CHAPTER 22

Mechanical Energy

CONTENTS

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1 INTRODUCTION

All objects have energy. The word energy comes from the Greek word energeia, meaning activity or operation. Energy is closely linked to mass and cannot be created or destroyed. In this chapter we will consider gravitational potential and kinetic energy.

2 POTENTIAL ENERGY

The potential energy of an object is generally defined as the energy an object has because of its position relative to other objects that it interacts with. There are different kinds of potential energy such as gravitational potential energy, chemical potential energy, electrical potential energy, to name a few. In this section we will be looking at gravitational potential energy.

DEFINITION

Potential energy

Potential energy is the energy an object has due to its position or state.

DEFINITION

Gravitational potential energy

Gravitational potential energy is the energy an object has due to its position in a gravitational field relative to some reference point.

Quantity: Gravitational potential energy (E_P) Unit name: Joule Unit symbol: J

In the case of Earth, *gravitational* potential energy is the energy of an object due to its position above the surface of the Earth. The symbol E_P is used to refer to gravitational potential energy. You will often find that the words potential energy are used where *gravitational potential energy* is meant. We can define gravitational potential energy as:

$$E_P = mgh$$

Where:

$E_P =$ potential energy (measured in joules, J)

$m =$ mass of the object (measured in kg)

$g =$ gravitational acceleration $\left(9,8 \text{ m} \cdot \text{s}^{-2}\right)$

$h =$ perpendicular height from the reference point (measured in m)

TIP

You may sometimes see potential energy written as PE. We will not use this notation in this book, but you may see it in other books.

You can treat gravitational acceleration, g, as a constant and you will learn more about it in Grade 11 and 12.

Let's look at the case of a suitcase, with a mass of 1 kg, which is placed at the top of a 2 m high cupboard. By lifting the suitcase against the force of gravity, we give the suitcase potential energy. We can calculate its gravitational potential energy using the equation defined above as:

$$\begin{aligned} E_P &= mgh \\ &= (1 \text{ kg}) \left(9,8 \text{ m} \cdot \text{s}^{-2}\right) (2 \text{ m}) = 19,6 \text{ J} \end{aligned}$$

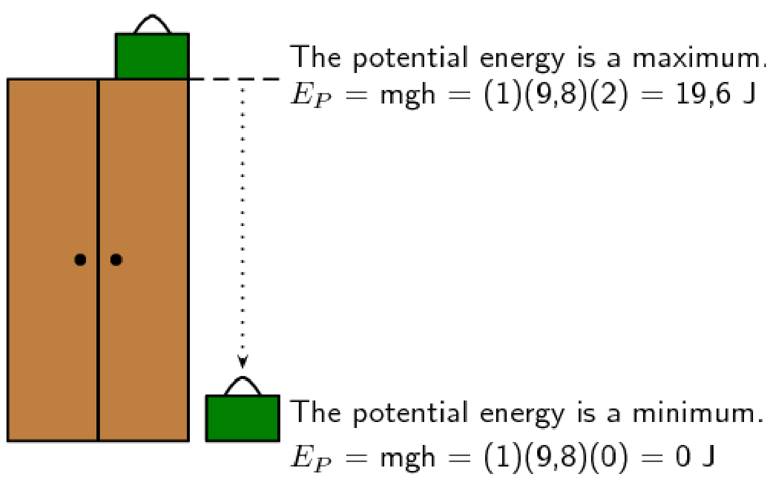
If the suitcase falls off the cupboard, it will lose its potential energy. Halfway down to the floor, the suitcase will have lost half its potential energy and will have only $9,8$ J left.

$$\begin{aligned} E_P &= mgh \\ &= (1 \text{ kg}) \left(9,8 \text{ m} \cdot \text{s}^{-2}\right) (1 \text{ m}) = 9,8 \text{ J} \end{aligned}$$

At the bottom of the cupboard the suitcase will have lost all its potential energy and its potential energy will be equal to zero.

$$\begin{aligned} E_P &= mgh \\ &= (1 \text{ kg}) \left(9,8 \text{ m} \cdot \text{s}^{-2}\right) (0 \text{ m}) = 0 \text{ J} \end{aligned}$$

This example shows us that objects have maximum potential energy at a maximum height and will lose their potential energy as they fall.



WORKED EXAMPLE 1: GRAVITATIONAL POTENTIAL ENERGY

QUESTION

A brick with a mass of 1 kg is lifted to the top of a 4 m high roof. It slips off the roof and falls to the ground. Calculate the gravitational potential energy of the brick at the top of the roof and on the ground once it has fallen.

SOLUTION

Step 1: Analyse the question to determine what information is provided

- The mass of the brick is $m = 1$ kg
- The height lifted is $h = 4$ m

All quantities are in SI units.

