   8.3 $100 - 50 + 30$

   8.4 $100 - (50 + 30)$

   8.5 $3(10 - 4) + 2$

   8.6 $10(5 + 7) + 3(18 - 8)$

   8.7 $250 - 10 \times (18 + 2) + 35$

   8.8 $(20 + 20) \times (20 - 10)$

   8.9 $(250 - 10) \times (18 + 2) + 35$

   8.10 $20 + 20 \times (20 - 10)$

   8.11 $200 + (100 \times 2(15 + 5))$

   8.12 $(200 + 100) \times 2 \times 15 + 5$

9. Evaluate each expression for $x = 10, y = 5$ and $z = 2$:

   9.1 $xy + z$

   9.2 $x(y + z)$

   9.3 $x + yz$

   9.4 $xy + xz$

   9.5 $xy - z$

9.6 $x(y - z)$

9.7 $x - yz$

9.8 $xy - yz$

9.9 $x + (y - z)$

9.10 $x - (y - z)$

9.11 $x - (y + z)$

9.12 $x - y - z$

9.13 $x + y - z$

9.14 $x - y + z$

11.5 Exercise 5

1. Calculate each of the following:

   1.1 $5(3 + 4)$

   1.2 $5 \times 3 + 5 \times 4$

   1.3 $6 \times 3 + (4 + 6)$

   1.4 $(6 + 4) + 3 \times 6$

   1.5 $3 \times (4 \times 5)$

   1.6 $(3 \times 4) \times 5$

2. Calculate each of the following:

   2.1 $5(x - y)$ for $x = 10$ and $y = 8$.

   2.2 $5x - 5y$ for $x = 10$ and $y = 8$

   2.3 $5(x - y)$ for $x = 100$ and $y = 30$

   2.4 $5x - 5y$ for $x = 100$ and $y = 30$

   2.5 $5(x - y + z)$ for $x = 10$, $y = 3$ and $z = 2$

   2.6 $5x - 5y + 5z$ for $x = 10$, $y = 3$ and $z = 2$

3. We say "multiplication distributes over addition". Does multiplication also distribute over subtraction? Give examples to support your answer.

4. We say "addition is commutative" and "multiplication is commutative" Is subtraction also commutative? Demonstrate your answer with an example.

5. Calculate $16 + 33 + 14 + 17$ in the easiest possible way.

6. Replace each of the following expressions with a simpler expression that will give the same answer. Do not do any calculations now. In each case, state why your replacement will be easier to do.

   6.1 $17 \times 43 + 17 \times 57$

   6.2 $7 \times 5 \times 8 \times 4 + 12 \times 8 \times 4 \times 7 - 9 \times 4 \times 5 \times 8$

   6.3 $43 \times 17 + 57 \times 17$

   6.4 $43x + 57x$ (for $x = 213$ or any other value)

7. Which properties of operations are utilised in each of the following questions?

   7.1 $17 \times 43 + 17 \times 57 = (53 + 47)(17)$

   7.2 $(8 \times 4 \times 7)(5 + 12) - 9 \times 4 \times 5 \times 8 = (8 \times 4)((5 + 12)(7) - 9(5)$

   7.3 $43 \times 17 + 57 \times 17 = 17(43 + 57)$

   7.4 $43x + 57x = 100x$

11.6 Exercise 6

1. Complete the following table by evaluating the expression for $x = 1$, $x = 10$, $x = 2$ and $x = -2$.

1.1

x	1	10	2	-2
$x(x+3)$				

1.2

x	1	10	2	-2
$x^2 + 3$				

2. Consider the expressions and their respective values for $x = 1, x = 10, x = 2$ and $x = -2$ given in the table:

x	1	10	2	-2
$x(x+3)$	4	130	10	-2
$x^2 + 3$	4	103	7	7

   2.1 Are the two expressions given in the table equivalent?

3. Complete the following table by evaluating the expression for $x = 1, x = 10, x = 2$ and $, x = -2$.

3.1

x	1	10	2	-2
$x^2 + 3x$				

4. Consider the expressions and their respective values for $x = 1, x = 10, x = 2$ and $x = -2$ given in the table:

x	1	10	2	-2
$x(x+3)$	4	130	10	-2
$x^2 + 3x$	4	130	10	-2

4.1 Are the two expressions equivalent?

5. Complete the following table by evaluating the expression for each x values:

5.1

x	10	2	5	1
$5x^2 + 2x^2$				

5.2

x	10	2	5	1
$7x^2$				

5.3

x	10	2	5	1
$13x - 8x$				

5.4

x	10	2	5	1
$5x$				

6. Complete the table by evaluating the expressions for each x values.

6.1

x	10	2	5	1
$3x^2 + 13x + 7 + 2x^2 - 8x - 12$				

6.2

x	10	2	5	1
$3x^2 + 2x^2 + 13x - 8x + 7 - 12$				

6.3

x	10	2	5	1
$5x^2 + 5x - 5$				

6.4

x	10	2	5	1
$5(x^2 + x - 1)$				

7. Consider the following table.

x	10	2	5	1
$3x^2 + 13x + 7 + 2x^2 - 8x - 12$	545	25	145	5
$3x^2 + 2x^2 + 13x - 8x + 7 - 12$	545	25	145	5

7.1 What do you observe about the values in the table?

7.2 How does the one expression in the table differ from the other one?

7.3 Combine like terms in $3x^2 + 2x^2 + 13x - 8x + 7 - 12$ to make a shorter equivalent expression.

7.4 Evaluate $5x^2 + 5x - 5$ for $x = 10, x = 2$ and $x = 5$.

7.5 Is $5x^2 + 5x - 5$ equivalent to $3x^2 + 13x + 7 + 2x^2 - 8x - 12$ Explain how you know whether it is or is not.

8. Simplify each expression:

8.1 $(3x^2 + 5x + 8) + (5x^2 + x + 4)$

8.2 $(7x^2+3x+5)+(2x^2-x-2)$

8.3 $(6x^2-7x-4)+(4x^2+5x+5)$

8.4 $(2x^2-5x-9)-(5x^2-2x-1)$

8.5 $(-2x^2+5x-3)+(-3x^2-9x+5)$

8.6 $(y^2+y+1)+(y^2-y-1)$

9. Complete the table. (Hint: Save yourself some work by simplifying first!).

9.1

x	$2,5$	$3,7$	$6,4$	$12,9$	35	$-4,7$	$-0,04$
$(3x+6,5)+(7x+3,5)$							

9.2

x	$2,5$	$3,7$	$6,4$	$12,9$	35	$-4,7$	$-0,04$
$(13x-6)+(26-12x)$							

10. Simplify:

10.1 $(2r^2+3r-5)+(7r^2-8r-12)$

10.2 $(2r^2+3r-5)-(7r^2-8r-12)$

10.3 $(2x+5xy+3y)-(12x-2xy-5y)$

10.4 $(2x+5xy+3y)+(12x-2xy-5y)$

11. Evaluate the following expression for $x=3, x=-2, x=5$ and $x=-3$

11.1 $2x(x^2-x-1)+5x(2x^2+3x-5)-3x(x^2+2x+1)$

11.2 $(3x^2-5x+7)-(7x^2+3x-5)+(5x^2-2x+8)$

12. Write an equivalent expression without brackets:

12.1 $3x^4-(x^2+2x)$

12.2 $3x^4-(x^2-2x)$

12.3 $3x^4+(x^2-2x)$

12.4 $x-(y+z-t)$

13. Write an equivalent expression without brackets, rearrange so that like terms are grouped together, and then combine the like terms:

13.1 $2y^2+(y^2-3y)$

13.2 $3x^2+(5x+x^2)$

13.3 $6x^2-(x^4+3x^2)$

13.4 $2t^2-(3t^2-5t^23$

13.5 $6x^2+3x-(4x^2+5x)$

13.6 $2r^2 - 5r + 7 + (3r^2 - 7r - 8)$

13.7 $5(x^2 + x) + 2(x^2 + 3x)$

13.8 $2x(x - 3) + 5x(x + 2)$

14. Write an equivalent expression without brackets and simplify the expressions as far as possible.

14.1 $3x^2 + x(x + 3)$

14.2 $5x + x(7 - 2x)$

14.3 $6r^2 - 2r(r - 5)$

14.4 $2a(a + 3) + 5a(a - 2)$

14.5 $6y(y + 1) - 3y(y + 2)$

14.6 $4x(2x - 3) - 3x(x + 2)$

14.7 $2x^2(x - 5) - x(3x^2 - 2)$

14.8 $x(x - 1) + x(2x + 3) - 2x(3x + 1)$

11.7 Exercise 7

1. Answer the following question.

1.1 Calculate 3×38 and 3×62, and add the two answers.

1.2 Add 38 and 62, then multiply the answer by 3.

2. Calculate the following:

2.1 10×56

2.2 $10 \times 16 + 10 \times 40$

3. Let $x = 12$ and $y = 2$.

3.1 Add x and y and multiply the answer by 3.

3.2 Calculate $3 \times x$ and $3 \times y$ and add the two answers.

4. Complete the following table:

4.1

x	12	50	5
y	4	30	10
$5x - 5y$			

4.2

x	12	50	5
y	4	30	10
$5(x - y)$			

4.3

x	12	50	5
y	4	30	10
$5x + 5y$			

4.4

x	12	50	5
y	4	30	10
$5(x + y)$			

5. Let $x = 10$ and $y = 20$.

   5.1 Evaluate $8(x + y)$ by first adding 10 and 20 and then multiplying by 8.

   5.2 Evaluate $8(x + y)$ by doing $8x + 8y$; in other words, first calculate 8×10 and 8×20

6. Evaluate $20(x - y)$ in two different ways, for $x = 5$ and $y = 3$.

7. The distributive property may be described as follows:
   $a(b + c) = ab + ac$ and
   $a(b - c) = ab - ac$
   where a, b and c can be any numbers.
   Use the distributive property in each case to make a different expression that is equivalent to the given expression:

   7.1 $a(b + c)$

   7.2 $a(b + c + d)$

   7.3 $x(x + 1)$

   7.4 $x(x^2 + x + 1)$

   7.5 $x(x^3 + x^2 + x + 1)$

   7.6 $x^2(x^2 - x + 3)$

   7.7 $2x^2(3x^2 + 2)$

   7.8 $3x^3(2x^2 + 4x - 5)$

   7.9 $-2x^4(x^3 - 2x^2 - 4x + 5$

   7.10 $a^2b(a^3 - a^2 + a + 1)$

   7.11 $x^2y^3(3x^2y + xy^2 - y)$

   7.12 $-2x(x^3 - y^3)$

   7.13 $2a^2b(3a^2 + 2a^2b^2 + 4b^2)$

   7.14 $2ab^2(3a^3 - 1)$

8. Expand the parts of each expression and simplify. Then evaluate the expression for $x = 5$.

   8.1 $5(x - 2) + 3(x + 4)$

8.2 $x(x+4)-4(x+4)$

8.3 $x(x-4)+4(x-4)$

8.4 $x(x^2+3x+9)-3(x^2+3x+9)$

8.5 $x(x^2-3x+9)+3(x^2-3x+9)$

8.6 $x^2(x^2-3x+4)-x(x^3+4x^2+2x+3)$

9. Write in expanded form:

9.1 $x(x^2+2xy+y^2)+y(x^2+2xy+y^2)$

9.2 $x^2y(x^2-2xy+y^2)-xy^2(2x^2-3xy-y^2)$

9.3 $ab^2c(b^2c^2-ac)+b^2c^4(a^2+abc^2)$

9.4 $p^2q(pq^2+p+q)+pq(p-q^2)$

11.8 Exercise 8

1. Evaluate each of the following expressions for $x=2, x=5$ and $x=10$

1.1 $(3x)^2$

1.2 $9x^2$

1.3 $(2x)^2$

1.4 $4x^2$

1.5 $(2x)^3$

1.6 $8x^3$

1.7 $(2x+3x)^2$

1.8 $(10x-7x)^2$

2. In each case, write an equivalent monomial without brackets:

2.1 $(5x)^2$

2.2 $(5x)^3$

2.3 $(20x)^2$

2.4 $(10x)^3$

2.5 $(2x+7x)^2$

2.6 $(20x-13x)^3$

3. Write down the square root of each of the following expressions:

3.1 $\sqrt{(7x)^2}$

3.2 $\sqrt{(9x^2}$

3.3 $\sqrt{(20x)^2}$

3.4 $\sqrt{100x^2}$

3.5 $\sqrt{(20x-15x)^2}$

3.6 $\sqrt{16x^2+9x^2}$

3.7 $\sqrt{(21x-16x)^2}$

3.8 $\sqrt{(5x)^2}$

4. Write down the cube root of each of the following expressions:

4.1 $\sqrt[3]{(7x)^3}$

4.2 $\sqrt[3]{27x^3}$

4.3 $\sqrt[3]{(20x)^3}$

4.4 $\sqrt[3]{1000x^3}$

4.5 $\sqrt[3]{(20x-15x)^3}$

4.6 $\sqrt[3]{125x^3}$

11.9 Exercise 9

1. Complete the following table:

1.1

x	20	10	5	-5	-10	-20
$(100x-5x^2)\div 5x$						

1.2

x	20	10	5	-5	-10	-20
$20-x$	0					

2. R 240 prize money must be shared equally between 20 netball players.

2.1 How much should each one get?

2.2 Mpho decided to do the calculations below. Do not do Mpho's calculations, but think about this: Will Mpho get that each netball player gets R 12 ?
$(140\div 20)+(100\div 20)$

2.3 Gert decided to do the calculations below. Without doing the calculations, say whether or not Gert will get that each netball player gets R 12?
$(240\div 12)+(240\div 8)$

3. Do the necessary calculations to find out whether the following statement is true or false:

3.1 $(140+100)\div 20=(140\div 20)+(100\div 20)$

3.2 $(240 \div (12 + 8) = (240 \div 12) + (240 \div 8)$

3.3 $(300 - 60) \div 20 = (300 \div 20) - (60 \div 20)$

4. Evaluate each expression for $x = 2$ and $x = 10$:

4.1 $(10x^2 + 5x) \div 5$

4.2 $(10x^2 \div 5) + (5x \div 5)$

4.3 $2x^2 + x$

4.4 $(10x^2 + 5x) \div 5x$

4.5 $(10x^2 \div 5x) + (5x \div 5x)$

4.6 $2x + 1$

5. Do not do any calculations. Which of the following expressions do you think will have the same value as $(10x^2 + 20x - 15) \div 5$ for $x = 10$ as well as $x = 2$?
$2x^2 + 20x - 15$; $10x^2 + 20x - 3$; $2x^2 + 4x - 3$

