



CHAPTER 9

Equations

CONTENTS

1	Introduction	1
2	Solving equations by inspection	1
3	Solving equations using additive and multiplicative inverses	3
4	Solving equations through inverse operations	3
5	Setting up equations	4
5.1	Constructing equations	4
5.2	Solving equations	4
5.3	Solving equations by using the laws of exponents	4
6	Solving by factorisation (Part 1)	6
6.1	Developing a strategy: Multiplying by zero	6
6.2	Taking out the highest common factor	7
7	Solving by factorisation (Part 2)	8
7.1	Solving by factorising trinomials	8
7.2	Solving by factorising the difference between two squares	8
8	Solving by factorisation (Part 3)	9
8.1	Solving by using properties of exponents	9
9	Set up equations to solve problems	10
9.1	The mathematical modelling process	10
9.2	Practise your modelling skills	11
10	Equations and ordered pairs	11
10.1	When unknowns become variables	11
10.2	Functions as sets of ordered pairs	11
11	Exercises	13
11.1	Exercise 1	13
11.2	Exercise 2	13
11.3	Exercise 3	16
11.4	Exercise 4	17
11.5	Exercise 5	17
11.6	Exercise 6	18

11.7 Exercise 7	19
11.8 Exercise 8	20
11.9 Exercise 9	20
11.10Exercise 10	21
11.11Exercise 11	22
11.12Exercise 12	22
11.13Exercise 13	22
11.14Exercise 14	23
11.15Exercise 15	23
11.16Exercise 16	24
11.17Exercise 17	24
11.18Exercise 18	25
11.19Exercise 19	25
12 Answers to Exercises	27
12.1 Exercise 1	27
12.2 Exercise 2	27
12.3 Exercise 3	29
12.4 Exercise 4	29
12.5 Exercise 5	30
12.6 Exercise 6	30
12.7 Exercise 7	31
12.8 Exercise 8	31
12.9 Exercise 9	32
12.10Exercise 10	32
12.11Exercise 11	33
12.12Exercise 12	33
12.13Exercise 13	33
12.14Exercise 14	33
12.15Exercise 15	34
12.16Exercise 16	34
12.17Exercise 17	35
12.18Exercise 18	35
12.19Exercise 19	35

April 20, 2021

Mathematical background

- Solving an equation means to find the value of the unknown that makes the equation true.
- The number of different solutions to an equation is not more than the degree of the equation, where the degree is the highest power of the unknown.
- The zero-product property means that if a product equals 0, at least one of the factors (or all of them) are 0.
- To solve a quadratic trinomial we need to have a factorised quadratic expression equal to zero. We then use the zero-product property to solve linear equations, for example $x^2 + 5x + 6 = 0$; $(x + 2)(x + 3) = 0$, then $x + 2 = 0$, so $x = -2$, or $x + 3 = 0$, so $x = -3$.
- A quadratic equation can also be a difference between two squares. The procedure is the same as for a trinomial. For example: $4x^2 - 25 = 0$; $(2x + 5)(2x - 5) = 0$; so $2x + 5 = 0$ or $2x - 5 = 0$; $x = -\frac{5}{2}$ or $\frac{5}{2}$.
- Setting up equations to describe situations is called mathematical modelling. A situation is described in mathematical terms. The mathematical problem is solved and the solution is interpreted and tested to see if it fits the original situation.
- A function is a set of ordered pairs of numbers. The input value is the first element and the output value is the second element of the ordered pair. When we set the formulae of two functions equal, we can solve the equation and find the ordered pair that belongs to both functions.

1 INTRODUCTION

In this chapter, you will find numbers that make statements true. This is called solution of equations. You will solve equations in two different ways, by inspection and by reversing them. You will find that two equations can have the same solution. Such equations are called equivalent equations. You will also discover that not all statements are algebraic equations. Some statements are algebraic identities and others are in fact algebraic impossibilities. You will learn what the difference is between these three types of statements.

2 SOLVING EQUATIONS BY INSPECTION

Note

"Searching" for the solution of an equation by using tables is called **solution by inspection**.

Two equations can have the same solution. For example, $5x = 10$ and $x + 2 = 4$ have the same solution; $x = 2$ is the solution for both equations.

An equation is a mathematical statement with two expressions, one on the left-hand side (LHS) and one on the right-hand (RHS) side of an equal sign, for example: $7x - 4 = 4x + 11$

Note

The "searching" for the solution of an equation is referred to as solving the equation by inspection.

Some statements are true for only certain values of the unknown. They are called equations. There are various ways to solve an equation. Some statements, called identities, are true for all the values of the unknown. When attempting to solve an identity you usually end up with something like $0 = 0$

For some statements there are no values for which they are true. They are called impossible.

Note

Statements like $21 - x = 2x + 3$ and $(x - 3)(x - 5) = 0$ which are only true for some values of x , are called equations.

Note

A statement like $2(x = 3) = 2x + 6$, which is true for all values of x you can think of, is called an identity.

Note

A statement like $2(x + 3) = 2x + 3$, where there are no values of x for which it is true, is called an impossibility.

A table of values can be used to solve an equation by inspection. Note that a potential answer is tested by substituting it first into the LHS and then into the RHS. If the two answers are equal, the value is a solution to the equation.

Note

Two equations are called **equivalent** if they have the same solution.

3 SOLVING EQUATIONS USING ADDITIVE AND MULTIPLICATIVE INVERSES

Note

If you add a number and then subtract the same number, you are back where you started. This is why addition and subtraction are called **inverse operations**. If you multiply by a number and then divide by the same number, you are back where you started. This is why multiplication and division are called **inverse operations**

4 SOLVING EQUATIONS THROUGH INVERSE OPERATIONS

A second method of solving equations is by using inverse operations. You have already worked with equivalent equations. Remember that a set of equations is equivalent if every equation in the set has the same solution. When we build up a set of equations, we create a set of equivalent equations. Using inverse operations to solve an equation in turn also creates a set of equivalent equations until we reach the solution, for example, $x = c$.