Step 2: Analyse the question to determine what is being asked

- We are asked to find the gain in potential energy of the brick as it is lifted onto the roof.
- We also need to calculate the potential energy once the brick is on the ground again.

Step 3: Use the definition of gravitational potential energy to calculate the value the potential energy of the brick at its highest point.

$$\begin{aligned} E_P &= mgh \\ &= (1 \text{ kg}) \left(9,8 \text{ m} \cdot \text{s}^{-2}\right) (4 \text{ m}) = 39,2 \text{ J} \end{aligned}$$

Step 4: Calculate the potential energy of the brick at its lowest point.

Again we use the definition of gravitational potential energy to solve this:

$$\begin{aligned} E_P &= mgh \\ &= (1 \text{ kg}) \left(9,8 \text{ m} \cdot \text{s}^{-2}\right) (0 \text{ m}) = 0 \text{ J} \end{aligned}$$

WORKED EXAMPLE 2: MORE GRAVITATIONAL POTENTIAL ENERGY

QUESTION

A netball player, who is $1,7$ m tall, holds a $0,5$ kg netball $0,5$ m above her head and shoots for the goal net which is $2,5\ m$ above the ground. What is the gravitational potential energy of the ball:

1. when she is about to shoot it into the net?
2. when it gets right into the net?
3. when it lands on the ground after the goal is scored?

SOLUTION

Step 1: Analyse the question to determine what information is provided

- the netball net is $2,5$ m above the ground
- the girl has a height of $1,7$ m
- the ball is $0,5$ m above the girl's head when she shoots for goal
- the mass of the ball is $0,5$ kg

Step 2: Analyse the question to determine what is being asked

We need to find the gravitational potential energy of the netball at three different positions:

- when it is above the girl's head as she starts to throw it into the net
- when it reaches the net
- when it reaches the ground

Step 3: Use the definition of gravitational potential energy to calculate the value for the ball when the girl shoots for goal

$$E_P = mgh$$

First we need to calculate h. The height of the ball above the ground when the girl shoots for goal is:

$$h = (1,7 + 0,5) = 2,2 \text{ m}$$

Now we can use this information in the equation for gravitational potential energy:

$$\begin{aligned} E_P &= mgh \\ &= (0,5 \text{ kg}) \left(9,8 \text{ m} \cdot \text{s}^{-2}\right) (2,2 \text{ m}) \\ &= 10,78 \text{ J} \end{aligned}$$

WORKED EXAMPLE 2: MORE GRAVITATIONAL POTENTIAL ENERGY

Step 4: Calculate the potential energy of the ball at the height of the net

Again we use the definition of gravitational potential energy to solve this:

$$\begin{aligned} E_P &= mgh \\ &= (0,5 \text{ kg}) \left(9,8 \text{ m} \cdot \text{s}^{-2}\right) (2,5 \text{ m}) \\ &= 12,25 \text{ J} \end{aligned}$$

Step 5: Calculate the potential energy of the ball on the ground

$$\begin{aligned} E_P &= mgh \\ &= (0,5 \text{ kg}) \left(9,8 \text{ m} \cdot \text{s}^{-2}\right) (0 \text{ m}) \\ &= 0 \text{ J} \end{aligned}$$

3 KINETIC ENERGY

DEFINITION

Kinetic energy

Kinetic energy is the energy an object has due to its motion.

Quantity: Kinetic energy energy (E_K) Unit name: Joule Unit symbol: J

Kinetic energy is the energy an object has because of its motion. This means that any moving object has kinetic energy. Kinetic energy is defined as:

$$E_K = \frac{1}{2}mv^2$$

Where:

$E_K =$ is the kinetic energy (measured in joules, J)

$m =$ mass of the object (measured in kg)

$v =$ velocity of the object (measured in $\text{m} \cdot \text{s}^{-1}$)

Therefore the kinetic energy E_K depends on the mass and velocity of an object. The faster it moves, and the more massive it is, the more kinetic energy it has. A truck of $2\ 000$ kg, moving at 100 km $\cdot$ h^{-1} will have more kinetic energy than a car of 500 kg, also moving at 100 km $\cdot$ h^{-1}.