6. Simplify the following expression: $(10x^2 + 20x - 15) \div 5$

7. Simplify:

7.1 $(2x + 2y) \div 2$

7.2 $(4x + 8y) \div 4$

7.3 $(20xy + 16x) \div 4x$

7.4 $(42x - 6) \div 6$

7.5 $(28x^4 - 7x^3 + x^2) \div x^2$

7.6 $(24x^2 + 16x) \div 8x$

7.7 $(30x^2 - 24x) \div 3x$

8. Simplify:

8.1 $(9x^2 + xy) \div xy$

8.2 $(48a - 30ab + 16ab^2) \div 2a$

8.3 $(3a^3 + a^2) \div a^2$

8.4 $(13a - 17ab) \div a$

8.5 $(3a^2 + 5a^3) \div a$

8.6 $(39a^2b + 13ab + ab^2$

9. Find a simpler equivalent expression for each expression (clearly, these expressions do not make sense if $x = 0$):

9.1 $\dfrac{16x^2 - 12x}{4x}$

9.2 $\dfrac{16x^3 - 12x}{4x}$

9.3 $\dfrac{16x^3 - 12x^2}{4x}$

9.4 $\dfrac{16x^3 - 12x^2}{4x^2}$

9.5 $\dfrac{16x^3 - 12x^2}{2x}$

9.6 $\dfrac{16x^3 - 12x}{8x}$

10. In each case check if the statement is true for $x = 10, x = 100, x = 5, x = 1$ and $x = -2$.

10.1 $\dfrac{x^2}{x} = x$

10.2 $\dfrac{x^3}{x} = x^2$

10.3 $\dfrac{x^3}{x^2} = x$

10.4 $\dfrac{5x^3}{x} = 5x^2$

10.5 $\dfrac{5x^3}{x} = 5^3$

10.6 $\dfrac{5x}{x^2} = \dfrac{5}{x}$

11. Complete the following table:

11.1

x	$1,5$	$2,8$	$-3,1$	$0,72$
$\dfrac{3x+12}{3}$				

11.2

x	$1,5$	$2,8$	$-3,1$	$0,72$
$\dfrac{18x^2+6}{6}$				

11.3

x	$1,5$	$2,8$	$-3,1$	$0,72$
$\dfrac{5x^2+7x}{x}$				

12. Simplify each expression to the equivalent form requiring the fewest operations:

12.1 $\dfrac{3a + a^2}{a}$

12.2 $\dfrac{x^3 + 2x^2 - x}{x}$

12.3 $\dfrac{2a + 12ab}{2a}$

12.4 $\dfrac{12x^2 + 10x}{2x}$

12.5 $\dfrac{21ab - 14a^2}{7a}$

12.6 $\dfrac{15a^2b + 30ab^2}{5ab}$

12.7 $\dfrac{7x^3 + 21x^2}{7x^2}$

12.8 $\dfrac{3x^2 + 9x}{3x}$

13. Solve the equation:

13.1 $\dfrac{3x^2 + 15x}{3x} = 20$

13.2 $\dfrac{30x - 18x^2}{6x} = 2$

14. Complete the following table:

14.1

x	$1,1$	$1,2$	$1,3$	$1,4$	$1,5$
$\dfrac{x^3 + 2x^2 - x}{x}$					

14.2

x	$1,1$	$1,2$	$1,3$	$1,4$	$1,5$
$\dfrac{7x^3 + 21x^2}{7x^2}$					

15. Simplify the following expressions:

15.1 $\dfrac{3x(5x + 4) + 6x(5x + 3)}{5x}$

15.2 $\dfrac{14x^2 - 28x}{7x} + \dfrac{24x - 18x^2}{3x}$

16. Explain why the equations below are true:

16.1 $\dfrac{100x - 5x^2}{5x} = 20 - x$ for all values of x, except $x = 0$.

16.2 $\dfrac{15x^2 - 10x}{5x}$ is equivalent to $3x - 2$, excluding $x = 0$.

11.10 Exercise 10

1. Describe how can you check if $(x + 2)(x + 3)$ is actually equivalent to $x^2 + 5x + 6$.

2. Complete the following table:

2.1

x	-1	0	0	1	2
y	0	1	2	-1	0
$(x - y)(x + 3y)$					

2.2

x	-1	0	0	1	2
y	0	1	2	-1	0
$x^2 + 2xy - 3y^2$					

3. Consider the following table.

x	-1	0	0	1	2
y	0	1	2	-1	0
$(x-y)(x+3y)$	1	-3	-12	-4	4
$x^2+2xy-3y^2$	1	-3	-12	-4	4

3.1 Are $(x-y)(x+3y)$ and $x^2+2xy-3y^2$ are equivalent?

4. Expand each of these expression:

4.1 $(x+3)(x+4)$

4.2 $(x+3)(4-x)$

4.3 $(x+3)(x-5)$

4.4 $(2x^2+1)(3x-4)$

4.5 $(x+y)(x+2y)$

4.6 $(a-b)(2a+3b)$

4.7 $(k^2+m)(k^2+2m)$

4.8 $(2x+3)(2x-3)$

4.9 $(5x+2)(5x-2)$

4.10 $(ax-by)(ax+by)$

5. Expand each of these expressions:

5.1 $(a+b)(a+b)$

5.2 $(a-b)(a-b)$

5.3 $(x+y)(x+y)$

5.4 $(x-y)(x-y)$

5.5 $(2a+3b)(2a+3b)$

5.6 $(2a-3b)(2a-3b)$

5.7 $(5x+2y)(5x+2y)$

5.8 $(5x-2y)(5x-2y)$

5.9 $(ax+b)(ax+b)$

5.10 $(ax-b)(ax-b)$

6. Expand the following expressions:

6.1 $(m+n)(m+n)$

6.2 $(m-n)(m-n)$

6.3 $(3x+2y)(3x+2y)$

6.4 $(3x-2y)(3x-2y)$

7. Expand:

7.1 $(ax+b)^2$

7.2 $(ax-b)^2$

7.3 $(2s+5)^2$

7.4 $(2s-5)^2$

7.5 $(ax+by)^2$

7.6 $(ax-by)^2$

7.7 $(2s+5r)^2$

7.8 $(2s-5r)^2$

8. Expand and simplify:

8.1 $(4x+3)(6x+4)+(3x+2)(8x+5)$

8.2 $(4x+3)(6x+4)-(3x+2)(8x+5)$

11.11 Exercise 11

1. Complete the following table:

1.1

x	13	-13	$2,5$	10
$(2x+3)(3x-5)$				

1.2

x	13	-13	$2,5$	10
$10x^2+5x-7+3x^2-4x-3$				

1.3

x	13	-13	$2,5$	10
$3(10x^2-5x+2)-5x(6x-4)$				

1.4

x	13	-13	$2,5$	10
$3(10x^2-5x+2)-5x(6x-4)$				

1.5

x	13	-13	$2,5$	10
$13x^2+x-10$				

1.6

x	13	-13	$2,5$	10
$6x^2-x-15$				

1.7

x	13	-13	$2,5$	10
$5x+6$				

2. Which expressions given in the table are equivalent?

x	13	-13	$2,5$	10
$(2x+3)(3x-5)$	986	1012	20	575
$10x^2+5x-7+3x^2-4x-3$	2200	2174	$73,75$	1300
$3(10x^2-5x+2)-5x(6x-4)$	71	-59	$18,5$	56
$13x^2+x-10$	2200	2174	$73,75$	1300
$6x^2-x-15$	986	1012	20	575
$5x+6$	71	-59	$18,5$	56

3. Complete the table:

3.1

x	1	2	3	4
$(2x+3)(5x-3)+(10x+9)(1-x)$				

3.2

x	1	2	3	4
$\frac{9x^2+30x}{3x}$				

3.3

x	1	2	3	4
$3x(10x-5)-5x(6x-4)$				

3.4

x	1	2	3	4
$5x(4x+3)-2x(7+13x)+2x(3x+2)$				

4. Describe any patterns that you observe in the following table.

x	1	2	3	4
$(2x+3)(5x-3)+(10x+9)(1-x)$	10	20	30	40
$\frac{9x^2+30x}{3x}$	13	16	19	22
$3x(10x-5)-5x(6x-4)$	5	10	15	20
$5x(4x+3)-2x(7+13x)+2x(3x+2)$	5	10	15	20

5. Complete the following table:

5.1

x	$1,5$	$2,5$	$3,5$	$4,5$
$(2x+3)(5x-3)+(10x+9)(1-x)$				

5.2

x	$1,5$	$2,5$	$3,5$	$4,5$
$\frac{9x^2+30x}{3x}$				

5.3

x	$1,5$	$2,5$	$3,5$	$4,5$
$3x(10x-5)-5x(6x-4)$				

5.4

x	$1,5$	$2,5$	$3,5$	$4,5$
$5x(4x+3)-2x(7+13x)+2x(3x+2)$				

11.12 Exercise 12

1. Calculate the value of each of the following expressions for $x = 12$:

   1. 1 $\frac{(x+2)(x+5)}{x+2}$

   1. 2 $\frac{(x-3)(x-4)}{x-4}$

   1. 3 $\frac{x(2x+1)}{2x+1}$

   1. 4 $\frac{(x+5)(x-5)}{x-5}$

2. Check if the following statements are identities by expanding the expressions on the right.

   2. 1 $x^2 - 9 = (x+3)(x-3)$

   2. 2 $x^2 + x - 6 = (x+3)(x-2)$

   2. 3 $x^2 + 4x + 3 = (x+3)(x+1)$

   2. 4 $x^2 + 3x = x(x+3)$

3. Write down the factors of each of the following expressions:

   3. 1 $x^2 + x - 6$

   3. 2 $x^2 + 3x$

   3. 3 $x^2 + 4x + 3$

   3. 4 $x^2 - 9$

4. Simplify the following quotients (algebraic fractions):

   4. 1 $\frac{x^2-9}{x+3}$

   4. 2 $\frac{x^2+x-6}{x+3}$

   4. 3 $\frac{x^2+x-6}{x-2}$

   4. 4 $\frac{x^2+4x+3}{(x+3)(x+1)}$

11.13 Exercise 13

1. Answer the following:

   1. 1 Is 5 a factor of 20.