The degree of an equation refers to the highest power of the unknown that appears in the equation, for example $x^3 + 5 = 13$ is an equation of the third degree, while $x^2 + 5x + 6 = 0$ is an expression of the second degree, and $2x + 9 = 14$ is an equation of the first degree. Even if an expression is written as a product of factors, as in $(x+2)(x-1)(x+1)$, it is an expression of the third degree, because the highest power of x we will get will be x^3 .

Note

When performing inverse operations to an equation, remember that these operations need to be performed on both sides of the equation. So you add and subtract by the same amount and you divide and multiply by the same amount on both sides of the equation.

We can start with a solution as an equation and then apply some operations to it to turn it into an equivalent but more complicated equation. For example:

Note

Remember that two are described as equivalent if they have the same solution.

Up to now you have only dealt with equations of the first degree. That means they contained only first powers of the unknown (x), for example $3x - 2 = 5x + 7$. In the following sections you will solve equations of the second degree: $x^2 + 1 = x + 13$.

When the expression part of the equation is written as the product of a monomial and a binomial (e.g. $x(x - 2) = 0$); or the product of two binomials (e.g. $(x - 2)(x + 3) = 0$), the result is also an equation of the second degree.

5 SETTING UP EQUATIONS

5.1 Constructing equations

You can easily make an equation that has 5 as the solution. Here is an example:

Start by writing the solution	$x = 5$
Add 3 to both sides	$x + 3 = 8$
Multiply both sides by 5	$5x + 15 = 40$

5.2 Solving equations

To make an equation, you can apply the same operation on both sides

To solve an equation, you can apply the inverse operation on both sides

	$x = 4$	
Multiply by 8	$8x = 32$	Divide by 8
Add 3	$8x + 3 = 35$	Subtract 3
Subtract $5x$	$3x + 3 = 35 - 5x$	Add $5x$

5.3 Solving equations by using the laws of exponents

You may need to look back at Chapter 5 to remember the laws of exponents.

One kind of exponential equation that you deal with in Grade 9 has one or more terms with a base that is raised to a power containing a variable.

Example: $2^x = 16$

Note

When we need to find the unknown value, we are asking the question: **"To what power must the base be raised for the statement to be true?"**

Example:

$2^x = 16$ Make sure that the terms with x are on their own on one side.

$2^x = 2^4$ Write the known term in the same base as the term with the exponent.

$x = 4$ Equate the exponents.

In the example above, we can equate the exponents because the two numbers are equal only when they are raised to the same power.

Example:

Solve for x: $3^x = \frac{1}{27}$

$3^x = 3^{-3}$ (Rewrite $\frac{1}{27}$ as a number to base 3)

$x = -3$ (Equate the exponents.)

In another kind of equation involving exponents, the variable is in the base.

Note

When we need to find the unknown value, we are asking the question: **"Which number must be raised to the given power for the statement to be true?"**

For these equations, you should remember what you know about the powers of numbers such as 2, 3, 4, 5 and 10.

6 SOLVING BY FACTORISATION (PART 1)

6.1 Developing a strategy: Multiplying by zero

The principle by which it is possible to solve a quadratic equation is that if a product of two numbers equals 0, then one or both of the numbers must be 0. This is the zero-product property.

Note

Zero-product property

If $a \times b = 0$

Then: $a = 0$ or $b = 0$

or $a = 0$ and $b = 0$

Some quadratic expressions are given as a product of two factors equal to 0, which means we can use the zero-product property.

Note

Each part of a product is called a factor of the expression. If $c = ab$, the a and b are factors of c . If $x^2 + 5x + 6 = (x + 2)(x + 3)$, then $(x + 2)$ and $x + 3$ are factors of $x^2 + 5x + 6$.

You can rewrite an equation so that it is in the form $expression = 0$; for example you can write

$$x^2 - 2x = 3x + 6 \text{ as } x^2 - 5x - 6 = 0.$$

You can factorise $x^2 - 5x - 6 = 0$ and then use the zero-product property to solve the equation, as shown below:

$$x^2 - 5x - 6 = 0$$

$$(x - 6)(x + 1) = 0$$

$$x = 6 \text{ or } x = -1$$

Note

Remember, when given an equation, for instance, $a \times b = 6$ there are more solutions to this equation than just saying that $a = 6$ or $b = 6$, other options need to be considered as well.

6.2 Taking out the highest common factor

In the previous chapter you practised removing a common factor. Remember that you can look for a common factor and to remove the highest possible common factor from all the terms. This is a way to simplify an expression which makes it possible to solve equations of the type $x^2 - 4x = 0$.

Factorise (common factor x): $x(x - 4) = 0$

Use the zero-product property: $x = 0$ or $x - 4 = 0$, so $x = 4$.

A common mistake that is made when solving equations such as $x^2 = 5x$, you may consider to divide by x on either side or get $x = 5$. This cannot be done, because one of the values of x , $x = 0$ (that will be eliminated), and division by 0 is undefined. You have to change the equation to get it equal to 0.

So, $x^2 = 5x$

$x^2 - 5x = 0$

$x(x - 5) = 0$ Factorise (common factor)

$x = 0$ or $x = 5$ Zero-product property

The process of writing a sum expression (polynomial) as a product (monomial) is called factorisation. This is the inverse of expansion. When looking at the expression $2x^2 - 6x$. $2x$ is a factor of both terms, therefore it is a factor of $2x^2 - 6x$. By division we get $\frac{2x^2 - 6x}{2x} = x - 3$. Hence $2x^2 - 6x = 2x(x - 3)$.

Note

It is unnecessary to write out the division step of this method. After finding the common factor, we write down the product form directly: $2x^2 - 6x = 2x(x - 3)$.