TIP

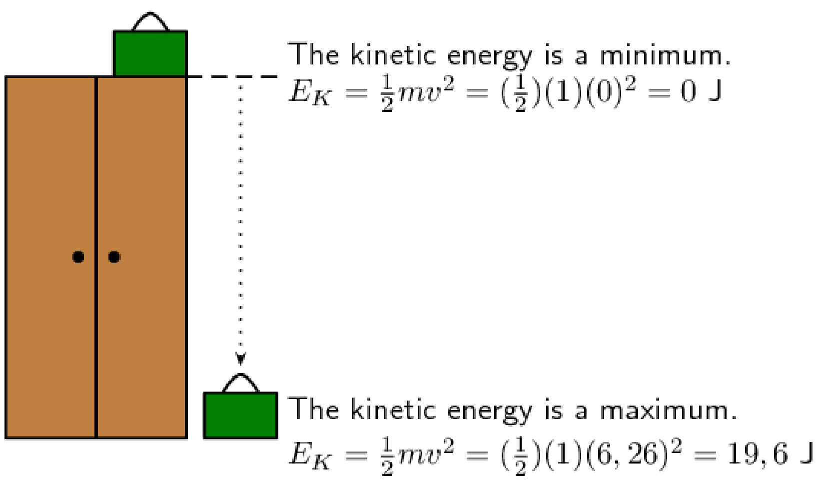
You may sometimes see kinetic energy written as KE. This is simply another way to write kinetic energy. We will not use this form in this book, but you may see it written like this in other books.

Consider the 1 kg suitcase on the cupboard that was discussed earlier. When it is on the top of the cupboard, it will not have any kinetic energy because it is not moving:

$$\begin{aligned} E_K &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(1 \text{ kg})\left(0 \text{ m} \cdot \text{s}^{-1}\right)^2 = 0 \text{ J} \end{aligned}$$

When the suitcase falls, its velocity increases (falls faster), until it reaches the ground with a maximum velocity. As its velocity increases, it will gain kinetic energy. Its kinetic energy will increase until it is a maximum when the suitcase reaches the ground. If it has a velocity of $6,26$ m$\cdot$s^{-1} when it reaches the ground, its kinetic energy will be:

$$\begin{aligned} E_K &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(1 \text{ kg})\left(6,26 \text{ m} \cdot \text{s}^{-1}\right)^2 = 19,6 \text{ J} \end{aligned}$$



WORKED EXAMPLE 3: CALCULATION OF KINETIC ENERGY

QUESTION

A 1 kg brick falls off a 4 m high roof. It reaches the ground with a velocity of $8,85\ \text{m} \cdot \text{s}^{-1}$. What is the kinetic energy of the brick when it starts to fall and when it reaches the ground?

SOLUTION

Step 1: Analyse the question to determine what information is provided

- The mass of the brick is $m = 1$ kg
- The velocity of the brick at the bottom $v = 8,85\ \text{m} \cdot \text{s}^{-1}$

These are both in the correct units so we do not have to worry about unit conversions.

Step 2: Analyse the question to determine what is being asked

We are asked to find the kinetic energy of the brick at the top and the bottom. From the definition we know that to work out E_K, we need to know the mass and velocity of the object and we are given both of these values.

Step 3: Calculate the kinetic energy at the top

Since the brick is not moving at the top, its kinetic energy is zero.

Step 4: Substitute and calculate the kinetic energy

$$\begin{aligned} E_K &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(1\ \text{kg})\left(8,85\ \text{m} \cdot \text{s}^{-1}\right)^2 = 39,2\ \text{J} \end{aligned}$$

WORKED EXAMPLE 4: KINETIC ENERGY OF 2 MOVING OBJECTS

QUESTION

A herder is herding his sheep into the kraal. A mother sheep and its lamb are both running at $2,7$ m·s^{-1} towards the kraal. The sheep has a mass of 80 kg and the lamb has a mass of 25 kg. Calculate the kinetic energy for each of the sheep and the lamb.

SOLUTION

Step 1: Analyse the question to determine what information is provided

- the mass of the mother sheep is 80 kg
- the mass of the lamb is 25 kg
- both the sheep and the lamb have a velocities of $2,7$ m · s^{-1}

Step 2: Analyse the question to determine what is being asked

We need to find the kinetic energy of the sheep and the kinetic energy of its lamb

Step 3: Use the definition to calculate the sheep's kinetic energy

$$\begin{aligned} E_K &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(80 \text{ kg}) \left(2,7 \text{ m} \cdot \text{s}^{-1}\right)^2 \\ &= 291,6 \text{ J} \end{aligned}$$

Step 4: Use the definition to calculate the lamb's kinetic energy

$$\begin{aligned} E_K &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(25 \text{ kg}) \left(2,7 \text{ m} \cdot \text{s}^{-1}\right)^2 \\ &= 91,13 \text{ J} \end{aligned}$$

Note: Even though the sheep and the lamb are running at the same velocity, due to their different masses, they have different amounts of kinetic energy. The sheep has more than the lamb because it has a higher mass.