   1. 2 Is 5 a factor of 30

   1. 3 Is 5 a factor of $30 + 20$?

   1. 4 Is 5 a factor of $30 - 20$

2. Answer the following:

2. 1 Is a a factor of ab?

2. 2 Is a a factor of ac?

2. 3 Is a a factor of $ab + ac$?

2. 4 Find another factor of $ab + ac$.

2. 5 Now try and simplify: $\frac{ab+ac}{a}$.

3. Complete the table:

(a) For each expression, find:	$3x + 6y$	$4a^3 + 2a$	$5x - 2x^2$	$ax^2 - bx^3$	$12a^2b + 18ab^2$
the factors of the first term	$3; x$				
the factors of the second term	$2; 3; y$				
the greatest common factor of the two terms	3				
(b) Write the expression in factor form	$3(x + 2y)$				

4. Study the example and then factorise the expressions that follow.

$$\begin{aligned}(a-b)x + (b-a)y &= (a-b)x - (a-b)y \\ &= (a-b)(x-y)\end{aligned}$$

4. 1 $(a - b)x + a - b$

4. 2 $(a - b)x - a + b$

4. 3 $(a + b)^2 - (a + b)$

4. 4 $(a + b)x - a - b$

4. 5 $3x(2x - 3) - (3 - 2x)$

4. 6 $(y^2 - 4y) + (3y - 12)$

11.14 Exercise 14

1. By copying and completing the tables below you will learn something that will help you to find the factors of expressions of the form $x^2 + (b + c)x + bc$, for example: $x^2 + 17x + 30$.

b	1	2	3	5	-1	-2	-3	-5
c	30	15	10	6	-30	-15	-10	-6
$b + c$								
bc								

b	-1	-2	-3	-5	1	2	3	5
c	30	15	10	6	-30	-15	-10	-6
$b+c$								
bc								

2. For each case below, find two numbers x and y so that their product xy is 30 and their sum $x+y$ is the given number:

2. 1 $xy = 30$ and $x + y = 13$

2. 2 $xy = 30$ and $x + y = -17$

2. 3 $xy = 30$ and $x + y = -11$

2. 4 $xy = 30$ and $x + y = 11$

3. Find x and y in each case:

3. 1 $xy = -30$ and $x + y = -13$

3. 2 $xy = 30$ and $x + y = 13$

3. 3 $xy = -30$ and $x + y = 13$

3. 4 $xy = -30$ and $x + y = -1$

3. 5 $xy = -30$ and $x + y = 1$

4. Find x and y in each case:

4. 1 $xy = 36$ and $x + y = 15$

4. 2 $xy = 40$ and $x + y = 22$

4. 3 $xy = 36$ and $x + y = 20$

4. 4 $xy = -40$ and $x + y = 18$

4. 5 $xy = 36$ and $x + y = -20$

4. 6 $xy = -40$ and $x + y = -18$

5. Evaluate each expression for $x = 2$.
Also expand each expression

5. 1 $(x + 5)(x - 2)$

5. 2 $(x + 5)(x + 2)$

5. 3 $(x - 5)(x - 2)$

5. 4 $(x - 5)(x + 2)$

6. Expand each product:

6. 1 $(x+3)(x+8)$

6. 2 $(x+2)(x+12)$

6. 3 $(x+4)(x+6)$

6. 4 $(x-1)(x+24)$

6. 5 $(x+3)(x-8)$

6. 6 $(x+2)(x-12)$

6. 7 $(x+4)(x-6)$

6. 8 $(x+1)(x-24)$

11.15 Exercise 15

1. Write the following out and fill in the missing parts of the factors (the dotted lines) in each case:

   1. 1 $(x+3)(x+...)=x^2+9x+18$

   1. 2 $(x+2)(x+...)=x^2+11x+18$

   1. 3 $x+3)(x-...)=x^2+9x-18$

   1. 4 $(...+...)(x+2)=x^2+5x+6$

2. Expand each product:

   2.1 $(x+p)(x+q)$

   2.2 $(x+p)(x-q)$

   2.3 $(x-p)(x-q)$

   2.4 $(x-p)(x+q)$

3. Try to factorise the following trinomials:

4. $x^2+8x+12$

5. $x^2-8x+12$

11.16 Exercise 16

1. Factorise the following trinomials. (Remember to check your answer by expanding the factors to test if you do get the original expression.)

   1. 1 $a^2+9a+14$

   1. 2 $x^2+3x-18$

   1. 3 $x^2-18x+17$

1.4 $y^2 + 17y + 30$

1.5 $y^2 - 13y - 30$

1.6 $y^2 + 7y - 30$

1.7 $x^2 + 2x - 15$

1.8 $m^2 + 4m - 21$

1.9 $x^2 - 6x + 9$

1.10 $b^2 + 15b + 56$

1.11 $a^2 - 2a - 63$

1.12 $a^2 - ab - 30b^2$

1.13 $x^2 - 5xy - 24y^2$

1.14 $x^2 - 13x + 40$

2. Study the example and then factorise the expressions.

$ac + bc + bd + ad = (ac + bc) + (bd + ad)$	(Order and group terms with common factors)
$= c(a + b) + d(b + a)$	(Take out the common factor)
$= (a + b)(c + d)$	(Write expression as a product)

2.1 $px + py + qx + qy$

2.2 $9x^2(x - 3) + (x - 3)$

2.3 $4(a + b) + 3p(a + b)$

2.4 $a^3(a + 1) + 3(a + 1)$

2.5 $x(y + 1) + (y + 1)$

2.6 $a(c - d) - b(c - d)$

3. Factorise:

3.1 $x^2 + 7x + 12$

3.2 $x^2 - 7x + 12$

11.17 Exercise 17

1.1.1 Copy and complete the following table and see if you can notice a pattern (rule) whereby you can predict the answers to the first column's calculations without squaring it:

a) $3^2 - 2^2$	$3+2$	$3-2$	$(3+2)(3-2)$
b) $4^2 - 3^2$	$4+3$	$4-3$	$(4+3)(4-3)$
c) $6^2 - 4^2$	$6+4$	$6-4$	$(6+4)(6-4)$
d) $9^2 - 3^2$	$9+3$	$9-3$	$(9+3)(9-3)$

Do you notice a pattern (rule) whereby you can predict the answers to such calculations? Now predict the answers to each of the following without squaring. Check your answers where necessary. Does the rule that you discovered in question $1.1.1$ also hold for the following cases?

1.1. 2 $17^2 - 13^2$

1.1. 3 $54^2 - 46^2$

1.1. 4 $28^2 - 22^2$

1.2. 1 Formulate your rule in symbols: $a^2 - b^2 = (...)(...)$

11.18 Exercise 18

1. Use the skills you learnt in the previous exercises to factorise the following:

   1. 1 $4a^2 - b^2$

   1. 2 $m^2 - 9n^2$

   1. 3 $25x^2 - 36y^2$

   1. 4 $121x^2 - 144y^2$

   1. 5 $16p^2 - 49q^2$

   1. 6 $64a^2 - 25b^2c^2$

   1. 7 $x^2 - 4$

   1. 8 $16x^2 - 36y^2$

2. Factorise:

   2. 1 $x^4 - 1$

   2. 2 $16a^4 - 81$

   2. 3 $1 - a^2b^2c^2$

   2. 4 $25x^{10} - 49y^8$

   2. 5 $2x^2 - 18$

2.6 $200 - 2b^2$

2.7 $3xy^2 - 48xa^2$

2.8 $5a^4 - 20b^2$

11.19 Exercise 19

1. Solve the following problems:

   1.1 Evaluate $\frac{x^2+5x+6}{x+2}$ if $x = 23$

   1.2 Solve $\frac{x^2+5x+6}{x+2} = 19$

2. Determine the value of each of the expressions:

   2.1 $\frac{x^2-9}{x+3}$

   2.2 $\frac{x^2+x-6}{x+3}$

11.20 Exercise 20

1. Copy and complete the following table by evaluating the value of the expression $\frac{x+2}{x-2}$ for the x -values given in the top row.

x	-2	0	2	4
$\frac{x+2}{x-2}$				

2. Refer to the table you have just completed.

   2.1 If $x = 2$, then $\frac{x+2}{x-2}$ will have the value $\frac{4}{0}$. What is the value of $\frac{4}{0}$?

   2.2 One way to determine the value of $\frac{4}{0}$, is to set it as $\frac{4}{0} = a$. Then $4 = 0 \times a$. Which values of a will make this statement true?

