CHAPTER 17

Electric Circuits

CONTENTS

March 5, 2021

1 POTENTIAL DIFFERENCE AND EMF

When a circuit is connected and complete, charge can move through the circuit. Charge will not move unless there is a reason, a force to drive it round the circuit. Think of it as though charge is at rest and something has to push it along. This means that work needs to be done to make charge move. A force acts on the charges, doing work, to make them move. The force is provided by the battery in the circuit.

A battery has the potential to drive charge round a closed circuit, the battery has potential energy that can be converted into electrical energy by doing work on the charge in the circuit to make it move.

Definition: Potential Difference

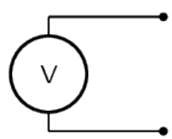
Potential difference is the work done per unit charge, $\frac{W}{q}$. The units of potential difference are the volt (V) which is defined as one joule per coulomb.

Quantity: Potential difference (V) Unit name: volt Unit symbol: V

1.1 Voltmeter

A voltmeter is an instrument for measuring the potential difference between two points in an electric circuit.

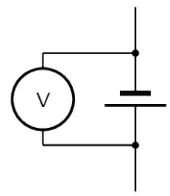
The symbol for a voltmeter is:



A voltmeter

1.2 EMF

When you measure the potential difference across (or between) the terminals of a battery that is not in a complete circuit you are measuring the emf of the battery. This is the maximum amount of work per coulomb of charge the battery can do to drive charge from one terminal, through the circuit, to the other terminal.

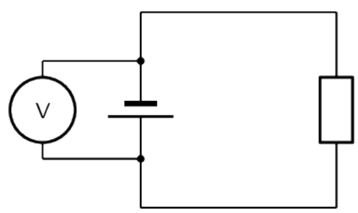


Did you know?

The volt is named after the Italian physicist Alessandro Volta (1745–1827).

Electrical potential difference is also called voltage.

When you measure the potential difference across (or between) the terminals of a battery that is **in a complete** circuit you are measuring the terminal potential difference of the battery. Although this is measured in volts it is not identical to the emf. The difference will be the work done to drive charge through the battery.



Batteries

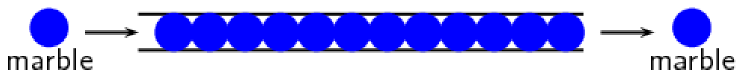
One lead of the voltmeter is connected to one end of the battery and the other lead is connected to the opposite end. The voltmeter may also be used to measure the voltage across a resistor or any other component of a circuit but must be connected in parallel.

2 CURRENT

2.1 Flow of charge

When we talk about current we talk about how much charge moves past a fixed point in circuit in one second. Think of charges being pushed around the circuit by the battery, there are charges in the wires but unless there is a battery they won't move.

When one charge moves the charges next to it also move. They keep their spacing as if you had a tube of marbles like in this picture or looked at a train and its carriages.



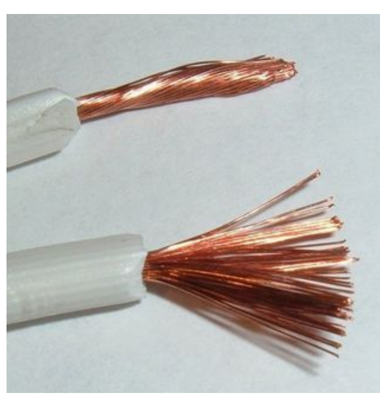
If you push one marble into the tube one must come out the other side, if a train locomotive moves all the carriages move immediately because they are connected. This is similar to charges in the wires of a circuit.

The idea is that if a battery started to drive charge in a circuit all the charges start moving instantaneously.

Did you know?

Benjamin Franklin made a guess about the direction of charge flow when rubbing smooth wax with rough wool. He thought that the charges flowed from the wax to the wool (i.e. positive to negative) which was opposite to the real direction. Due to this, electrons are said to have a negative charge and so objects which Ben Franklin called "negative" (meaning, shortage of charge) really have an excess of electrons. By the time the true direction of electron flow was discovered, the convention of "positive" and "negative" had already been so well accepted in the scientific world that no effort was made to change it.

Copper wire

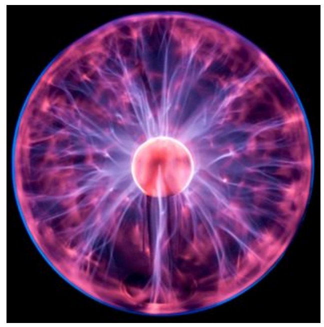


Definition: Potential Difference

Current Current is the rate at which charges moves past a fixed point in a circuit. The units of current are the ampere (A) which is defined as one coulomb per second.

Quantity: Current (I) Unit name: ampere Unit symbol: A

Plasma ball



We use the symbol I to show current and it is measured in amperes (A). One ampere is one coulomb of charge moving in one second (C·s^{-1}).

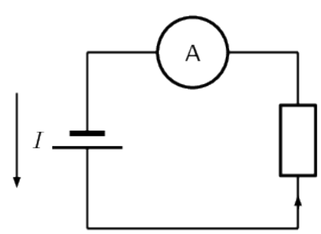
$$I = \frac{Q}{\Delta t}$$

When current flows in a circuit we show this on a diagram by adding arrows. The arrows show the direction of flow in a circuit. By convention we say that charge flows from the positive terminal on a battery to the negative terminal.

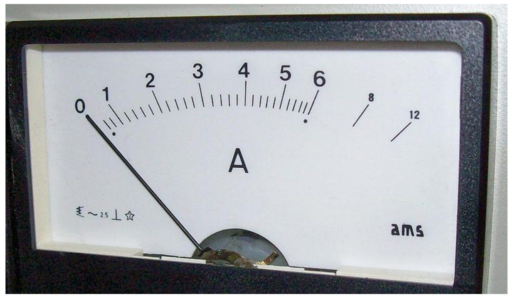
If the voltage is high enough a current can be driven through almost anything. In the plasma ball example on the left, a voltage is created that is high enough to get charge to flow through the gas in the ball. The voltage is very high but the resulting current is very low. This makes it safe to touch.