3.1 Checking units

According to the equation for kinetic energy, the unit should be kg · m · s^{-2}. We can prove that this unit is equal to the joule, the unit for energy.

$$\begin{aligned}(\text{kg})\left(\text{m}\cdot\text{s}^{-1}\right)^2 &= \left(\text{kg}\cdot\text{m}\cdot\text{s}^{-2}\right)\cdot\text{m}\\ &= \text{N}\cdot\text{m}\left(\text{because Force (N)} = \text{mass (kg)} \times \text{acceleration}\ \left(\text{m}\cdot\text{s}^{-2}\right)\right)\\ &= \text{J}\,(\text{Work (J)} = \text{Force (N)} \times \text{distance (m)})\end{aligned}$$

We can do the same to prove that the unit for potential energy is equal to the joule:

$$\begin{aligned}(\text{kg})\left(\text{m}\cdot\text{s}^{-2}\right)(\text{m}) &= \text{N}\cdot\text{m}\\ &= \text{J}\end{aligned}$$

WORKED EXAMPLE 5: MIXING UNITS AND ENERGY CALCULATIONS

QUESTION

A bullet, having a mass of 150 g, is shot with a muzzle velocity of 960 m· s^{-1}. Calculate its kinetic energy.

SOLUTION

Step 1: Analyse the question to determine what information is provided

- We are given the mass of the bullet m = 150g. This is not the unit we want mass to be in. We need to convert to kg.

$$\begin{aligned}\text{Mass in grams} \div 1000 &= \text{Mass in kg}\\ 150\text{ g} \div 1000 &= 0{,}150\text{ kg}\end{aligned}$$

- We are given the initial velocity with which the bullet leaves the barrel, called the muzzle velocity, and it is $v = 960$ m · s^{-1}

Step 2: Analyse the question to determine what is being asked

We are asked to find the kinetic energy.

Step 3: Substitute and calculate

We just substitute the mass and velocity (which are known) into the equation for kinetic energy:

$$\begin{aligned}E_K &= \frac{1}{2}mv^2\\ &= \frac{1}{2}(0{,}150\text{ kg})\left(960\text{ m}\cdot\text{s}^{-1}\right)^2\\ &= 69\ 120\text{ J}\end{aligned}$$

4 MECHANICAL ENERGY

DEFINITION

Mechanical energy

Mechanical energy is the sum of the gravitational potential energy and the kinetic energy of a system.

Quantity: Mechanical energy energy (E_M) Unit name: Joule Unit symbol: J

Mechanical energy, E_M, is simply the sum of gravitational potential energy (E_P) and the kinetic energy (E_K). Mechanical energy is defined as:

$$E_M = E_P + E_K$$

$$E_M = mgh + \frac{1}{2}mv^2$$

TIP

You may see mechanical energy written as U. We will not use this notation in this book, but you should be aware that this notation is sometimes used.

WORKED EXAMPLE 6: MECHANICAL ENERGY

QUESTION

Calculate the total mechanical energy for a ball of mass $0,15$ kg which has a kinetic energy of 20 J and is 2 m above the ground.

SOLUTION

Step 1: Analyse the question to determine what information is provided

The ball has a mass $m = 0,15$ kg, and is at a height $h = 2$ m, with a kinetic energy $E_K = 20$ J

Step 2: Analyse the question to determine what is being asked

We need to find the total mechanical energy of the ball.

Step 3: Use the definition to calculate the total mechanical energy

$$\begin{aligned} E_M = E_P + E_K &= mgh + \frac{1}{2}mv^2 \\ &= mgh + 20 \\ &= (0,15 \text{ kg})\left(9,8 \text{ m}\cdot\text{s}^{-1}\right)(2 \text{ m}) + 20 \text{ J} \\ &= 2,94 \text{ J} + 20 \text{ J} \\ &= 22,94 \text{ J} \end{aligned}$$

5 CONSERVATION OF MECHANICAL ENERGY

DEFINITION

Conservation of Energy

The Law of Conservation of Energy: Energy cannot be created or destroyed, but is merely changed from one form into another.

So far we have looked at two types of energy: gravitational potential energy and kinetic energy. The sum of the gravitational potential energy and kinetic energy is called the mechanical energy. In a closed system, one where there are no external dissipative forces acting, the mechanical energy will remain constant. In other words, it will not change (become more or less). This is called the Law of Conservation of Mechanical Energy.

TIP

In problems involving the use of conservation of energy, the path taken by the object can be ignored. The only important quantities are the object's velocity (which gives its kinetic energy) and height above the reference point (which gives its gravitational potential energy).

DEFINITION

Conservation of mechanical energy

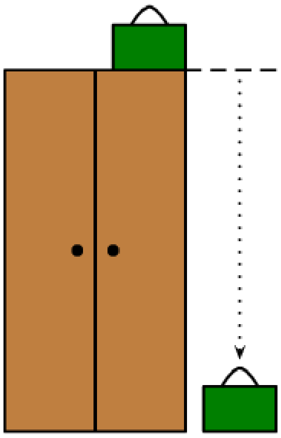
Law of Conservation of Mechanical Energy: The total amount of mechanical energy, in a closed system in the absence of dissipative forces (e.g. friction, air resistance), remains constant.