11.21 Exercise 21

1. Are the following statements true? If not, correct the statement.

   1.1 $\frac{x}{x} = 1$ for all values of x

   1.2 $\frac{x^3}{x^2} = x$ for all values of x

   1.3 $\frac{x-3}{x-3} = 1$ for all values of x

   1.4 $\frac{x^2+x}{x(x+1)} = 1$ for all values of x

2. For which values of the variables will each expression be undefined?

2. 1 $\frac{7(y+5)}{y+2}$

2. 2 $\frac{3x+2}{x+4}$

2. 3 $\frac{2x+1}{x^2-1}$

2. 4 $\frac{2x^2-1}{(x-2)(x+3)}$

11.22 Exercise 22

1. Simplify each of the following algebraic fractions by factorising the numerator and then using the property $\frac{ax}{a} = x$ if $a \neq 0$.

   1. 1 $\frac{3xy+y^2}{3x+y}$

   1. 2 $\frac{a^2b+ab^2}{a+b}$

   1. 3 $\frac{3x^2y-6x^2y^2}{3xy}$

   1. 4 $\frac{10x^4+15x^3}{5x^2}$

2. Simplify each of the following algebraic fractions by factorising the numerator and then using the property $\frac{ax}{a} = x$ if $a \neq 0$. (See if you can factorise the trinomials.)

   2. 1 $\frac{x^2+5x+6}{x+2}$

   2. 2 $\frac{x^2+2x-8}{x-2}$

   2. 3 $\frac{x^2-5x-50}{x+5}$

   2. 4 $\frac{x^2-16x+15}{x-15}$

3. Simplify each of the following algebraic fractions by factorising the numerator and then using the property $\frac{ax}{a} = x$ if $a \neq 0$:

   3. 1 $\frac{x^2-4}{x-2}$

   3. 2 $\frac{4x^2-1}{2x+1}$

11.23 Exercise 23

1. Factorise the following expressions completely:

   1. 1 $4a + 6b$

   1. 2 $x^2 + 8x + 7$

   1. 3 $c^2 - 9$

   1. 4 $y^2 - 8y + 15$

   1. 5 $-3ab + b$

   1. 6 $-3a(b-1) + (b-1)$

1. 7 $dfg^2 + d^2g - df^2g$

1. 8 $x^2 + 6x + 8$

1. 9 $a^2 + 5a + 6$

1. 10 $x^2 - 8x - 20$

1. 11 $x^5y^3 - x^3y^5$

1. 12 $x^3y - xy^3$

1. 13 $4 - 4y + y^2$

1. 14 $3a^2 + 18a - 21$

1. 15 $6a^2 - 54$

1. 16 $-a^2 - 11a - 30$

1. 17 $2a^2 + 10a - 72$

1. 18 $5x^3 - 15x^2 - 200x$

1. 19 $(x + 2)^2 - y^2$

1. 20 $(x + y)^2 - a^2$

1. 21 $(a^2 - 2a + 1) - b^2$

1. 22 $1 - (a^2 - 2ab + b^2)$

1. 23 $(a - b)x + (b - a)y$

1. 24 $a(2x - y) + (y - 2x)$

1. 25 $2x^2y^{10} - 8x^{10}y^2$

1. 26 $(a + b)^3 - 4(a + b)$

1. 27 $(a + b)^2 - a - b$

1. 28 $(x + y)(a - b) + (-x - y)(b - a)$

2. Simplify each of the following algebraic fractions as far as possible:

1. 1 $\frac{16-9x^2}{4+3x}$

1. 2 $\frac{25x^2-36}{5x^2+6x}$

1. 3 $\frac{x^3+x^2-30x}{x+6}$

1. 4 $\frac{2x^2+5x+3}{2x+3}$

1. 5 $\frac{ab+bc}{abc}$

1. 6 $\frac{pa+pb}{a+b}$

12 ANSWERS FOR EXERCISES

12.1 Exercise 1

1.1	Add 5 to a number and then multiply the answer by 3	$\boxed{+5} \longrightarrow \boxed{\times 3} \longrightarrow$	$3(x+5)$ or $(x+5)\times 3$
1.2	Subtract 3 from a number and multiply the answer by 5	$\boxed{-3} \longrightarrow \boxed{\times 5} \longrightarrow$	$(x-3)\times 5$ or $5(x-3)$
1.3	Multiply a number by 2, add 3 and multiply the answer by 3	$\boxed{\times 2} \longrightarrow \boxed{+3} \longrightarrow \boxed{\times 3}$	$3(2x+3)$

2.1 $3(10x-5)$

2.2 $3(10(x-5))$

3.1 150

3.2 1950

3.3 90

3.4 250

3.5 90

3.6 250

12.2 Exercise 2

1.1 1

1.2 1

1.3 Yes. It will be $-3x^0$, which is simply -3

2.1

Expression	Type of expression	Symbol	Constant	Coefficient of ...
$6s^3+s^2+5$	Trinomial	s	5	s^2 is: 1

2.2

Expression	Type of expression	Symbol	Constant	Coefficient of ...
$\frac{k}{3}+12$	Binomial	k	12	the first term is: $\frac{1}{3}$

2.3

Expression	Type of expression	Symbol	Constant	Coefficient of ...
$4p^10$		p	0	p^10 is: 4

12.3 Exercise 3

1.1

x	-2	-1	0	1	2
$3x+2$	-4	-1	2	5	8

1.2

x	-2	-1	0	1	2
$2x-3$	-7	-5	-3	-1	1

1.3

x	-2	-1	0	1	2
$3x+2+2x-3$	-11	-6	-1	4	9

1.4

x	-2	-1	0	1	2
$2x-3+3x+2$	-11	-6	-1	4	9

1.5

x	-2	-1	0	1	2
$5x-1$	-11	-6	-1	4	9

1.6

x	-2	-1	0	1	2
$(3x+2)(2x-3)$	28	5	-6	-5	8

1.7

x	-2	-1	0	1	2
$3x(2x-3)+2(2x-3)$	28	5	-6	-5	8

1.8

x	-2	-1	0	1	2
$6x^2-5x-6$	28	5	-6	-5	8

1.9

x	-2	-1	0	1	2
$\frac{(3x+2)(2x-3)}{3x+2}$	-7	-5	-3	-1	1

1.10

x	-2	-1	0	1	2
$\frac{6x^2-5x-6}{3x+2}$	-7	-5	-3	-1	1

2. List 1: $2x-3$ and $\frac{(3x+2)(2x-3)}{3x+2}$ and $\frac{6x^2-5x-6}{3x+2}$
   List 2: $3x+2+2x-3$ and $2x-3+3x+2$ and $5x-1$
   List 3: $(3x+2)(2x-3)$ and $3x(2x-3)+2(2x-3)$ and $6x^2-5x-6$

3.1

x	2	3	5	10	-5	-10
$12x - 7 + 3x + 10 - 5x$	23	33	53	103	-47	-97

3.2

x	2	3	5	10	-5	-10
$10x + 3$	23	33	53	103	-47	-97

4. Yes, they have the same numerical value for any given value of x. If you simplify the longer expression by combining the like terms, it gives you the shorter expression.