2.2 Ammeter

An ammeter is an instrument used to measure the rate of flow of electric current in a circuit. Since one is interested in measuring the current flowing through a circuit component, the ammeter must be connected in series with the measured circuit component.



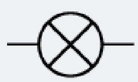
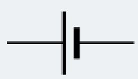
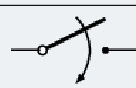
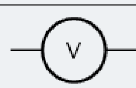
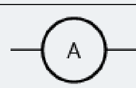
Ammeter



ACTIVITY

Constructing circuits

Construct circuits to measure the emf and the terminal potential difference for a battery. Some common elements (components) which can be found in electrical circuits include light bulbs, batteries, connecting leads, switches, resistors, voltmeters and ammeters. You have learnt about many of these already. Below is a table with the items and their symbols:

Component	Symbol	Usage
light bulb		glows when charge moves through it
battery		provides energy for charge to move
switch		allows a circuit to be open or closed
resistor		resists the flow of charge
	OR	
		
voltmeter		measures potential difference
ammeter		measures current in a circuit
connecting lead		connects circuit elements together

Experiment with different combinations of components in the circuits.

The table below summarises the use of each measuring instrument that we discussed and the way it should be connected to a circuit component.

TIP

A battery **does not** produce the same amount of current no matter what is connected to it. While the voltage produced by a battery is constant, the amount of current supplied depends on the circuit.

Instrument	Measured Quantity	Proper Connection
Voltmeter	Voltage	In Parallel
Ammeter	Current	In Series

ACTIVITY

Using meters

If possible, connect meters in circuits to get used to the use of meters to measure electrical quantities. If the meters have more than one scale, always connect to the first so that the meter will not be damaged by having to measure values that exceed its limits.

WORKED EXAMPLE 1: CALCULATING CURRENT I

QUESTION

An amount of charge, of 45 C, moves past a point in a circuit in 1 second, what is the current in the circuit?

SOLUTION

Step 1: Analyse the question

We are given an amount of charge and a time and asked to calculate the current. We know that current is the rate at which charge moves past a fixed point in a circuit so we have all the information we need. We have quantities in the correct units already.

Step 2: Apply the principles

We know that:

$$I = \frac{Q}{\Delta t}$$
$$I = \frac{45 \text{ C}}{1 \text{ s}}$$
$$I = 45 \text{ C}\cdot\text{s}^{-1}$$
$$I = 45 \text{ A}$$

Step 3: Quote the final result

The current is 45 A.

WORKED EXAMPLE 2: CALCULATING CURRENT II

QUESTION

An amount of charge, of 53 C, moves past a fixed point in a circuit in 2 s, what is the current in the circuit?

SOLUTION

Step 1: Analyse the question

We are given an amount of charge and a time and asked to calculate the current. We know that current is the rate at which charge moves past a fixed point in a circuit so we have all the information we need. We have quantities in the correct units already.

Step 2: Apply the principles

We know that:

$$I = \frac{Q}{\Delta t}$$
$$I = \frac{53 \text{ C}}{2 \text{ s}}$$
$$I = 26,5 \text{ C} \cdot \text{s}^{-1} \text{ C·s}^{-1}$$
$$I = 26,5 \text{ A}$$

Step 3: Quote the final result

The current is $26,5$ A.

WORKED EXAMPLE 3: CALCULATING CURRENT III

QUESTION

95 electrons move past a fixed point in a circuit in one tenth of a second, what is the current in the circuit?

SOLUTION

Step 1: Analyse the question

We are given a number of charged particles that move past a fixed point and the time that it takes. We know that current is the rate at which charge moves past a fixed point in a circuit so we have to determine the charge. In the last chapter we learnt that the charge carried by an electron is $1,6$×10–19 C.

Step 2: Apply the principles: determine the charge

We know that each electron carries a charge of $1,6$×10–19 C, therefore the total charge is:

$$Q = 95 \times 1,6 \times 10^{-19} \text{ C}$$
$$= 1,52 \times 10^{-17} \text{ C}$$

WORKED EXAMPLE 3: CALCULATING CURRENT III (continued)

Step 3: Apply the principles: determine the charge We know that:

$$I = \frac{Q}{\Delta t}$$
$$I = \frac{1,52 \times 10^{-17}\ \text{C}}{\frac{1}{10}\ \text{s}}$$
$$I = \frac{1,52 \times 10^{-17}\ \text{C}}{1} \times \frac{1}{\frac{1}{10}\ \text{s}}$$
$$I = \frac{1,52 \times 10^{-17}\ \text{C}}{1} \times \frac{10}{1\ \text{s}}$$
$$I = 1,52 \times 10^{-16}\ \text{C·s}^{-1}$$
$$I = 1,52 \times 10^{-16}\ \text{A}$$

Step 4: Quote the final result

The current is $1,52 \times 10^{-16}$ A.

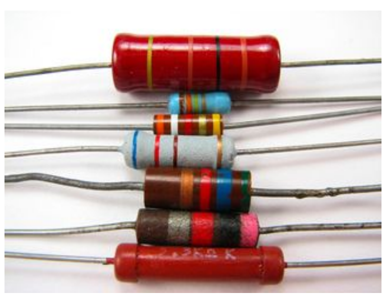
3 RESISTANCE

Resistance is a measure of "how hard" it is to "push" electricity through a circuit element. Resistance can also apply to an entire circuit.

3.1 What causes resistance?

On a microscopic level, electrons moving through the conductor collide (or interact) with the particles of which the conductor (metal) is made. When they collide, they transfer kinetic energy. The electrons therefore lose kinetic energy and slow down. This leads to resistance. The transferred energy causes the resistor to heat up. You can feel this directly if you touch a cellphone charger when you are charging a cell phone - the charger gets warm because its circuits have some resistors in them!

Examples of resistors



DEFINITION

Resistance Resistance slows down the flow of charge in a circuit. The unit of resistance is the ohm (Ω) which is defined as a volt per ampere of current.

Quantity: Resistance R Unit: ohm Unit symbol: ⍰

$$1 \text{ ohm} = 1 \frac{\text{volt}}{\text{ampere}}$$

DID YOU KNOW?

Fluorescent lightbulbs do not use thin wires; they use the fact that certain gases glow when a current flows through them. They are much more efficient (much less resistance) than lightbulbs.