This means that potential energy can become kinetic energy, or vice versa, but energy cannot "disappear". For example, in the absence of air resistance, the mechanical energy of an object moving through the air in the Earth's gravitational field, remains constant (is conserved).

5.1 Using the law of conservation of energy

Mechanical energy is conserved (in the absence of friction). Therefore we can say that the sum of the E_P and the E_K anywhere during the motion must be equal to the sum of the the E_P and the E_K anywhere else in the motion.

We can now apply this to the example of the suitcase on the cupboard. Consider the mechanical energy of the suitcase at the top and at the bottom. We can say:



$$\begin{aligned} E_{M1} &= E_{M2} \\ E_{P1} + E_{K1} &= E_{P2} + E_{K2} \\ mgh + \frac{1}{2}mv^2 &= mgh + \frac{1}{2}mv^2 \\ (1\text{ kg})\left(9,8\text{ m}\cdot\text{s}^{-2}\right)(2\text{ m}) + 0 &= 0 + \frac{1}{2}(1\text{ kg})\left(v^2\right) \\ 19,6 &= \frac{1}{2}\left(v^2\right) \\ v^2 &= 39,2\text{ m}^2\cdot\text{s}^{-2} \\ v &= 6,26\text{ m}\cdot\text{s}^{-1} \end{aligned}$$

The suitcase will strike the ground with a velocity of $6,26\text{ m}\cdot\text{s}^{-1}$.

From this we see that when an object is lifted, like the suitcase in our example, it gains potential energy. As it falls back to the ground, it will lose this potential energy, but gain kinetic energy. We know that energy cannot be created or destroyed, but only changed from one form into another. In our example, the potential energy that the suitcase loses, is changed to kinetic energy.

The suitcase will have maximum potential energy at the top of the cupboard and maximum kinetic energy at the bottom of the cupboard. Halfway down it will have half kinetic energy and half potential energy. As it moves down, the potential energy will be converted (changed) into kinetic energy until all the potential energy is gone and only kinetic energy is left. The $19,6$ J of potential energy at the top will become $19,6$ J of kinetic energy at the bottom.

ACTIVITY

Conversion of energy

Materials

A length of plastic pipe with diameter approximately 20 mm, a marble, some masking tape and a measuring tape.

To do (1)

- First put one end of the pipe on the table top so that it is parallel to the top of the table and tape it in position with the masking tape.
- Lift the other end of the pipe upwards and hold it at a steady height not too high above the table.
- Measure the vertical height from the table top to the top opening of the pipe.
- Now put the marble at the top of the pipe and let it go so that it travels through the pipe and out the other end.

Questions

- What is the velocity (i.e. fast, slow, not moving) of the marble when you first put it into the top of the pipe and what does this mean for its gravitational potential and kinetic energy?
- What is the velocity (i.e. fast, slow, not moving) of the marble when it reaches the other end of the pipe and rolls onto the desk? What does this mean for its gravitational potential and kinetic energy?

To do (2)

- Now lift the top of the pipe as high as it will go.
- Measure the vertical height of the top of the pipe above the table top.
- Put the marble into the top opening and let it roll through the pipe onto the table.

Questions

- What is the velocity (i.e. fast, slow, not moving) of the marble when you put it into the top of the pipe, and what does this mean for its gravitational potential and kinetic energy?
- Compared to the first attempt, what was different about the height of the top of the tube? How do you think this affects the gravitational potential energy of the marble?
- Compared to your first attempt, was the marble moving faster or slower when it came out of the bottom of the pipe the second time? What does this mean for the kinetic energy of the marble?

The activity with the marble rolling down the pipe shows very nicely the conversion between gravitational potential energy and kinetic energy. In the first instance, the pipe was held relatively low and therefore the gravitational potential energy was also relatively low. The kinetic energy at this point was zero since the marble wasn't moving yet. When the marble rolled out of the other end of the pipe, it was moving relatively slowly, and therefore its kinetic energy was also relatively low. At this point its gravitational potential energy was zero since it was at zero height above the table top.

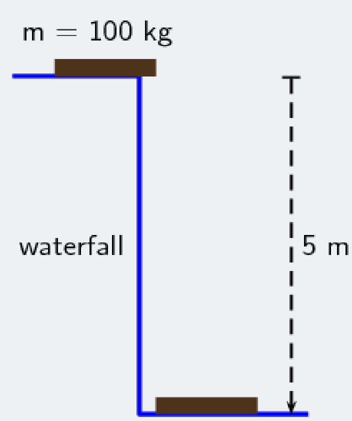
In the second instance, the marble started off higher up and therefore its gravitational potential energy was higher. By the time it got to the bottom of the pipe, its gravitational potential energy was zero (zero height above the table) but its kinetic energy was high since it was moving much faster than the first time. Therefore, the gravitational potential energy was converted completely to kinetic energy (if we ignore friction with the pipe).