5. First simplify the expression by combining the like terms, then substitute for x.

12.4 Exercise 4

1.1 $4x + xy - 3y$

1.2 $70 - 7x + 50x + 30$

2.1 60

2.2 60

2.3 40

3.1 13

3.2 13

3.3 10

3.4 60

4.1 17

4.2 27

4.3 17

5.1 $4 \times 3 + 5$

5.2 $5 + 4 \times 3$

6.1 80

6.2 80

6.3 20

6.4 -20

7.1 -11

7.2 21

7.3 27

7.4 19

8.1 120

8.2 120

8.3 80

8.4 20

8.5 20

8.6 150

8.7 85

8.8 400

8.9 4835

8.10 220

8.11 4200

8.12 9005

9.1 52

9.2 70

9.3 20

9.4 70

9.5 48

9.6 30

9.7 0

9.8 40

9.9 13

9.10 7

9.11 3

9.12 3

9.13 13

9.14 7

12.5 Exercise 5

1.1 35

1.2 35

1.3 28

1.4 28

1.5 60

1.6 60

2.1 10

2.2 10

2.3 350

2.4 350

2.5 45

2.6 45

3. Yes. Since $5(3-2)=5(3)-5(2)$ and $2(6-4)=2(6)-2(4)$

4. No.$10-3\neq 3-10$

5. $16+14=30$ and $33+17=50$. Then, $16+14+17+33=50+30=80$

6.1 $17(43+57)$

6.2 $(8\times 4)((5+12)(7)-9(5))$

6.3 $17(43+57)$

6.4 $100x$

7.1 distributive(reversed)

7.2 associative, distributive (reversed)

7.3 distributive (reversed)

7.4 distributive (reversed)

12.6 Exercise 6

1.1

x	1	10	2	-2
$x(x+3)$	4	130	10	-2

1.2

x	1	10	2	-2
x^2+3	4	103	7	7

2.1 No. The two expressions are not equivalent. The table shows different answers for some of the input numbers.

3.1

x	1	10	2	-2
x^2+3x	4	130	10	-2

4.1 Yes. The two expressions are equivalent. The table shows the same answers for each input number.

5.1

x	10	2	5	1
$5x^2+2x^2$	700	28	175	7

5.2

x	10	2	5	1
$7x^2$	700	28	175	7

5.3

x	10	2	5	1
$13x-8x$	50	10	25	5

5.4

x	10	2	5	1
$5x$	50	10	25	5

6.1

x	10	2	5	1
$3x^2+13x+7+2x^2+8x-12$	545	25	145	5

6.2

x	10	2	5	1
$3x^2+2x^2+13x-8x+7-12$	545	25	145	5

6.3

x	10	2	5	1
$5x^2+5x-5$	545	25	145	5

6.4

x	10	2	5	1
$5(x^2+x-1)$	545	25	145	5

7.1 The values in the table are all the same.

7.2 Both have the same terms; they are just arranged differently.

7.3 $5x^2+5x-5$

7.4 $545, 25$ and 145

7.5 Yes. The one expression is just an accumulation of all the like terms in the other expression.

8.1 $8x^2 + 6x + 12$

8.2 $9x^2 + 2x + 3$

8.3 $10x^2 - 2x + 1$

8.4 $-3x^2 - 3x - 8$

8.5 $-5x^2 - 4x + 2$

8.6 $2y^2$

9.1

x	$2,5$	$3,7$	$6,4$	$12,9$	35	$-4,7$	$-0,04$
$(3x + 6,5) + (7x + 3,5)$	35	47	74	139	360	-37	$9,6$

9.2

x	$2,5$	$3,7$	$6,4$	$12,9$	35	$-4,7$	$-0,04$
$(13x - 6) + (26 - 12x)$	$22,5$	$23,7$	$26,4$	$32,9$	55	$15,3$	$19,96$