7 SOLVING BY FACTORISATION (PART 2)

7.1 Solving by factorising trinomials

The fact that we can usually factorise a quadratic expression to get a product of two factors means we can use the zero-product property. But this means that the RHS (or the LHS) of the equation must be 0. There may be some manipulation needed to get the equation in to that form.

For example, $(x - 3)(x + 2) = 6$

Expand the brackets on the LHS: $x^2 - x - 6 = 6$

Add -6 to both sides: $x^2 - x - 12 = 0$

Factorise: $(x - 4)(x + 3) = 0$

Now use the zero product property: $(x - 4) = 0$ or $(x + 3) = 0$

Solve the two simple equations: $x = 4$ or $x = -3$

Note

A common error that is made is that the RHS is ignored if it is not equal to 0, the equations should be equal to 0 before factorising.

The product of the first terms of the factors must be equal to the x^2 term of the trinomial. The product of the last terms of the factors must be equal to the last term (the constant term) of the trinomial.

7.2 Solving by factorising the difference between two squares

The pattern for factorising the difference of two squares is $a^2 - b^2 = (a - b)(a + b)$.

Remember from the previous chapter: If p and q are perfect squares, also referred to as algebraic squares, then:

Note

An expression of the form $a^2 - b^2$ is called the difference between two squares. To factorise a difference between squares, we use the identity: $a^2 - b^2 = (a + b)(a - b)$ where a and b represent numbers or algebraic expressions.

8 SOLVING BY FACTORISATION (PART 3)

8.1 Solving by using properties of exponents

Both sides of the equation have to be written as powers of the same base, to which end the number not written as a power is written as a product of its prime factors.

Note

All number can be written as the product of their prime factors: $16 = 4 \times 4 = 2 \times 2 \times 2 \times 2 = 2^4$.
Factorise the number until all the factors are prime numbers.

In the equation $2^x = 16$, the letter symbol (x) is the exponent. Equations with the letter symbol as an exponent are referred to as exponential equations.

Note

If the base of the LHS is the same as the base of the RHS, then the exponent on the LHS must be equal to the exponent on the RHS. If $a^x = a^y$, then $x = y$.

There are certain techniques to remember when factorising:

1. Removing a common factor
2. Factorising a quadratic equation
3. Using the zero-product property
4. Using the properties of exponents

9 SET UP EQUATIONS TO SOLVE PROBLEMS

9.1 The mathematical modelling process

Consider this problem involving a practical solution:

Printing Shop A charges 45c per page and R12 for binding a book.

Printing Shop B charges 35c per page and R15 for binding a book.

For a book with the same amount of pages, will the shops charge the same?

You can write an equation to describe the problem. Let the number of pages for which the works costs the same be x . Then:

$$45x + 1200 = 35x + 1500$$

Note

The equation represents a mathematical problem that can be solved without necessarily keeping the practical situation in mind. It is called a mathematical model of the practical solution.

We work with numbers, so we solve problems that can be quantified. We describe the quantities we want to calculate in terms of unknowns, for example x .

Now solve the equation.

$$45x + 1200 = 35x + 1500$$

$$45x - 35x = 1500 - 1200$$

$$10x = 300$$

$$x = 30$$

We may now ask what the solution to the mathematical problem (" $x = 30$ ") means in terms of the practical situation. When the equation was set up above, the symbol x was used as a placeholder for the number of pages in a book for which the two shops would charge the same. So, what does the solution tell you?

Now check to see if the two shops will charge the same for a book with 30 pages.

At shop A, 30 pages will cost $30 \times 45c = 1350c = R13,50$. Binding is R12; total cost is R25,50.

At shop B, 30 pages will cost $30 \times 35c = 1050c = R10,50$. Binding is R15; total cost is R25,50.

The solution to the mathematical problem is also a solution to the practical problem.

Note

The mathematical solution may be interpreted to establish what it means in the practical situation.

Note

The mathematical solution should be tested in the practical solution, because mistakes may have been made.

When people work like this, we say they do mathematical modelling.

9.2 Practise your modelling skills

We should decide what the unknown quantity is and consign a variable to it, for example: let the age be x , or let the number of days be x . Remember to always make an interpretation.

10 EQUATIONS AND ORDERED PAIRS

10.1 When unknowns become variables

In the previous we dealt with equations which had fixed or limited solutions. They only had one letter symbol, which in this case acted as a placeholder for the value/s which will make the statement true.

If the value of the expression (output number) is also expressed as a variable, y , it means we could choose values for y , for example 27, and solve the equation $27 = 5x + 2$. For each value of x , we can generate a value for y . The sets of values are called ordered pairs meaning there is a first value, the input value, and a second value, the output value.

10.2 Functions as sets of ordered pairs

We already know that a function can be represented in different forms. A function is the set of all the ordered pairs that satisfy the description of the function. In the case of a relationship that is a function, there is only one unique y value for every x value.

Note

In the function indicated by $y = 5x + 2$, the letter symbol in the formula (x) represents the input or independent variable while the other letter symbol (y) represents the output or dependent variable.

The ordered pairs represent all the input values and their corresponding output values given either in a table:

or as a set of brackets, for example: $(1; 2)$, $(2; 4)$, $(3; 6)$, ... and so on.

Note

If there is precisely one value of y for each value of x , we say that y is a function of x .