In the case of the pipe being held higher, the gravitational potential energy at the start was higher, and the kinetic energy (and velocity) of the marble was higher at the end. In other words, the total mechanical energy was higher and and only depended on the height you held the pipe above the table top and not on the distance the marble had to travel through the pipe.

WORKED EXAMPLE 7: USING THE LAW OF CONSERVATION OF MECHANICAL ENERGY

QUESTION

During a flood a tree trunk of mass 100 kg falls down a waterfall. The waterfall is 5 m high.



If air resistance is ignored, calculate:

- the potential energy of the tree trunk at the top of the waterfall.
- the kinetic energy of the tree trunk at the bottom of the waterfall.
- the magnitude of the velocity of the tree trunk at the bottom of the waterfall.

WORKED EXAMPLE 7: USING THE LAW OF CONSERVATION OF MECHANICAL ENERGY (CONTINUED)

SOLUTION

Step 1: Analyse the question to determine what information is provided

- The mass of the tree trunk $m = 100$ kg
- The height of the waterfall $h = 5$ m.

These are all in SI units so we do not have to convert.

Step 2: Analyse the question to determine what is being asked

- Potential energy at the top
- Kinetic energy at the bottom
- Velocity at the bottom

Step 3: Calculate the potential energy at the top of the waterfall

$$\begin{aligned} E_P &= mgh \\ &= (100 \text{ kg})\left(9,8 \text{ m} \cdot \text{s}^{-2}\right)(5 \text{ m}) \\ &= 4\,900 \text{ J} \end{aligned}$$

Step 4: Calculate the kinetic energy at the bottom of the waterfall

The total mechanical energy must be conserved.

$$E_{K1} + E_{P1} = E_{K2} + E_{P2}$$

Since the trunk's velocity is zero at the top of the waterfall, $E_{K1} = 0$.
At the bottom of the waterfall, $h = 0$m, so $E_{P2} = 0$.
Therefore $E_{P1} = E_{K2}$ or in words:
The kinetic energy of the tree trunk at the bottom of the waterfall is equal to the potential energy it had at the top of the waterfall. Therefore $E_K = 4\,900\ J$

Step 5: Calculate the velocity at the bottom of the waterfall

To calculate the velocity of the tree trunk we need to use the equation for kinetic energy.

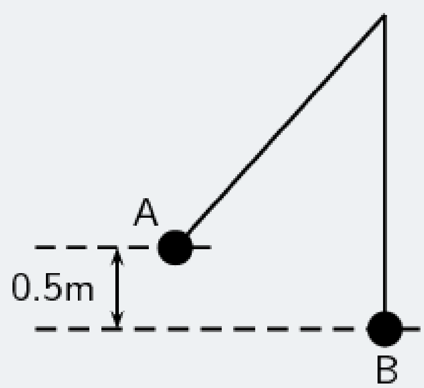
$$\begin{aligned} E_K &= \frac{1}{2}mv^2 \\ 4\,900 &= \frac{1}{2}(100 \text{ kg})\left(v \text{ m} \cdot \text{s}^{-1}\right)^2 \\ 98 &= v^2 \\ v &= 9,899...\text{m} \cdot \text{s}^{-1} \\ v &= 9,90 \text{ m} \cdot \text{s}^{-1} \end{aligned}$$

WORKED EXAMPLE 8: PENDULUM

QUESTION

A 2 kg metal ball is suspended from a rope as a pendulum. If it is released from point A and swings down to the point B (the bottom of its arc):

1. show that the velocity of the ball is independent of its mass,
2. calculate the velocity of the ball at point B.



SOLUTION

Step 1: Analyse the question to determine what information is provided

- The mass of the metal ball is $m = 2$ kg
- The change in height going from point A to point B is $h = 0,5$ m
- The ball is released from point A so the velocity at point, $v_A = 0$ m · s^{-1}.

All quantities are in SI units.

Step 2: Analyse the question to determine what is being asked

- Prove that the velocity is independent of mass.
- Find the velocity of the metal ball at point B.