10.1 $9r^2 - 5r - 17$

10.2 $-5r^2 + 11r + 7$

10.3 $-10x + 7xy + 8y$

10.4 $14x + 3xy - 2y$

11.1 $x = 3 : 216$
$x = -2 : 16$
$x = 5 : 1150$
$x = -3 : -90$

11.2 $x = 3 : -1$
$x = -2 : 44$
$x = 5 : -5$
$x = -3 : 59$

12.1 $3x^4 - x^2 - 2x$

12.2 $3x^4 - x^2 + 2x$

12.3 $3x^4 + x^2 - 2x$

12.4 $x - y - z + t$

13.1 $3y^2 - 3y$

13.2 $4x^2 + 5x$

13.3 $3x^2 - x^4$

13.4 $-t^2 + 5t^3$

13.5 $2x^2 - 2x$

13.6 $5r^2 - 12r - 1$

13.7 $7x^2 + 11x$

13.8 $7x^2 + 4x$

14.1 $4x^2 + 3x$

14.2 $12x - 2x^2$

14.3 $4r^2 + 10r$

14.4 $7a^2 - 4a$

14.5 $3y^2$

14.6 $5x^2 - 18x$

14.7 $-x^3 - 10x^2 + 2x$

14.8 $-3x^2$

12.7 Exercise 7

1.1 300

1.2 300

2.1 560

2.2 560

3.1 42

3.2 42

4.1

x	12	50	5
y	4	30	10
$5x - 5y$	40	100	-25

4.2

x	12	50	5
y	4	30	10
$5(x-y)$	40	100	-25

4.3

x	12	50	5
y	4	30	10
$5x+5y$	80	400	75

4.4

x	12	50	5
y	4	30	10
$5(x+y)$	80	400	75

5.1 240

5.2 240

6. 40

7.1 $ab+ac$

7.2 $ab+ac+ad$

7.3 x^2+x

7.4 x^3+x^2+x

7.5 $x^4+x^3+x^2+x$

7.6 $x^4-x^3+3x^2$

7.7 $6x^4+4x^2$

7.8 $6x^5+12x^4-15x^3$

7.9 $-2x^7+4x^6+8x^5-10x^4$

7.10 $a^5b-a^4b+a^3b+a^2b$

7.11 $3x^4y^4+x^3y^5-x^2y^4$

7.12 $-2x^4+2xy^3$

7.13 $6a^4b+4a^4b^3+8a^2b^3$

7.14 $6a^4b^2-2ab^2$

8.1 $8x+2; 8(5)+2=42$

8.2 $x^2-16; 5^2-16=9$

8.3 $x^2 - 16; 5^2 - 16 = 9$

8.4 $x^3 - 27; 5^3 - 27 = 98$

8.5 $x^3 + 27; 5^3 + 27 = 152$

8.6 $-7x^3 + 2x^2 - 3x; -27(5)^3 + 2(5)^2 - 3(5) = -840$

9.1 $x^3 + 3x^2y + 3xy^2 + y^3$

9.2 $x^4y - 4x^3y^2 + 4x^2y^3 + xy^4$

9.3 $ab^4c^3 - a^2b^2c^2 + ab^3c^6$

9.4 $p^3q^3 + p^3q + p^2q^2 + p^2q - pq^3$

12.8 Exercise 8

1.1 $9x^2 : 36; 225; 900$

1.2 $9x^2 : 36; 225; 900$

1.3 $4x^2 : 16; 100; 400$

1.4 $4x^2 : 16; 100; 400$

1.5 $8x^3 : 64; 1000; 8000$

1.6 $8x^3 : 64; 1000; 8000$

1.7 $25x^2 : 100; 625; 2500$

1.8 $9x^2 : 36; 225; 900$

2.1 $25x^2$

2.2 $125x^3$

2.3 $400x^2$

2.4 $1000x^3$

2.5 $81x^2$

2.6 $343x^3$

3.1 $7x$

3.2 $3x$

3.3 $20x$

3.4 $10x$

3.5 $5x$

3.6 $5x$

3.7 $5x$

3.8 $5x$

4.1 $7x$

4.2 $27x$

4.3 $20x$

4.4 $10x$

4.5 $5x$

4.6 $5x$

12.9 Exercise 9

1.1

x	20	10	5	-5	-10	-20
$(100x - 5x^2) \div 5x$	0	10	15	25	30	40

1.2

x	20	10	5	-5	-10	-20
$20 - x$	0	10	15	25	30	40

2.1 R 12

2.2 Yes

2.3 No

3.1 True

3.2 False

3.3 True

4.1 $x = 2 : 10$
$x = 10 : 210$

4.2 $x = 2 : 10$
$x = 10 : 210$

4.3 $x = 2 : 10$

$x = 10 : 210$

4.4 $x = 2 : 5$

$x = 10 : 21$

4.5 $x = 2 : 5$

$x = 10 : 21$

4.6 $x = 2 : 5$

$x = 10 : 21$

5. $2x^2 + 4x - 3$

6. $2x^2 + 4x - 3$

7.1 $x + y$

7.2 $x + 2y$

7.3 $5y + 4$

7.4 $7x - 1$

7.5 $28x^2 - 7x + 1$

7.6 $3x + 2$

7.7 $10x - 8$

8.1 $\frac{9x}{y} + 1$

8.2 $24 - 15b + 8b^2$

8.3 $3a + 1$

8.4 $13 - 17b$

8.5 $3a + 5a^2$

8.6 $39a + 13 + b$

9.1 $4x - 3$

9.2 $4x^2 - 3$

9.3 $4x^2 - 3x$

9.4 $4x - 3$

9.5 $8x^2 - 6x$

9.6 $2x^2 - 1\frac{1}{2}$

10.1 True

10.2 True

10.3 True

10.4 True

10.5 False

10.6 True

11.1

x	$1,5$	$2,8$	$-3,1$	$0,72$
$\frac{3x+12}{3}$	$5,5$	$6,8$	$0,9$	$4,72$

11.2

x	$1,5$	$2,8$	$-3,1$	$0,72$
$\frac{18x^2+6}{6}$	$7,75$	$24,52$	$29,83$	$2,5552$

11.3

x	$1,5$	$2,8$	$-3,1$	$0,72$
$\frac{5x^2+7x}{x}$	$14,5$	21	$-8,5$	$10,6$

12.1 $3 + a$

12.2 $x^2 + 2x - 1$

12.3 $1 + 6b$

12.4 $6x + 5$

12.5 $3b - 2a$

12.6 $3a + 6b$

12.7 $x + 3$

12.8 $x + 3$

13.1 $x = 15$

13.2 $x = 1$

14.1

x	$1,1$	$1,2$	$1,3$	$1,4$	$1,5$
$\frac{x^3+2x^2-x}{x}$	$2,41$	$2,84$	$3,29$	$3,76$	$4,25$

14.2

x	$1,1$	$1,2$	$1,3$	$1,4$	$1,5$
$\dfrac{7x^3+21x^2}{7x^2}$	$4,1$	$4,2$	$4,3$	$4,4$	$4,5$

14.3

x	$1,1$	$1,2$	$1,3$	$1,4$	$1,5$
$\dfrac{50x^2+5x}{5x}$	12	13	14	15	16

15.1 $9x+6$

15.2 $-4x+4$

16.1 LHS simplifies to RHS using distributive property. Division by 0 is not allowed.

16.2 LHS simplifies to RHS using distributive property. Division by 0 is not allowed.

12.10 Exercise 10

1.1 Replace x with various values in both expressions and check whether the outputs are the same.

2.1

x	-1	0	0	1	2
y	0	1	2	-1	0
$(x-y)(x+3y)$	1	-3	-12	-4	4

2.2

x	-1	0	0	1	2
y	0	1	2	-1	0
$x^2+2xy-3y^2$	1	-3	-12	-4	4

3.1 Yes. The values in the table are the same.

4.1 $x^2+7x+12$

4.2 $-x^2+x+12$

4.3 $x^2-2x-15$

4.4 $6x^3-8x^2+3x-4$

4.5 $x^2+3xy+2y^2$

4.6 $2a^2+ab-3b^2$

4.7 $k^4+3k^2m+2m^2$

4.8 $4x^2-9$

4.9 $25x^2-4$

4.10 $a^2x^2-b^2y^2$

5.1 $a^2 + 2ab + b^2$

5.2 $a^2 - 2ab + b^2$

5.3 $x^2 + 2xy + y^2$

5.4 $x^2 - 2xy + y^2$

5.5 $4a^2 + 12ab + 9b^2$

5.6 $4a^2 - 12ab + 9b^2$

5.7 $25x^2 + 20xy + 4y^2$

5.8 $25x^2 - 20xy + 4y^2$

5.9 $a^2x^2 + 2abx + b^2$

5.10 $a^2x^2 - 2abx + b^2$

6.1 $m^2 + 2mn + n^2$

6.2 $m^2 - 2mn + n^2$

6.3 $9x^2 + 12xy + 4y^2$

6.4 $9x^2 - 12xy + 4y^2$

7.1 $a^2x^2 + 2abx + b^2$

7.2 $a^2x^2 - 2abx + b^2$

7.3 $4s^2 + 20s + 25$

7.4 $4s^2 - 20s + 25$

7.5 $a^2x^2 + 2abxy + b^2y^2$

7.6 $a^2x^2 - 2abxy + b^2y^2$

7.7 $4s^2 + 20rs + 25r^2$

7.8 $4s^2 - 20rs + 25r^2$

8.1 $48x^2 + 65x + 22$

8.2 $3x + 2$

12.11 Exercise 11

1.1

x	13	-13	$2,5$	10
$(2x+3)(3x-5)$	986	1012	20	575

1.2

x	13	-13	$2,5$	10
$10x^2+5x-7+3x^2-4x-3$	2200	2174	$73,75$	1300

1.3

x	13	-13	$2,5$	10
$3(10x^2-5x+2)-5x(6x-4)$	71	-59	$18,5$	56

1.4

x	13	-13	$2,5$	10
$13x^2+x-10$	2200	2174	$73,75$	1300

1.5

x	13	-13	$2,5$	10
$6x^2-x-15$	986	1012	20	575

1.6

x	13	-13	$2,5$	10
$5x+6$	71	-59	$18,5$	56

2. $10x^2+5x-7+3x^2-4x-3$ and $13x^2+x-10$
$(2x+3)(3x-5)$ and $6x^2-x-15$
$3(10x^2-5x+2)-5x(6x-4)$ and $5x+6$

3.1

x	1	2	3	4
$(2x+3)(5x-3)+(10x+9)(1-x)$	10	20	30	40

3.2

x	1	2	3	4
$\frac{9x^2+30x}{3x}$	13	16	19	22

3.3

x	1	2	3	4
$3x(10x-5)-5x(6x-4)$	5	10	15	20

3.4

x	1	2	3	4
$5x(4x+3)-2x(7+13x)+2x(3x+2)$	5	10	15	20

4. There is a constant increase every time x increases by 1

5.1

x	$1,5$	$2,5$	$3,5$	$4,5$
$(2x+3)(5x-3)+(10x+9)(1-x)$	15	25	35	45

5.2

x	$1,5$	$2,5$	$3,5$	$4,5$
$\frac{9x^2+30x}{3x}$	$14,5$	$17,5$	$20,5$	$23,5$

5.3

x	$1,5$	$2,5$	$3,5$	$4,5$
$3x(10x-5)-5x(6x-4)$	$7,5$	$12,5$	$17,5$	$22,5$

5.4

x	$1,5$	$2,5$	$3,5$	$4,5$
$5x(4x+3)-2x(7+13x)+2x(3x+2)$	$7,5$	$12,5$	$17,5$	$22,5$

12.12 Exercise 12

1. 1 17

1. 2 9

1. 3 12

1. 4 17

2. 1 It is an identity

2. 2 It is an identity

2. 3 It is an identity

2. 4 It is an identity

3. 1 $(x+3)(x-2)$

3. 2 $x(x+3)$

3. 3 $(x+3)(x+1)$

3. 4 $(x-3)(x+3)$

4. 1 $x-3$

4. 2 $x-2$

4. 3 $x+3$

4. 4 $x+3$

12.13 Exercise 13

1. 1 Yes

1. 2 Yes

1. 3 Yes

1. 4 Yes

2. 1 Yes

2. 2 Yes

2. 3 Yes

2. 4 $b+c$

2. 5 $b+c$

3. 1

(a) For each expression, find:	$3x+6y$	$4a^3+2a$	$5x-2x^2$	ax^2-bx^3	$12a^2b+18ab^2$
the factors of the first term	$3; x$	$2; 2; a^3$	$5; x$	$a; x; x$	$2^2; 3; a^2; b$
the factors of the second term	$2; 3; y$	$2; a$	$2; x; x$	$b; x; x; x$	$2; 3; a; b^2$
the greatest common factor of the two terms	3	$2a$	x	x^2	$6ab$
(b) Write the expression in factor form	$3(x+2y)$	$2a\left(2a^2+1\right)$	$x(5-2x)$	$x^2(a-bx)$	$6ab(2a+3b)$