Light bulb filament

All conductors have some resistance. For example, a piece of wire has less resistance than a light bulb, but both have resistance.

A lightbulb is a very thin wire surrounded by a glass housing The high resistance of the small wire (filament) in a lightbulb causes the electrons to transfer a lot of their kinetic energy in the form of heat. The heat energy is enough to cause the filament to glow white-hot which produces light.

The wires connecting the lamp to the cell or battery hardly even get warm while conducting the same amount of current. This is because of their much lower resistance due to their larger cross-section (they are thicker).

An important effect of a resistor is that it converts electrical energy into other forms of heat energy. Light energy is a by-product of the heat that is produced.

DID YOU KNOW?

There is a special type of conductor, called a **superconductor** that has no resistance, but the materials that make up all known superconductors only start superconducting at very low temperatures. The "highest" temperature superconductor is mercury barium calcium copper oxide ($HgBa_2Ca_2Cu_3O_x$) which is superconducting for temperatures of $-140°$C and colder.

Physical attributes affecting resistance [NOT IN CAPS]

The physical attributes of a resistor affect its total resistance.

- **Length**: if a resistor is increased in length its resistance will increase. Typically if you increase the length of a resistor by a certain factor you will increase the resistance by the same factor.
- **Width and height or cross-sectional area**: if a resistor provides a larger pathway by being made wider or broader then more current can flow through it. If the total surface area through which current flows (cross-sectional area) is increased by a factor the resistance typically decreases by the same factor.

Extension: For a single resistor this can be summarised as

$$R \propto \frac{L}{A}$$

where **L** is the length and **A** is the cross-sectional area.

Why do batteries go flat?

A battery stores chemical potential energy. When it is connected in a circuit, a chemical reaction takes place inside the battery which converts chemical potential energy to electrical energy which powers the electrons to move through the circuit. All the circuit elements (such as the conducting leads, resistors and lightbulbs) have some resistance to the flow of charge and convert the electrical energy to heat and, in the case of the lightbulb, light. Since energy is always conserved, the battery goes flat when all its chemical potential energy has been converted into other forms of energy.

3.2 Resistors in electric circuits

It is important to understand what effect adding resistors to a circuit has on the total resistance of a circuit and on the current that can flow in the circuit.

4 SERIES RESISTORS

When we add resistors in series to a circuit:

- There is only one path for current to flow which ensures that the **current is the same at every point in the circuit**.
- The voltage is **divided** across the resistors. The voltage across the battery in the circuit is equal to the sum of voltages across the series resistors:

$$V_{battery} = V_1 + V_2 + \ldots$$

- The resistance to the flow of current **increases**. The total resistance, R_S, is given by:

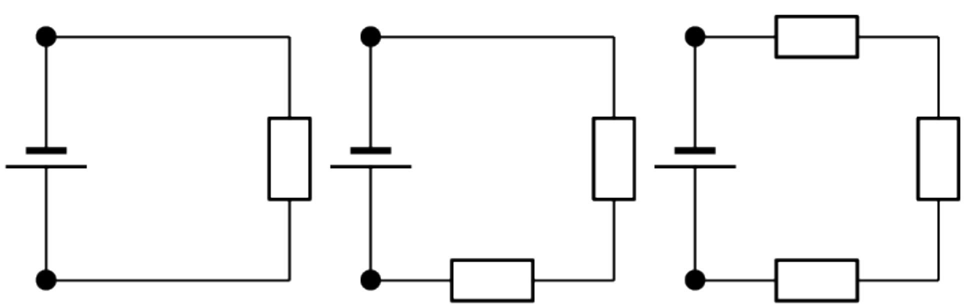
$$R_S = R_1 + R_2 + \ldots$$

We will revisit each of these features of series circuits in more detail below.

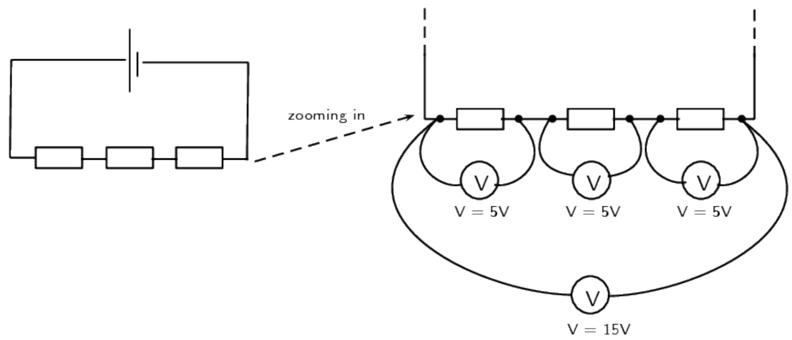
When resistors are in series, one after the other, there is a potential difference across each resistor. The total potential difference across a set of resistors in series is the sum of the potential differences across each of the resistors in the set.

$$V_{\text{battery}} = V_1 + V_2 + \ldots$$

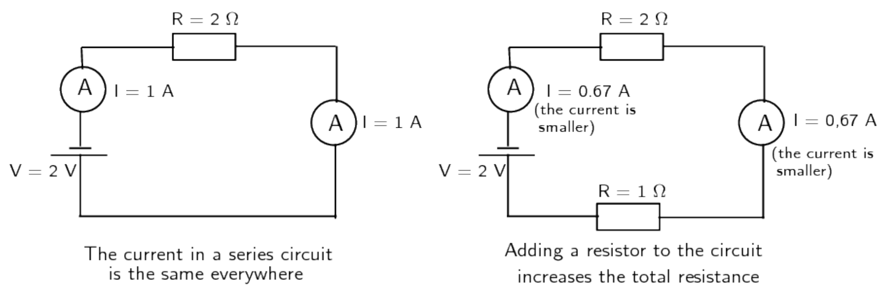
Look at the circuits below. If we measured the potential difference between the black dots in all of these circuits it would be the same as we saw earlier. So we now know the total potential difference is the same across one, two or three resistors. We also know that some work is required to make charge flow through each resistor.



Let us look at this in a bit more detail. In the picture below you can see what the different measurements for 3 identical resistors in series could look like. The total voltage across all three resistors is the sum of the voltages across the individual resistors. Resistors in series are known as **voltage dividers** because the total voltage across all the resistors is divided amongst the individual resistors.