Step 3: Apply the Law of Conservation of Mechanical Energy to the situation

Since there is no friction, mechanical energy is conserved. Therefore:

WORKED EXAMPLE 8: PENDULUM (CONTINUED)

$$E_{M1} = E_{M2}$$
$$E_{P1} + E_{K1} = E_{P2} + E_{K2}$$
$$mgh_1 + \frac{1}{2}m\left(v_1\right)^2 = mgh_2 + \frac{1}{2}m\left(v_2\right)^2$$
$$mgh_1 + 0 = 0 + \frac{1}{2}m\left(v_2\right)^2$$
$$mgh_1 = \frac{1}{2}m\left(v_2\right)^2$$

The mass of the ball m appears on both sides of the equation so it can be eliminated so that the equation becomes:

$$gh_1 = \frac{1}{2}(v_2)^2$$
$$2gh_1 = (v_2)^2$$

This proves that the velocity of the ball is independent of its mass. It does not matter what its mass is, it will always have the same velocity when it falls through this height.

Step 4: Calculate the velocity of the ball at point B

We can use the equation above, or do the calculation from "first principles":

$$(v_2)^2 = 2gh_1$$
$$(v_2)^2 = (2)\left(9,8\text{ m·s}^{-2}\right)(0,5\text{ m})$$
$$(v_2)^2 = 9,8\text{ m}^2\text{·s}^{-2}$$
$$v_2 = \sqrt{9,8\text{ m}^2\text{·s}^{-2}}$$
$$v_2 = 3,13\text{ m·s}^{-1}$$

Alternatively you can do:

$$E_{K1} + E_{P1} = E_{K2} + E_{P2}$$
$$mgh_1 + \frac{1}{2}m(v_1)^2 = mgh_2 + \frac{1}{2}m(v_2)^2$$
$$mgh_1 + 0 = 0 + \frac{1}{2}m(v_2)^2$$
$$(v_2)^2 = \frac{2mgh_1}{m}$$
$$(v_2)^2 = \frac{2\,(2\text{ kg})\left(9,8\text{ m·s}^{-2}\right)(0,5\text{ m})}{2\text{ kg}}$$
$$v_2 = \sqrt{9,8\text{ m}^2\text{·s}^{-2}}$$
$$v_2 = 3,13\text{ m·s}^{-1}$$

WORKED EXAMPLE 9: THE ROLLER COASTER

QUESTION

A roller coaster ride at an amusement park starts from rest at a height of 50 m above the ground and rapidly drops down along its track. At some point, the track does a full 360° loop which has a height of 20 m, before finishing off at ground level. The roller coaster train itself with a full load of people on it has a mass of 850 kg.

Roller coaster

If the roller coaster and its track are frictionless, calculate:

1. the velocity of the roller coaster when it reaches the top of the loop
2. the velocity of the roller coaster at the bottom of the loop (i.e. ground level)

SOLUTION

Step 1: Analyse the question to determine what information is provided

- The mass of the roller coaster is $m = 850$ kg
- The initial height of the roller coaster at its starting position is $h_1 = 50$ m
- The roller coaster starts from rest, so its initial velocity $v_1 = 0$ m · s^{-1}
- The height of the loop is $h_2 = 20$ m
- The height at the bottom of the loop is at ground level, $h_3 = 0$ m

We do not need to convert units as they are in the correct form already.

WORKED EXAMPLE 9: THE ROLLER COASTER (CONTINUED)

Step 2: Analyse the question to determine what is being asked

- the velocity of the roller coaster at the top of the loop
- the velocity of the roller coaster at the bottom of the loop

Step 3: Calculate the velocity at the top of the loop

From the conservation of mechanical energy, We know that at any two points in the system, the total mechanical energy must be the same. Let's compare the situation at the start of the roller coaster to the situation at the top of the loop:

$$E_{M1} = E_{M2}$$
$$E_{K1} + E_{P1} = E_{K2} + E_{P2}$$
$$0 + mgh_1 = \frac{1}{2}m(v_2)^2 + mgh_2$$

We can eliminate the mass, m, from the equation by dividing both sides by m.

$$gh_1 = \frac{1}{2}(v_2)^2 + gh_2$$
$$(v_2)^2 = 2\left(gh_1 - gh_2\right)$$
$$(v_2)^2 = 2\left(\left(9,8 \text{ m·s}^{-2}\right)(50 \text{ m}) - \left(9,8 \text{ m·s}^{-2}\right)(20 \text{ m})\right)$$
$$v_2 = 24,25 \text{ m·s}^{-1}$$

Step 4: Calculate the velocity at the bottom of the loop

Again we can use the conservation of energy and the total mechanical energy at the bottom of the loop should be the same as the total mechanical energy of the system at any other position. Let's compare the situations at the start of the roller coaster's trip and the bottom of the loop:

$$E_{M1} = E_{M3}$$
$$E_{K1} + E_{P1} = E_{K3} + E_{P3}$$
$$\frac{1}{2}m_1(0)^2 + mgh_1 = \frac{1}{2}m(v_3)^2 + mg\left(0\right)$$
$$mgh_1 = \frac{1}{2}m(v_3)^2$$
$$(v_3)^2 = 2gh_1$$
$$(v_3)^2 = 2\left(9,8 \text{ m·s}^{-2}\right)(50 \text{ m})$$
$$v_3 = 31,30 \text{ m·s}^{-1}$$