11 EXERCISES

11.1 Exercise 1

1. Six equations are listed below the table. Use the table to find out for which of the given values of x it will be true that the left-hand side of the equation is equal to the right-hand side.

x	-3	-2	-1	0	1	2	3	4
$2x + 3$	-3	-1	1	3	5	7	9	11
$x + 4$	1	2	3	4	5	6	7	8
$9 - x$	12	11	10	9	8	7	6	5
$3x - 2$	-11	-8	-5	-2	1	4	7	10
$10x - 7$	-37	-27	-17	-7	3	13	23	33
$5x + 3$	-12	-7	-2	3	8	13	18	23
$10 - 3x$	19	16	13	10	7	4	1	-2

1.1 $2x + 3 = 5x + 3$

1.2 $5x + 3 = 9 - x$

1.3 $2x + 3 = x + 4$

1.4 $10x - 7 = 5x + 3$

1.5 $3x - 2 = x + 4$

1.6 $9 - x = 2x + 3$

2. Which of the equations in question 1 have the same solutions? Explain.

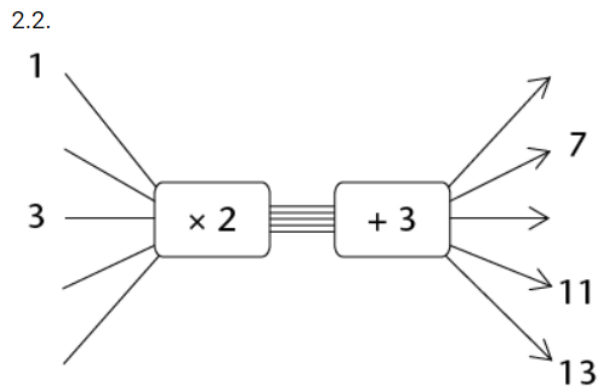
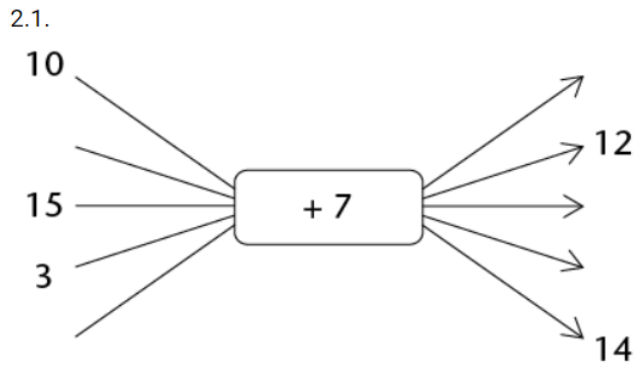
11.2 Exercise 2

1. In each case find the value of x :

1.1. $x \rightarrow \boxed{+7} \rightarrow 10$

1.2. $x \rightarrow \boxed{\times 2} \rightarrow \boxed{+3} \rightarrow 13$

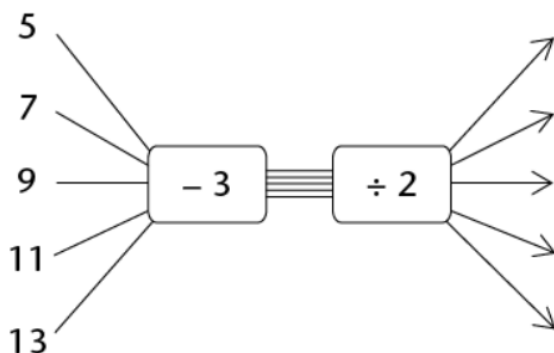
2. Complete the flow diagrams. You have to fill in all the missing numbers.



3. Use your answers for question 2 to check your answers for question 1.

4. Describe the instructions in flow diagram 2.2 in words, and also with a symbolic expression.

5. Complete the flow diagram.



6. Compare the input numbers and the output numbers of the flow diagrams in question 2(b) and question 5. What do you notice?

7.

7.1 Add 5 to any number and then subtract 5 from your answer. What do you get?

7.2 Multiply any number by 10 and then divide the answer by 10. What do you get?

The expression $5x - 3$ says "multiply by 5 then subtract 3". This instruction can also be given with a flow diagram:



The equation $5x - 3 = 47$ can also be written as a flow diagram:



8. Solve the equations below. You may do this by using the inverse operations. You may write a flow diagram to help you to see the operations.

8.1 $2x + 5 = 23$

8.2 $3x - 5 = 16$

8.3 $5x - 60 = -5$

8.4 $\frac{1}{3}x + 11 = 19$

8.5 $10(x + 3) = 88$

8.6 $2(x - 13) = 14$

11.3 Exercise 3

1. What is the solution of the equation $5x + 15 = 40$?
2. Make your own equation with the solution $x = 3$.
3. Bongile worked like this to make the equation $2(x + 8) = 30$, but he rubbed out part of his work:

Start by writing the solution $x =$
Add 8 to both sides $= 15$
Multiply both sides by 2 $2(x + 8) = 30$

Complete Bongile's writing to solve the equation $2(x + 8) = 30$.

4. This is how Bongile made a more difficult equation:

Start by writing the solution $x =$
Multiply by 3 on both sides $3x =$
Subtract 9 from both sides $3x - 9 = 6$
Add $2x$ to both sides $5x - 9 = 2x + 6$

4.1 What was on the right-hand side before Bongile subtracted 9?

4.2 What is the solution of $5x - 9 = 2x + 6$?

5. Bongile started with a solution and he ended up with an equation. Fill in the steps that Bongile took to make the equation, and solve the equation:

$x =$
 $8x =$
 $8x + 3 =$
 $3x + 3 = 35 - 5x$

11.4 Exercise 4

Use any appropriate method to solve the equations below.

1.

1.1 $5x + 3 = 24 - 2x$

1.2 $2x + 4 = -9$

1.3 $3-x = x-3$

1.4 $6(2x + 1) = 0$

2.

2.1 $4(1 - 2x) = 12 - 7x$

2.2 $8(1 - 3x) = 5(4x + 6)$

2.3 $7x - 10 = 3x + 7$

2.4 $1, 6x + 7 = 3, 5x + 3, 2$

11.5 Exercise 5

1.

1.1 Which of the following rules will produce the number pattern given in the second row of the table below?

- A. Term value = $8n$ where n is the term number
- B. Term value = $6n - 1$ where n is the term number
- C. Term value = $6n + 2$ where n is the term number
- D. Term value = $10n - 2$ where n is the term number
- E. Term value = $5n + 3$ where n is the term number

Term number	1	2	3	4	5	6	7	8	9
Term value	8	13	18	23	28	33	38	43	48

1.2 The sixth term of the sequence has the value 33. Which term will have the value 143? You may set up and solve an equation to find out.