Consider the diagram below. On the left there is a circuit with a single resistor and a battery. **No matter where we measure the current, it is the same in a series circuit**. On the right, we have added a second resistor in series to the circuit. The total resistance of the circuit has increased and you can see from the reading on the ammeter that the current in the circuit has decreased.



GENERAL EXPERIMENT

Voltage dividers

Aim

Test what happens to the current and the voltage in series circuits when additional resistors are added.

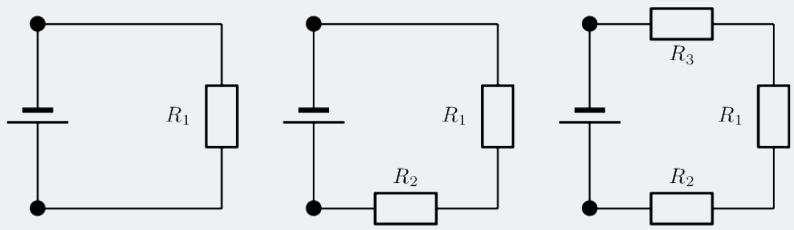
Apparatus

- A battery
- A voltmeter
- An ammeter
- Wires
- Resistors

GENERAL EXPERIMENT (continued)

Method

- Construct each circuit shown below
- Measure the voltage across each resistor in the circuit.
- Measure the current before and after each resistor in the circuit.



Results and conclusions

- Compare the sum of the voltages across all the resistors in each of the circuits.
- Compare the various current measurements within the same circuit.

The total resistance is equal to R_1 in the first circuit, to $R_1 + R_2$ in the second circuit and $R_1 + R_2 + R_3$ in the third circuit. In general, for resistors in series, the total resistance is given by:

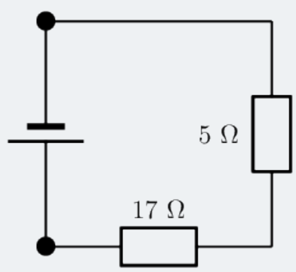
$$R_S = R_1 + R_2 + \ldots$$

We know that the potential energy lost across a resistor is proportional to the resistance of the component because the higher the resistance the more work must be done to drive charge through the resistor.

WORKED EXAMPLE 4: SERIES RESISTORS I

QUESTION

A circuit contains two resistors in series. The resistors have resistance values of $5\ \Omega$ and $17\ \Omega$.



What is the total resistance in the circuit?

WORKED EXAMPLE 4: SERIES RESISTORS I (continued)

SOLUTION

Step 1: Analyse the question

We are told that the circuit is a series circuit and that we need to calculate the total resistance. The values of the two resistors have been given in the correct units, Ω.

Step 2: Apply the relevant principles

The total resistance for resistors in series is the sum of the individual resistances. We can use

$$R_S = R_1 + R_2 + \ldots$$

We have only two resistors and we now the resistances. In this case we have that:

$$\begin{aligned} R_S &= R_1 + R_2 + \ldots \\ R_S &= R_1 + R_2 \\ &= 5\ \Omega + 17\ \Omega \\ &= 22\ \Omega \end{aligned}$$

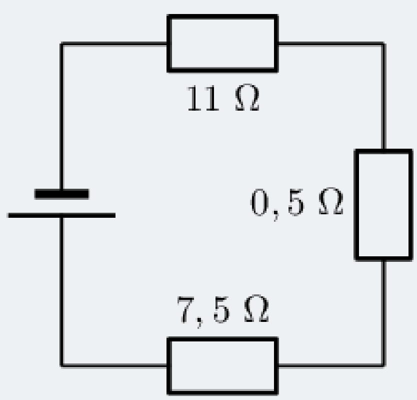
Step 3: Quote the final result

The total resistance of the resistors in series is $22\ \Omega$

WORKED EXAMPLE 5: SERIES RESISTORS II

QUESTION

A circuit contains three resistors in series. The resistors have resistance values of $0,5\ \Omega$, $7,5\ \Omega$ and $11\ \Omega$.



What is the total resistance in the circuit?

WORKED EXAMPLE 5: SERIES RESISTORS II (continued)

SOLUTION

Step 1: Analyse the question

We are told that the circuit is a series circuit and that we need to calculate the total resistance. The values of the three resistors have been given in the correct units, Ω.

Step 2: Apply the relevant principles

The total resistance for resistors in series is the sum of the individual resistances. We can use

$$R_S = R_1 + R_2 + \ldots$$

We have three resistors and we now the resistances. In this case we have that:

$$\begin{aligned} R_S &= R_1 + R_2 + \ldots \\ R_S &= R_1 + R_2 + R_3 \\ &= 0,5\,\Omega + 7,5\,\Omega + 11\,\Omega \\ &= 19\,\Omega \end{aligned}$$

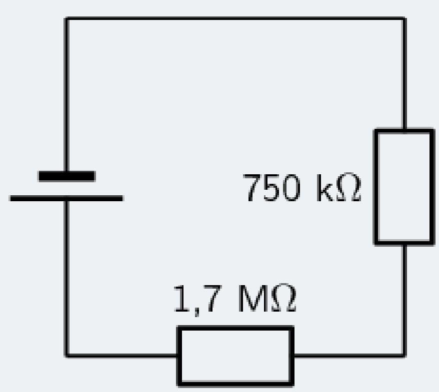
Step 3: Quote the final result

The total resistance of the resistors in series is $19\,\Omega$

WORKED EXAMPLE 6: SERIES RESISTORS III

QUESTION

A circuit contains two resistors in series. The resistors have resistance values of 750 kΩ and $1,7$ MΩ.



What is the total resistance in the circuit?

WORKED EXAMPLE 6: SERIES RESISTORS III (continued)

SOLUTION

Step 1: Analyse the question

We are told that the circuit is a series circuit and that we need to calculate the total resistance. The values of the two resistors have been given in the correct units, Ω.