4. 1 $(a-b)(x+1)$

4. 2 $(a-b)(x-1)$

4. 3 $(a+b)(a+b-1)$

4. 4 $(a+b)(x-1)$

4. 5 $(2x-3)(3x+1)$

4. 6 $(y-4)(y+3)$

12.14 Exercise 14

1.

b	1	2	3	5	-1	-2	-3	-5
c	30	15	10	6	-30	-15	-10	-6
$b+c$	31	17	13	11	-31	-17	-13	-11
bc	30	30	30	30	30	30	30	30

b	-1	-2	-3	-5	1	2	3	5
c	30	15	10	6	-30	-15	-10	-6
$b+c$	29	13	7	1	-29	-13	-7	-1
bc	-30	-30	-30	-30	-30	-30	-30	-30

2. 1 3 and 10

2. 2 -2 and -15

2. 3 -5 and -6

2. 4 5 and 6

3. 1 -15 and 2

3. 2 -10 and -3

3. 3 3 and 10

3.4 5 and -6

3.5 -5 and 6

4.1 3 and 12

4.2 20 and 2

4.3 18 and 2

4.4 20 and -2

4.5 -18 and -2

4.6 -20 and 2

5.1 value is 0 for $x = 2, x^2 + 3x - 10$

5.2 Value is 28 for $x = 2, x^2 + 7x + 10$

5.3 Value is 0 for $x = 2, x^2 - 7x + 10$

5.4 Value is -12 for $x = 2, x^2 - 3x - 10$

6.1 $x^2 + 11x + 24$

6.2 $x^2 + 14x + 24$

6.3 $x^2 + 10x + 24$

6.4 $x^2 + 25x + 24$

6.5 $x^2 - 5x - 24$

6.6 $x^2 - 10x - 24$

6.7 $x^2 - 2x - 24$

6.8 $x^2 - 23x - 24$

12.15 Exercise 15

1.1 $(x+3)(x+6)$

1.2 $(x+2)(x+9)$

1.3 $(x+3)(x-6)$

1.4 $(x+3)(x+2)$

2.1 $x^2 + px + 4x + pq$ or $x^2 + (p+q)x + pq$

2.2 $x^2 + px - qx - pq$ or $x^2 + (p-q)x - pq$

2.3 $x^2 - px + qx - pq$ or $x^2 - (p-q)x - pq$

2.4 $x^2 - px - qx + pq$ or $x^2 - (p+q)x + pq$

3.1 $(x+2)(x+6)$

3.2 $(x-2)(x-6)$

12.16 Exercise 16

1. 1 $(a+7)(a+2)$

1. 2 $(x-3)(x+6)$

1. 3 $(x-17)(x-1)$

1. 4 $(y+15)(y+2)$

1. 5 $(y-15)(y+2)$

1. 6 $(y+10)(y-3)$

1. 7 $(x+5)(x-3)$

1. 8 $(m+7)(m-3)$

1. 9 $(x-3)(x-3)$

1. 10 $(b+7)(b+8)$

1. 11 $(a-9)(a+7)$

1. 12 $(a+5b)(a-6b)$

1. 13 $(x-8y)(x+3y)$

1. 14 $(x-8)(x-5)$

2. 1 $(x+y)(p+q)$

2. 2 $(x-3)(9x^2+1)$

2. 3 $(a+b)(4+3p)$

2. 4 $(a+1)(a^3+3)$

2. 5 $(y+1)(x+1)$

2. 6 $(c-d)(a-b)$

3. 1 $(x+3)(x+4)$

3. 2 $(x-4)(x-3)$

12.17 Exercise 17

1.1. 1

1.1. 2 120

1.1. 3 800

1.1. 4 300

1.2. 1 $(a+b)(a-b)$

12.18 Exercise 18

1. 1 $(2a+b)(2a-b)$

1. 2 $(m+3n)(m-3n)$

1. 3 $(5x+6y)(5x-6y)$

1. 4 $(11x+12y)(11x-12y)$

1. 5 $(4p+7q)(4p-7q)$

1. 6 $(8a+5bc)(8a-5bc)$

1. 7 $(x+2)(x-2)$

1. 8 $(4x+6y)(4x-6y)$

2. 1 $(x^2+1)(x+1)(x-1)$

2. 2 $(4a^2+9)(2a+3)(2a-3)$

2. 3 $(1+abc)(1-abc)$

2. 4 $(5x^5+7y^4)(5x^5-7y^4)$

2. 5 $2(x+3)(x-3)$

2. 6 $2(10+b)(10-b)$

2. 7 $3x(y+4a)(y-4a)$

2. 8 $5(a^2+2b)(a^2-2b)$

12.19 Exercise 19

1. 1 26

1. 2 16

2. 1 33

2. 2 34

12.20 Exercise 20

1.

x	-2	0	2	4
$\frac{x+2}{x-2}$	0	-1	Undefined	3

2. 1 Undefined

2. 2 There is no such value.

12.21 Exercise 21

1. 1 Not true. Corrected statement: $\frac{x}{x} = 1$ if $x \neq 0$

1. 2 Not true. Corrected statement: $\frac{x^3}{x^2} = x$ if $x \neq 0$

1. 3 Not true. Corrected statement: $\frac{x-3}{x-3} = 1$ if $x \neq 3$

1. 4 Not true. Corrected statement: $\frac{x^2+x}{x(x+1)} = 1$ if $x \neq 0$ or -1

2. 1 $y = -2$

2. 2 $x = -4$

2. 3 $x = 1$ and -1

2. 4 $x = 2$ and $x = -3$

12.22 Exercise 22

1. 1 y

1. 2 ab

1. 3 $x - 2xy$

1. 4 $2x^2 + 3x$

2. 1 $x+3$

2. 2 $x+4$

2. 3 $x-10$

2. 4 $x-1$

3. 1 $x+2$

3. 2 $2x-1$

12.23 Exercise 23

1. 1 $2(2a+3b)$

1. 2 $(x+7)(x+1)$

1. 3 $(c+3)(c-3)$

1. 4 $(y-5)(y-3)$

1. 5 $b(-3a+1)$

1. 6 $(b-1)(-3a+1)$

1. 7 $dg(fg+d-f^2)$

1. 8 $(x+4)(x+2)$

1. 9 $(a+3)(a+2)$

1. 10 $(x-10)(x+2)$

1. 11 $x^3y^3(x^2-y^2)$

1. 12 $xy(x^2-y^2)$

1. 13 $(2-y)(2-y)$

1. 14 $3(a+7)(a-1)$

1. 15 $6(a+3)(a-3)$

1. 16 $-(a+5)(a+6)$

1. 17 $2(a+9)(a-4)$

1. 18 $5x(x-8)(x+5)$

1. 19 $(x+2+y)(x+2-y)$

1. 20 $(x+y+a)(x+y-a)$

1. 21 $(a-1+b)(a-1-b)$

1. 22 $(1+a-b)(1-a+b)$

1. 23 $(a-b)(x-y)$

1. 24 $(2x-y)(a-1)$

1. 25 $2x^2y^2\left(y^4+2x^4\right)\left(y^4-2x^4\right)$

1. 26 $(a+b)(a+b+2)(a+b-2)$

1. 27 $(a+b)(a+b-1)$

1. 28 $2(x+y)(a-b)$

2. 1 $4-3x$

2. 2 $\frac{5x-6}{x}$

2. 3 x^2-5x

2. 4 $x+1$

2. 5 $\frac{a+c}{ac}$

2. 6 p