1.3 Apply rule E to your answer, to check whether your answer is correct.

2.

2.1 Write the rule that will produce the number pattern in the second row of this table. You may have to experiment to find out what the rule is.

Term number	1	2	3	4	5	6	7	8	9
Term value	5	8	11	14	17	20	23	26	29

2.2 Which term will have the value 221?

3. The rule for number pattern A is $4n + 11$, and the rule for pattern B is $7n - 34$.

3.1 Complete the table below for the two patterns.

Term number	1	2	3	4	5	6	7	8	9
Pattern A									
Pattern B									

3.2 For which value of n are the terms of the two patterns equal?

11.6 Exercise 6

1. Consider this situation:

To rent a room in a certain building, you have to pay a deposit of R400 and then R80 per day.

1.1 How much money do you need to rent the room for 10 days?

1.2 How much money do you need to rent the room for 15 days?

2. Which of the following best describes the method that you used to do question 1(a) and (b)?

A. Total cost = $R400 + R80$

B. Total cost = $400(\text{number of days} + 80)$

C. Total cost = $80 \times \text{number of days} + 400$

D. Total cost = $(80 + 400) \times \text{number of days}$

3. For how many days can you rent the room described in question 1, if you have R2 800 to pay?

4. To rent a certain room, you have to pay a deposit of R300 and then R120 per day.

4.1 For how many days can you rent the room if you can pay a total of R1 740? (If you experience trouble in setting up the equation, it may help you to decide first how you will work out what it will cost to rent the room for 6 days.)

-
- 4.2 What will it cost to rent the room for 10 days, 11 days and 12 days?
- 4.3 For how many days can you rent the room if you have R3 300 available?
- 4.4 For how many days can you rent the room if you have R3 000 available?
5. Ben and Thabo decide to do some calculations with a certain number. Ben multiplies the number by 5 and adds 12. Thabo gets the same answer as Ben when he multiplies the number by 9 and subtracts 16. What is the number they worked with?
6. The cost of renting a certain car for a period of x days can be calculated with the following formula:
 $Rentalcostinrand = 260x + 310$
What information about renting this car will you get, if you solve the equation
 $260x + 310 = 2910$?
7. Sarah paid a deposit of R320 for a stall at a market, and she also pays R70 per day rental for the stall. She sells fruit and vegetables at the stall, and finds that she makes about R150 profit each day. After how many days will she have earned as much as she has paid for the stall, in total?

11.7 Exercise 7

1. Solve for x :

1.1 $5^{x-1} = 125$

1.2 $2^{x+3} = 8$

1.3 $10^x = 10000$

1.4 $4^{x+2} = 64$

1.5 $7^{x+1} = 1$

1.6 $x^0 = 1$

2. Solve for x .

2.1 $7^x = \frac{1}{49}$

2.2 $10^x = 0.001$

2.3 $6^x = \frac{1}{216}$

2.4 $10^{x-1} = 0.001$

2.5 $4^{-x} = \frac{1}{16}$

2.6 $7^x = 7^{-3}$

11.8 Exercise 8

1. Complete the table below and answer the questions that follow:

	x	2	3	4	5
(a)	x^3	$2^3 = 8$			
(b)	x^5	$2^5 = 32$			
(c)	x^4	$2^4 = 16$			

1.1 $x^3 = 64$

1.2 $x^5 = 32$

1.3 $x^4 = 256$

1.4 $x^3 = 8$

1.5 $x^4 = 16$

1.6 $x^5 = 3125$

2. Solve for x and give a reason:

2.1 $x^3 = 216$

2.2 $x^2 = 324$

2.3 $x^4 = 10000$

2.4 $8^x = 512$

2.5 $18^x = 324$

2.6 $6^x = 216$

11.9 Exercise 9

1. Substitute the given values for x into the given equation and determine if the Left Hand side is equal to the Right Hand Side as well as which of the given values for x yields LHS=RHS

Equation	LHS if $x = 4$	Is LHS = RHS?	LHS if $x = 5$	Is LHS = RHS?	LHS if $x = 6$	Is LHS = RHS?	Correct solution
$3x - 4 = 11$							
Equation	LHS if $x = 4$	Is LHS = RHS?	LHS if $x = 5$	Is LHS = RHS?	LHS if $x = 6$	Is LHS = RHS?	Correct solution
$2x + 7 = 19$							
Equation	LHS if $x = 4$	Is LHS = RHS?	LHS if $x = 5$	Is LHS = RHS?	LHS if $x = 6$	Is LHS = RHS?	Correct solution
$13 - 5x = -7$							

2. For the following, input a +, - or = to make the given equation true for the given value of x

2.1 $2x \dots 7 = 15$ for $x = 4$

2.2 $3 \dots 2x = 11$ for $x = -4$

2.3 $-x \dots 7 = 3$ for $x = 4$

2.4 $28 \dots 5x = 3$ for $x = 5$

11.10 Exercise 10

1. Complete the following calculations

1.1 $3 - 3$

1.2 $-9\,765 + 9\,765$

1.3 $-a + a$

1.4 $13a - 13a$

2. What do you notice about the following equations?

$$3 - 3 = 0$$

$$-9765 + 9765 = 0$$

$$-a + a = 0$$

$$13a - 13a = 0$$

3. Complete the following calculations

3.1 $3 \div 3$

3.2 $3 \times \frac{1}{3}$

3.3 $\frac{1}{x} \times x$

3.4 $\frac{x}{3} \times \frac{3}{x}$

4. What do you notice from the given equations?

$$3 \div 3 = 1$$

$$3 \times \frac{1}{3} = 1$$

$$\frac{1}{x} \times x = 1$$

$$\frac{x}{3} \times \frac{3}{x} = 1$$

5. Solve the following for x

5.1 $2(x + 4) + 9 = 15$

5.2 $5(x - 2) = 7(2 - x)$

5.3 $\frac{2x}{3} - 2 = 12$

5.4 $\frac{3y-3}{2} + \frac{5}{2} = \frac{5y}{3}$

11.11 Exercise 11

- Can you find two numbers x and y so that if you multiply them the answer is 0? i.e. $xy = 0$
- Complete the following tables as in the example and find the factors of the given equations