Step 2: Apply the relevant principles

The total resistance for resistors in series is the sum of the individual resistances. We can use

$$R_S = R_1 + R_2 + \ldots$$

We have only two resistors and we know the resistances. In this case we have that:

$$\begin{aligned} R_S &= R_1 + R_2 + \ldots \\ R_S &= R_1 + R_2 \\ &= 750 \text{ k}\Omega + 1,7 \text{ M}\Omega \\ &= 750 \times 10^3 \ \Omega + 1,7 \times 10^6 \ \Omega \\ &= 0,75 \times 10^6 \ \Omega + 1,7 \times 10^6 \text{ M}\Omega \\ &= 2,45 \times 10^6 \ \Omega \\ &= 2,45 \text{ M}\Omega \end{aligned}$$

Step 3: Quote the final result

The total resistance of the resistors in series is $2,45$ MΩ.

5 PARALLEL RESISTORS

When we add resistors in parallel to a circuit:

- There are more paths for current to flow which ensures that the **current splits across the different paths**.
- The voltage is **the same** across the resistors. The voltage across the battery in the circuit is equal to the voltage across each of the parallel resistors:

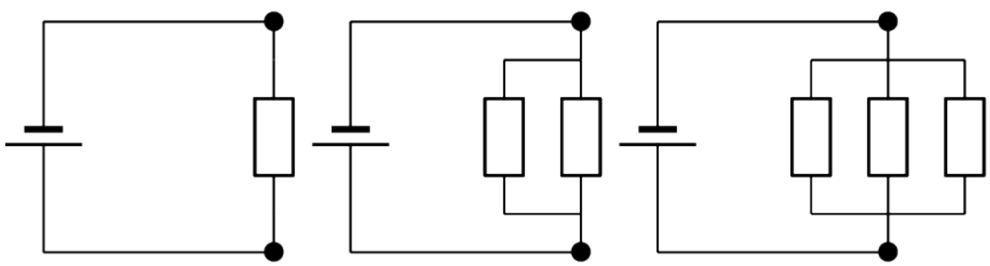
$$V_{\text{battery}} = V_1 = V_2 = V_3 \ldots$$

- The resistance to the flow of current **decreases**. The total resistance, RP, is given by:

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

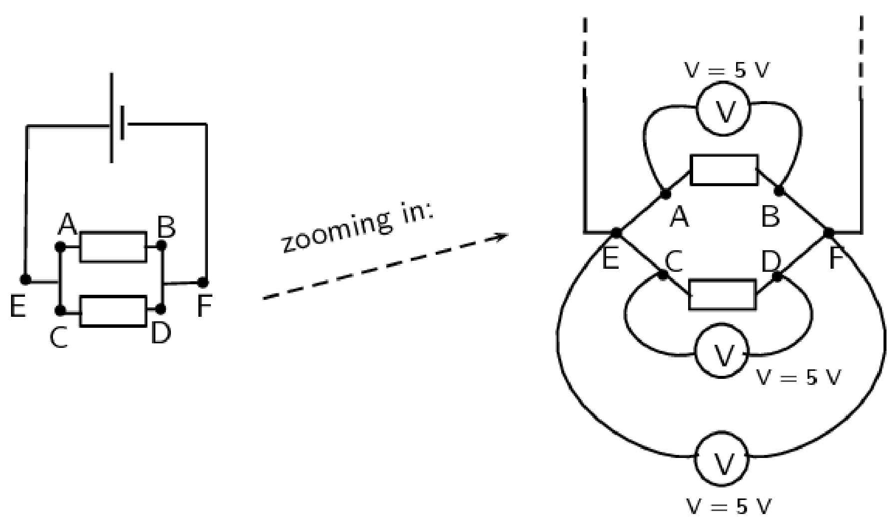
When resistors are connected in parallel the start and end points for all the resistors are the same. These points have the same potential energy and so the potential difference between them is the same no matter what is put in between them. You can have one, two or many resistors between the two points, the potential difference will not change. You can ignore whatever components are between two points in a circuit when calculating the difference between the two points.

Look at the following circuit diagrams. The battery is the same in all cases, all that changes is more resistors are added between the points marked by the black dots. If we were to measure the potential difference between the two dots in these circuits we would get the same answer for all three cases.



Lets look at two resistors in parallel more closely. When you construct a circuit you use wires and you might think that measuring the voltage in different places on the wires will make a difference. This is not true. The potential difference or voltage measurement will only be different if you measure a different set of components. All points on the wires that have no circuit components between them will give you the same measurements.

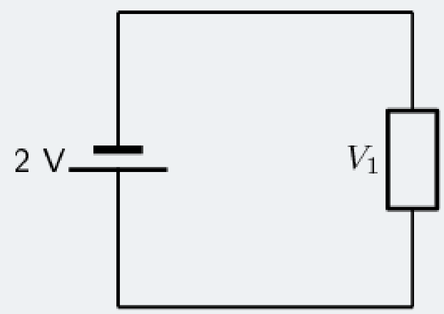
All three of the measurements shown in the picture below (i.e. A–B, C–D and E–F) will give you the same voltage. The different measurement points on the left have no components between them so there is no change in potential energy. Exactly the same applies to the different points on the right. When you measure the potential difference between the points on the left and right you will get the same answer.



WORKED EXAMPLE 7: VOLTAGES I

QUESTION

Consider this circuit diagram:



What is the voltage across the resistor in the circuit shown?

SOLUTION

Step 1: Check what you have and the units

We have a circuit with a battery and one resistor. We know the voltage across the battery. We want to find that voltage across the resistor.

$$V_{\text{battery}} = 2 \text{ V}$$

Step 2: Applicable principles

We know that the voltage across the battery must be equal to the total voltage across all other circuit components.

$$V_{\text{battery}} = V_{\text{total}}$$

There is only one other circuit component, the resistor.

$$V_{\text{total}} = V_1$$

This means that the voltage across the battery is the same as the voltage across the resistor.

$$V_{\text{battery}} = V_{\text{total}} = V_1$$
$$V_1 = 2 \text{ V}$$

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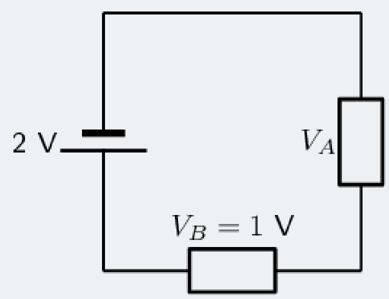
WORKED EXAMPLE 8: VOLTAGES II

QUESTION

Consider this circuit diagram:



What is the voltage across the unknown resistor in the circuit shown?

SOLUTION

Step 1: Check what you have and the units

We have a circuit with a battery and two resistors. We know the voltage across the battery and one of the resistors. We want to find that voltage across the resistor.

$$V_{\text{battery}} = 2 \text{ V}$$
$$V_B = 1 \text{ V}$$

Step 2: Applicable principles

We know that the voltage across the battery must be equal to the total voltage across all other circuit components that are in series

$$V_{\text{battery}} = V_{\text{total}}$$

he total voltage in the circuit is the sum of the voltages across the individual resistors

$$V_{\text{total}} = V_A + V_B$$

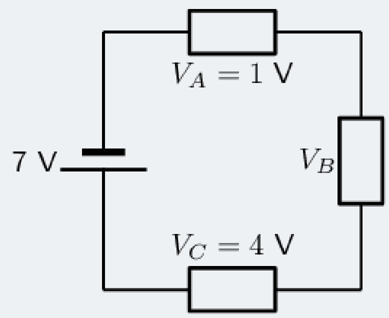
Using the relationship between the voltage across the battery and total voltage across the resistors

$$V_{\text{battery}} = V_{\text{total}}$$
$$V_{\text{battery}} = V_1 + V_{\text{resistor}}$$
$$2 \text{ V} = V_1 + 1 \text{ V}$$
$$V_1 = 1 \text{ V}$$

WORKED EXAMPLE 9: VOLTAGES III

QUESTION

Consider this circuit diagram:



What is the voltage across the resistor in the circuit shown?

SOLUTION

Step 1: Check what you have and the units

We have a circuit with a battery and three resistors. We know the voltage across the battery and two of the resistors. We want to find that voltage across the unknown resistor.

$$V_{\text{battery}} = 7\text{ V}$$
$$V_{\text{known}} = V_A + V_C$$
$$= 1\text{ V} + 4\text{ V}$$

Step 2: Applicable principles

We know that the voltage across the battery must be equal to the total voltage across all other circuit components.

$$V_{\text{battery}} = V_{\text{total}}$$

The total voltage in the circuit is the sum of the voltages across the individual resistors

$$V_{\text{total}} = V_B + V_{\text{known}}$$

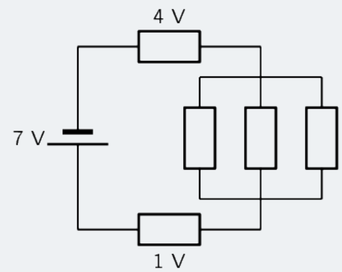
Using the relationship between the voltage across the battery and total voltage across the resistors

$$V_{\text{battery}} = V_{\text{total}}$$
$$V_{\text{battery}} = V_B + V_{\text{known}}$$
$$7\text{ V} = V_B + 5\text{ V}$$
$$V_B = 2\text{ V}$$

WORKED EXAMPLE 10: VOLTAGES IV

QUESTION

Consider this circuit diagram:



What is the voltage across the parallel resistor combination in the circuit shown?

Hint: the rest of the circuit is the same as the previous problem.

SOLUTION

Step 1: Quick Answer

The circuit is the same as the previous example and we know that the voltage difference between two points in a circuit does not depend on what is between them so the answer is the same as above $V_{parallel}$ = 2 V.

Step 2: Check what you have and the units - long answer

We have a circuit with a battery and five resistors (two in series and three in parallel). We know the voltage across the battery and two of the resistors. We want to find that voltage across the parallel resistors, $V_{parallel}$.

$$\begin{aligned} V_{\text{battery}} &= 7 \text{ V} \\ V_{\text{known}} &= 1 \text{ V} + 4 \text{ V} \end{aligned}$$

Step 3: Applicable principles

We know that the voltage across the battery must be equal to the total voltage across all other circuit components.

$$V_{\text{battery}} = V_{\text{total}}$$

Voltages only add algebraically for components in series. The resistors in parallel can be thought of as a single component which is in series with the other components and then the voltages can be added.

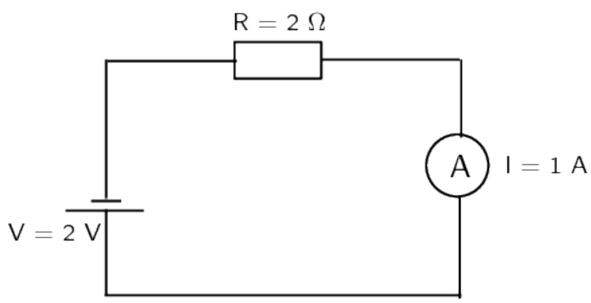
$$V_{\text{total}} = V_{\text{parallel}} + V_{\text{known}}$$

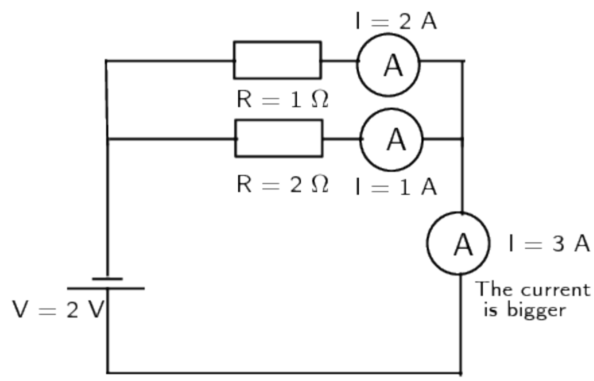
Using the relationship between the voltage across the battery and total voltage across the resistors

$$\begin{aligned} V_{\text{battery}} &= V_{\text{total}} \\ V_{\text{battery}} &= V_{\text{parallel}} + V_{\text{known}} \\ 7 \text{ V} &= V_{\text{parallel}} + 5 \text{ V} \\ V_{\text{parallel}} &= 2 \text{ V} \end{aligned}$$

In contrast to the series case, when we add resistors in parallel, we create more paths along which current can flow. By doing this we decrease the total resistance of the circuit!