Equation	Factors	Product	First possible solution	Second possible solution
$x(x - 2) = 0$	x and $(x - 2)$	0	$x = 0$	$x = 2$

Equation	Factors	Product	First possible solution	Second possible solution
2.1 $x(x + 5) = 0$				

Equation	Factors	Product	First possible solution	Second possible solution
2.2 $2x(3x - 12) = 0$				

Equation	Factors	Product	First possible solution	Second possible solution
2.3 $(x + 2)(x + 1) = 0$				

11.12 Exercise 12

- Determine the values of x which will make the following statements true

1.1 $x^2 + 2x^2 = 6x$

1.2 $x^2 = -3x$

1.3 $\frac{6x}{3} + x = -4x^2$

1.4 $x = x(2 - x)$

11.13 Exercise 13

- Find the factors of the following equations

1.1 $x^2 + 9x = -14$

1.2 $x^2 + 3x = 18$

1.3 $x^2 - 18x = -17$

1.4 $x^2 + 30 = 11x$

1.5 $x^2 = 13x + 30$

1.6 $x^2 + 7x = 30$

11.14 Exercise 14

1. Determine the values of x or a that will make the following statements true by using the difference between two squares

1.1 $x^2 = 4$

1.2 $x^2 = 16$

1.3 $4a^2 = 9$

1.4 $81 = 9n^2$

1.5 $25x^2 = 36$

1.6 $121x^2 = 144$

1.7 $16p^2 = 49$

1.8 $64a^2 = 25$

11.15 Exercise 15

1. Write the following numbers as the product of their prime factors

1.1 128

1.2 243

1.3 125

1.4 2 401

2. Determine the values of x which will make the following statements true

2.1 $2^x = 2^7$

2.2 $3^x = 3^5$

2.3 $5^x = 5^3$

2.4 $7^x = 7^4$

3. Determine the values of x which will make the following statements true

3.1 $2^x = 128$

3.2 $3^x = 243$

3.3 $5^x = 125$

3.4 $7^x = 7^{2401}$

3.5 $2^x + 9 = 7^{25}$

3.6 $27(3^x) = 3$

11.16 Exercise 16

1. Determine the values of the unknown (x or m or b , etc.) which will make the following statements true:

1.1 $\frac{6x}{3} + x = -4x^2$

1.2 $x = x(2 - x)$

1.3 $x^2 + 2x = 15$

1.4 $m^2 + 4m = 21$

1.5 $x^2 + 3 = 4x$

1.6 $b^2 - 16b = -15$

1.7 $1 = a^2$

1.8 $25x^2 = 49$

1.9 $2^x - 25 = -9$

1.10 $81(3^x) = 3$

11.17 Exercise 17

1. Answer the following questions.

1.1 Louis is six years older than Karin and Karin is four years older than Heidi. The sum of their ages is 53 years. How old is Heidi?

1.2 The sum of two numbers is 15 . Three times the smaller number is 5 more than the larger number. Calculate the two numbers. (Hint: Let the smaller number be x .)

1.3 The sum of three consecutive even numbers is 108. What are the numbers?

2. Firm A calculates the cost of a job using the formula: $\text{cost} = 500 + 30t$, where t is the number of days it takes to complete the job. <>

Firm B calculates the cost of the same job using the formula: $\text{cost} = 260 + 48t$

2.1 What would Firm A charge for a job that takes ten days?

2.2 How long would Firm B take to complete a job for which their charge is R 596?

2.3 There is a specific job for which firms charge the same and take the same time to complete. How long does this job take?

11.18 Exercise 18

1. Answer the following about the equation

$$y = 5x + 2$$

1.1 How many letter symbols does the equation have?

1.2 Is it possible to solve this equation?

1.3

x	12	10	5	6	-5	-10
$5x+2$						

11.19 Exercise 19

1. Copy and complete each table by writing the ordered pairs in brackets below the table, in the table, as shown in the example. Then choose two more input numbers and write down two additional ordered pairs that belong to each given function.

$y = 4x + 5$

x	-2	0	1	2	5	10	20
y	-3	5	9	13	25	45	85

$(-2; -3), (0; 5), (1; 9), (2; 13), (5; 25)$ and $(10; 45)$ and $(20; 85)$

1.1 $y = x^2 + 9$

x	5		0	-3	
y	3	18			34

1.2 $y = 3x - 2$:

x	5	1	0	-3	
y					-17

$(5; 13), (1; 1), (0; -2), (-3; -11), (-5; -17)$, and $(...; ...)$ and $(...; ...)$

1.3 $y = 5x - 4$:

x	-5	-3	1	2	
y					21

$(-5; -29), (-3; -19), (1; 1), (2; 6), (5; 21)$, and $(...; ...)$ and $(...; ...)$

1.4 $y = 5x - 4$:

x	1	2	3	4	
y					-3

$(1; 21), (2; 6), (3; 3), (4; 0), (5; -3)$, and $(...; ...)$ and $(...; ...)$

1.5 $y = x^2 + 2$

x	-12	-7	-2	3	
y					102

$(-12; 146), (-7; 51), (-2; 6), (3; 11), (10; 102)$, and $(...; ...)$ and $(...; ...)$