Take a look at the diagram below. On the left we have the same circuit as in the previous section with a battery and a resistor. The ammeter shows a current of 1 A. On the right we have added a second resistor in parallel to the first resistor. This has increased the number of paths (branches) the charge can take through the circuit - the total resistance has decreased. You can see that the current in the circuit has increased. Also notice that the current in the different branches can be different.



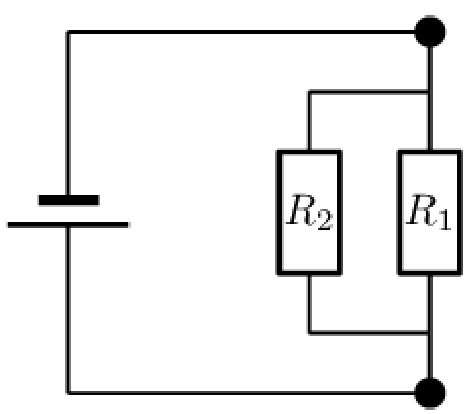


Adding a resistor to the circuit in parallel decreases the total resistance

The total resistance of a number of parallel resistors is NOT the sum of the individual resistances as the overall resistance decreases with more paths for the current. The total resistance for parallel resistors is given by:

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

Let us consider the case where we have two resistors in parallel and work out what the final resistance would be. This situation is shown in the diagram below:



Applying the formula for the total resistance we have:

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

There are only two resistors:

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$$

Add the fractions:

$$\frac{1}{R_P} = \frac{1}{R_1} \times \frac{R_2}{R_2} + \frac{1}{R_2} \times \frac{R_1}{R_1}$$

$$\frac{1}{R_P} = \frac{R_2}{R_1 R_2} + \frac{R_1}{R_1 R_2}$$

Reaarange:

$$\frac{1}{R_P} = \frac{R_2 + R_1}{R_1 R_2}$$

$$\frac{1}{R_P} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_P = \frac{R_1 R_2}{R_1 + R_2}$$

For any two resistors in parallel, we know that:

$$R_P = \frac{\text{product of resistances}}{\text{sum of resistances}} = \frac{R_1 R_2}{R_1 + R_2}$$

GENERAL EXPERIMENT

Current dividers

Aim

Test what happens to the current and the voltage in series circuits when additional resistors are added.

Apparatus

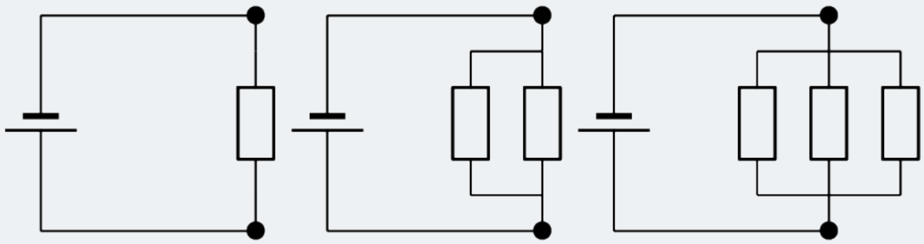
- A battery
- A voltmeter
- An ammeter
- Wires
- Resistors

Method

- Connect each circuit shown below
- Measure the voltage across each resistor in the circuit.

GENERAL EXPERIMENT (continued)

- Measure the current before and after each resistor in the circuit and before and after the parallel branches.



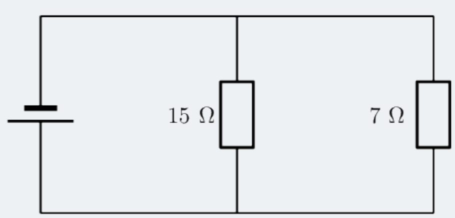
Results and conclusions

- Compare the currents through individual resistors with each other.
- Compare the sum of the currents through individual resistors with the current before the parallel branches.
- Compare the various voltage measurements across the parallel resistors.

WORKED EXAMPLE 11: PARALLEL RESISTORS I

QUESTION

A circuit contains two resistors in parallel. The resistors have resistance values of $15\ \Omega$ and $7\ \Omega$.



What is the total resistance in the circuit?

SOLUTION

Step 1: Analyse the question

We are told that the resistors in the circuit are in parallel circuit and that we need to calculate the total resistance. The values of the two resistors have been given in the correct units, Ω.

WORKED EXAMPLE 11: PARALLEL RESISTORS I (continued)

Step 2: Apply the relevant principles

The total resistance for resistors in parallel has been shown to be the product of the resistances divided by the sum. We can use

$$R_P = \frac{R_1 R_2}{R_1 + R_2}$$

We have only two resistors and we now the resistances. In this case we have that:

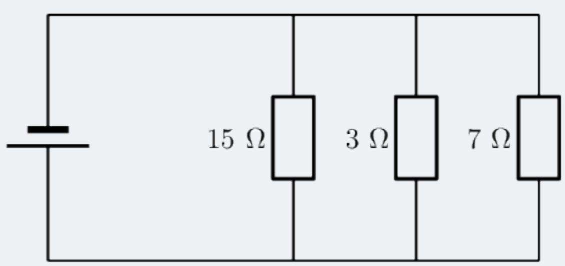
$$\begin{aligned} R_P &= \frac{R_1 R_2}{R_1 + R_2} \\ &= \frac{(15\ \Omega)(7\ \Omega)}{15\ \Omega + 7\ \Omega} \\ &= \frac{105\Omega^2}{22\ \Omega} \\ &= 4,77\ \Omega \end{aligned}$$

Step 3: Quote the final result

The total resistance of the resistors in parallel is $4,77\ \Omega$

WORKED EXAMPLE 12: PARALLEL RESISTORS II

QUESTION We add a third parallel resistor to the configuration (setup) in the previous example. The additional resistor has a resistance of $3\ \Omega$.



What is the total resistance in the circuit?

SOLUTION

Step 1: Analyse the question

We are told that the resistors in the circuit are in parallel circuit and that we need to calculate the total resistance. The values of the two resistors have been given in the correct units, Ω.