1.6 $y = x^2 + 2$

x	-12	-7	-2	3	
y					102

$(-12; 146), (-7; 51), (-2; 6), (3; 11), (10; 102)$ and $(\dots; \dots)$ and $(\dots; \dots)$

1.7 $y = x^2 + 2$

x	-12	-7	-2	3	
y					102

$(-12; 146), (-7; 51), (-2; 6), (3; 11), (10; 102)$ and $(\dots; \dots)$ and $(\dots; \dots)$

1.8 $y = 2^x + 2$

x	0	1	2	3	
y					18

$(0; 3), (1; 4), (2; 6), (3; 10), (4; 18)$ and $(\dots; \dots)$ and $(\dots; \dots)$

2. Answer the following with regards to ordered pairs

2.1 Which ordered pair belongs to both $y = 3x - 2$ and $y = 5x - 4$?

2.2 Which ordered pair belongs to both $y = 12 - 3x$ and $y = 5x - 4$?

2.3 Which ordered pair belongs to both $y = 5x + 7$ and $y = 3x + 25$?

12 ANSWERS TO EXERCISES

12.1 Exercise 1

1.1 $x = 0$

1.2 $x = 1$

1.3 $x = 1$

1.4 $x = 2$

1.5 $x = 3$

1.6 $x = 2$

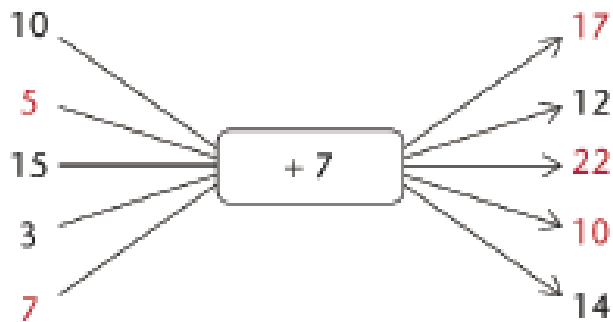
2. 1.2 and 1.3 have the same solution, $x = 1$; 1.4 and 1.6 have the same solution, $x = 2$

12.2 Exercise 2

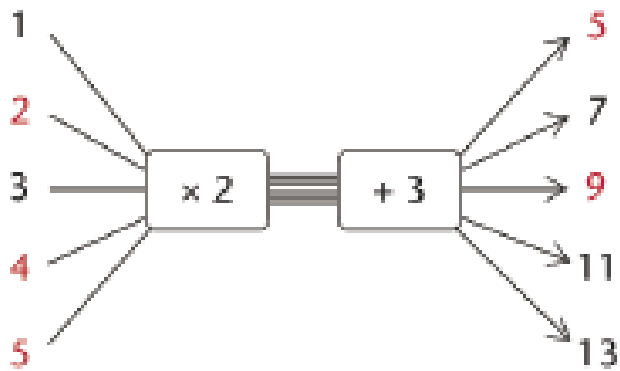
1.1 $x = 3$

1.2 $x = 5$

2.1

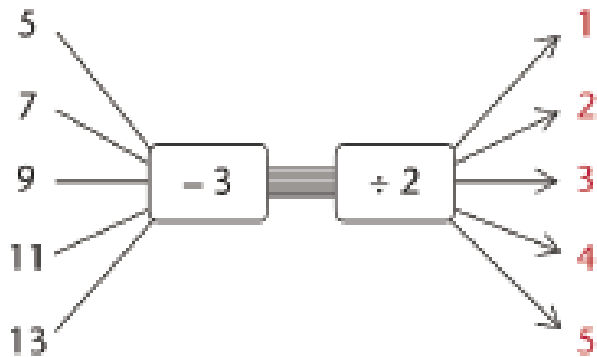


2.2



3. Multiply a number by 2 and add 3 to the answer: $2x + 3$

4.



5. The input numbers in question 2.2 are the output numbers in question 5 and the output numbers in question 2.2 are the input numbers in question 5.

6.1 You get the number you started with.

6.2 You get the number you started with.

7.1 $x = 9$

7.2 $x = 7$

7.3 $x = 11$

7.4 $x = 24$

7.5 $x = 5, 8$

7.6 $x = 20$

12.3 Exercise 3

1. $x = 5$

2. Learners' answers will differ. Below is a sample answer:

$$x = 3$$

Subtract 5 on both sides $x - 5 = -2$

multiply both sides by -5 $-5x + 25 = 10$

3. Start by writing the solution $x = 7$

Add 8 to both sides $x + 8 = 15$

Multiply both sides by 2 $2(x + 8) = 30$

4. 1 15

4. 2 $x = 5$

5 $x = 4$

Multiply by 8 $8x = 32$

Add 3 $8x + 3 = 35$

Subtract $5x$ $3x + 3 = 35 - 5x$

12.4 Exercise 4

1. 1 $x = 3$

1. 2 $x = -6\frac{1}{2}$

1. 3 $x = 3$

1. 4 $x = -\frac{1}{2}$

2. 1 $x = -8$

2. 2 $x = -\frac{1}{2}$

2. 3 $x = 4\frac{1}{4}$

2. 4 $x = 2$

12.5 Exercise 5

1.1 Rule E produces the number pattern in the second row of the table.

1.2 $n = 28$.

1.3 Term value = $5n + 3$, so term 28 = $28 \times 5 + 3 = 140 + 3 = 143$.

2.1 The rule is: Term value = $3n + 2$.

2.2 $n = 73$.

3.1

Term number	1	2	3	4	5	6	7	8	9
Pattern A	15	19	23	27	31	35	39	43	47
Pattern B	-27	-20	-13	-6	1	8	15	22	29

3.2 $n = 15$.