WORKED EXAMPLE 12: PARALLEL RESISTORS II (continued)

Step 2: Apply the relevant principles

The total resistance for resistors in parallel has been given as

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

We have three resistors and we now the resistances. In this case we have that:

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

There are three resistors:

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Add the fractions:

$$\frac{1}{R_P} = \frac{1}{R_1} \times \frac{R_2R_3}{R_2R_3} + \frac{1}{R_2} \times \frac{R_1R_3}{R_1R_3} + \frac{1}{R_3} \times \frac{R_1R_2}{R_1R_2}$$

$$\frac{1}{R_P} = \frac{R_2R_3}{R_1R_2R_3} + \frac{R_1R_3}{R_1R_2R_3} + \frac{R_1R_2}{R_1R_2R_3}$$

Rearrange:

$$\frac{1}{R_P} = \frac{R_2R_3 + R_1R_3 + R_2R_3}{R_1R_2R_3}$$

$$R_P = \frac{R_1R_2R_3}{R_2R_3 + R_1R_3 + R_2R_3}$$

$$R_P = \frac{(15\Omega)(7\Omega)(3\Omega)}{(7\Omega)(3\Omega) + (15\Omega)(3\Omega) + (7\Omega)(15\Omega)}$$

$$R_P = \frac{315\Omega^3}{21\Omega^2 + 45\Omega^2 + 105\Omega^2}$$

$$R_P = \frac{315\Omega^3}{171\Omega^2}$$

$$R_P = 1,84\Omega$$

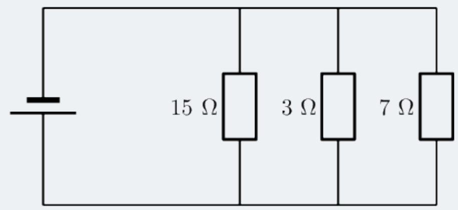
Step 3: Quote the final result

The total resistance of the resistors in parallel is $1,84$ Ω

WORKED EXAMPLE 13: PARALLEL RESISTORS III

QUESTION

We add a third parallel resistor to the first parallel worked example configuration (setup). The additional resistor has a resistance of $3\ \Omega$.



What is the total resistance in the circuit?

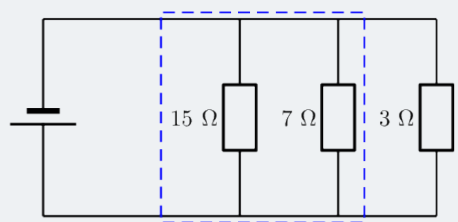
SOLUTION

Step 1: Analyse the question

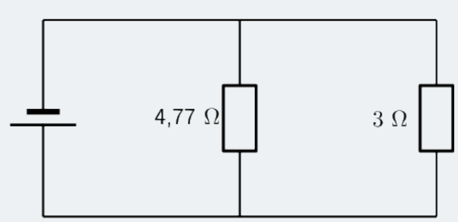
We are told that the resistors in the circuit are in parallel circuit and that we need to calculate the total resistance. The value of the additional resistor has been given in the correct units, Ω.

Step 2: Apply the relevant principles

We can swap the resistors around without changing the circuit:



We have already calculated the total resistance of the two resistors in the dashed box to be $4,77\ \Omega$. We can replace these two resistors with a single resistor of $4,77\ \Omega$ to get:



WORKED EXAMPLE 13: PARALLEL RESISTORS III (continued)

Step 3: Calculate the total resistance for the next pair of resistors

Then we use the formula for two parallel resistors again to get the total resistance for this new circuit:

$$\begin{aligned} R_P &= \frac{R_1 R_2}{R_1 + R_2} \\ &= \frac{(4,77\ \Omega)(3\ \Omega)}{4,77\ \Omega + 3\ \Omega} \\ &= \frac{14,31\Omega^2}{11,77\Omega} \\ &= 1,84\ \Omega \end{aligned}$$

Step 4: Quote the final result

The total resistance of the resistors in parallel is $1,84\ \Omega$. This is the same result as when we added all three resistors together at once.

6 CHAPTER SUMMARY

- The potential difference across the terminals of a battery when it is **not** in a complete circuit is the electromotive force (emf) measured in volts (V).
- The potential difference across the terminals of a battery when it is in a complete circuit is the terminal potential difference measured in volts (V).
- Voltage is a measure of required/done to move a certain amount of charge and is equivalent to J·C^{-1}.
- Current is the rate at which charge flows and is measured in amperes (A) which is equivalent to C·s^{-1}.
- Conventional current flows from the positive terminal of a battery, through a circuit, to the negative terminal.
- Ammeters measure current and must be connected in series.
- Voltmeters measure potential difference (voltage) and must be connected in parallel.
- Resistance is a measure of how much work must be done for charge to flow through a circuit element and is measured in ohms (Ω) and is equivalent to V·A^{-1}.
- Resistance of circuit elements is related to the material from which they are made as well as the physical characteristics of length and cross-sectional area.
- Current is constant through resistors in series and they are called voltage dividers as the sum of the voltages is equal to the voltage across the entire set of resistors.

- The total resistance of resistors in series is the sum of the individual resistances, $R_S = R_1 + R_2 + \ldots$
- Voltage is constant across resistors in parallel and they are called current divides because the sum of the current through each is the same as the total current through the circuit configuration.
- The total resistance of resistors in parallel is calculated by using $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$ which is $R_P = \frac{R_1 R_2}{R_1 + R_2}$ for two parallel resistors.

Physical Quantities		
Quantity	**Unit name**	**Unit symbol**
Potential difference (VV)	volt	VV
emf	volt	VV
Voltage (VV)	volt	VV
Current (II)	ampere	AA
Resistance (RR)	ohm	Ω

7 EXERCISES

7.1 Exercise 1

1. What is the unit of resistance called and what is its symbol?
2. Explain what happens to the total resistance of a circuit when resistors are added in series?
3. Explain what happens to the total resistance of a circuit when resistors are added in parallel?
4. Why do batteries go flat?

8 ANSWERS TO EXERCISES

8.1 Exercise 1

1. Unit = ohm
   Symbol = Ω
2. It increases the more you add resistors.
3. Decreases
4. Batteries are made up of chemicals. These chemicals react with each other to convert the chemical energy to electric energy. When all the chemical substrates have reacted, i.e. the substrates are depleted, the battery is flat.