12.6 Exercise 6

1.1 R1 200

1.2 R1 600

2. C. Total cost = $80 \times$ number of days + 400

3. Different methods can be used.

For example, $R2\ 800 - R400 = R2\ 400$, then $R2\ 400 \div R80 = 30$, or

Let x number of days. Then $80x + 400 = 2\ 800$

$$\text{so} \quad 80x = 2\ 400$$

$$\text{so} \quad x = 2\ 400 \div 80$$

$$= 30\text{days}$$

4.1 12 days

4.2 $120 \times 10 + 300 = R1\ 500$

$$120 \times 11 + 300 = R1\ 620$$

$$120 \times 12 + 300 = R1\ 740$$

4.3 25 days

4.4 22 days

5. Let the number be x . $x = 7$.

6. 10 days

7. 4 days.

12.7 Exercise 7

1.1 $x = 4$

1.2 $x = 0$

1.3 $x = 4$

1.4 $x = 1$

1.5 $x = -1$

1.6 x can be any number except 0.

2.1 $x = -2$

2.2 $x = -3$

2.3 $x = -3$

2.4 $x = -2$

2.5 $x = 2$

2.6 $x = -3$

12.8 Exercise 8

1.

x	2	3	4	5
x^3	$2^3 = 8$	$3^3 = 27$	$4^3 = 64$	$5^3 = 125$
x^5	$2^5 = 32$	$3^5 = 243$	$4^5 = 1\ 024$	$5^5 = 3\ 125$
x^4	$2^4 = 16$	$3^4 = 81$	$4^4 = 256$	$5^4 = 625$

1.1 $x = 4$

1.2 $x = 2$

1.3 $x = 4$

1.4 $x = 2$

1.5 $x = 2$

1.6 $x = 5$

2.1 $x = 6(6^3 = 216)$

2.2 $x = 18(18^2 = 324)$

2.3 $x = 10(10^4 = 10\ 000)$

2.4 $x = 3(8^3 = 512)$

2.5 $x = 2(18^2 = 324)$

2.6 $x = 3(6^3 = 216)$

12.9 Exercise 9

1.1 $x = 5$

1.2 $x = 6$

1.3 $x = 4$

2.1 +

2.2 -

2.3 +

2.4 -

12.10 Exercise 10

1.1 0

1.2 0

1.3 0

1.4 0

2. Using the inverse operation for addition or subtraction produces zero as the answer

3.1 1

3.2 1

3.3 1

3.4 1

4. Using the inverse operation for multiplication or division produces 1 as the answer

5.1 $x = -1$

5.2 $x = 2$

5.3 $x = 21$

5.4 $y = 6$

12.11 Exercise 11

1. One or both of the factors must be zero $x = 0$ or $y = 0$ $x = 0$ and $y = 0$
2. 2.1 $x = 0$ and $x = -5$
2.2 $x = 0$ and $x = 4$
2.3 $x = -2$ and $x = 2$

12.12 Exercise 12

- 1.1 $x = 0$ or $x = 2$
- 1.2 $x = 0$ or $x = -3$
- 1.3 $x = 0$ or $x = -\frac{3}{4}$
- 1.4 $x = 0$ or $x = 1$

12.13 Exercise 13

- 1.1 $x = -7$ or $x = -2$
- 1.2 $x = 3$ or $x = -6$
- 1.3 $x = 17$ or $x = 1$
- 1.4 $x = 6$ or $x = 5$
- 1.5 $x = -2$ or $x = 15$
- 1.6 $x = 3$ or $x = -10$

12.14 Exercise 14

- 1.1 $x = -2$ or $x = 2$
- 1.2 $x = -4$ or $x = 4$ $a = -\frac{3}{2}$ or $a = \frac{3}{2}$ $n = -3$ or $n = 3$
- 1.3 $x = -\frac{6}{5}$ or $x = \frac{6}{5}$
- 1.4 $x = -\frac{12}{11}$ or $x = \frac{12}{11}$
- 1.5 $p = -\frac{7}{4}$ or $p = \frac{7}{4}$
- 1.6 $a = -\frac{5}{8}$ or $a = \frac{5}{8}$

12.15 Exercise 15

1.1 2^7

1.2 3^5

1.3 5^3

1.4 7^4

2.1 $x = 7$

2.2 $x = 5$

2.3 $x = 3$

2.4 $x = 4$

3.1 $x = 7$

3.2 $x = 5$

3.3 $x = 3$

3.4 $x = 4$

3.5 $x = 4$

3.6 $x = -2$

12.16 Exercise 16

1.1 $x = 0$ or $x = -\frac{3}{4}$

1.2 $x = 0$ or $x = 1$

1.3 $x = 3$ or $x = -5$

1.4 $m = -7$ or $m = 3$

1.5 $x = 3$ or $x = 1$

1.6 $b = 15$ or $b = 1$

1.7 $a = -1$ or $a = 1$

1.8 $x = -\frac{7}{5}$ or $x = \frac{7}{5}$

1.9 $x = 4$

1.10 $x = -3$

12.17 Exercise 17

1. 1.1 Heidi is 13 years old
- 1.2 Small number: 5
Large number: 10
- 1.3 1st number: 34
2nd number: 36
3rd number: 38
2. 2.1 R 800
- 2.2 7 days
- 2.3 $13\frac{1}{3}$ days

12.18 Exercise 18

- 1.1 Two
- 1.2 No, there are two variables
- 1.3

x	12	10	20	5	6	-5	-10
$5x + 2$	62	52	102	27	32	-23	-48

12.19 Exercise 19

- 1.1 $(-7; 58)$ and $(-9; 90)$
- 1.2 $(-7; -23)$ and $(-9; -29)$
- 1.3 $(7; 31)$ and $(9; 41)$
- 1.4 $(7; -9)$ and $(9; -15)$
- 1.5 $(15; 227)$ and $(20; 402)$
- 1.6 $(15; 227)$ and $(20; 402)$
- 1.7 $(15; 227)$ and $(20; 402)$
- 1.8 $(5; 34)$ and $(6; 66)$
- 2.1 $(1; 1)$
- 2.2 $(2; 6)$
- 2.3 $(9; 52)$