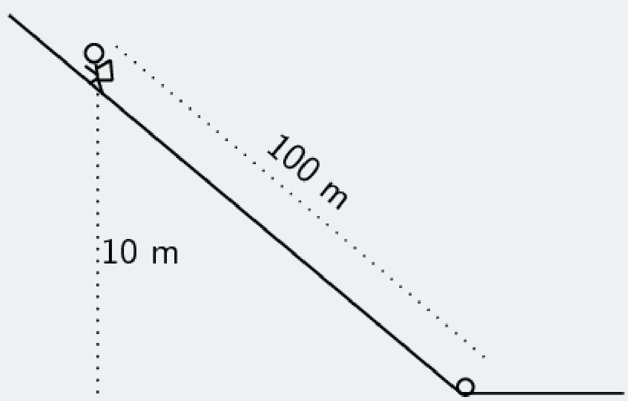
WORKED EXAMPLE 10: AN INCLINED PLANE

QUESTION

A mountain climber who is climbing a mountain in the Drakensberg during winter, by mistake drops her water bottle which then slides 100 m down the side of a steep icy slope to a point which is 10 m lower than the climber's position. The mass of the climber is 60 kg and her water bottle has a mass of 500 g.

1. If the bottle starts from rest, how fast is it travelling by the time it reaches the bottom of the slope? (Neglect friction.)

2. What is the total change in the climber's potential energy as she climbs down the mountain to fetch her fallen water bottle? i.e. what is the difference between her potential energy at the top of the slope and the bottom of the slope?



SOLUTION

Step 1: Analyse the question to determine what information is provided

- the distance travelled by the water bottle down the slope, $d = 100$ m
- the difference in height between the starting position and the final position of the water bottle is $h = 10$ m
- the bottle starts sliding from rest, so its initial velocity is $v_1 = 9,8$ m $\cdot$ s^{-1}
- the mass of the climber is 60 kg
- the mass of the water bottle is 500 g. We need to convert this mass into kg : 500 g $= 0,5$ kg

WORKED EXAMPLE 10: AN INCLINED PLANE (CONTINUED)

Step 2: Analyse the question to determine what is being asked

- What is the velocity of the water bottle at the bottom of the slope?
- What is the difference between the climber's potential energy when she is at the top of the slope compared to when she reaches the bottom?

Step 3: Calculate the velocity of the water bottle when it reaches the bottom of the slope

$$
\begin{aligned}
E_{M1} &= E_{M2} \\
E_{K1} + E_{P1} &= E_{K2} + E_{P2} \\
\frac{1}{2}m\left(v_1\right)^2 + mgh_1 &= \frac{1}{2}m\left(v_2\right)^2 + mgh_2 \\
0 + mgh_1 &= \frac{1}{2}m\left(v_2\right)^2 + 0 \\
\left(v_2\right)^2 &= \frac{2mgh}{m} \\
\left(v_2\right)^2 &= 2gh \\
\left(v_2\right)^2 &= (2)\left(9,8\ \text{m}\cdot\text{s}^{-2}\right)(10\ \text{m}) \\
v_2 &= 14\ \text{m}\cdot\text{s}^{-1}
\end{aligned}
$$

Note: the distance that the bottle travelled (i.e. $100\ m$) does not play any role in calculating the energies. It is only the height difference that is important in calculating potential energy.

Step 4: Calculate the difference between the climber's potential energy at the top of the slope and her potential energy at the bottom of the slope

At the top of the slope, her potential energy is:

$$
\begin{aligned}
E_{P1} &= mgh_1 \\
&= (60\ \text{kg})\left(9,8\ \text{m}\cdot\text{s}^{-2}\right)(10\ \text{m}) \\
&= 5\ 880\ \text{J}
\end{aligned}
$$

At the bottom of the slope, her potential energy is:

$$
\begin{aligned}
E_{P1} &= mgh_1 \\
&= (60\ \text{kg})\left(9,8\ \text{m}\cdot\text{s}^{-2}\right)(0\ \text{m}) \\
&= 0\ \text{J}
\end{aligned}
$$

Therefore the difference in her potential energy when moving from the top of the slope to the bottom is:

$$E_{P1} - E_{P2} = 5\ 880 - 0 = 5\ 880\ \text{J}$$

6 CHAPTER SUMMARY

- The gravitational potential energy of an object is the energy the object has because of its position in the gravitational field relative to some reference point.
- The kinetic energy of an object is the energy the object has due to its motion.
- The mechanical energy of an object is the sum of the potential energy and kinetic energy of the object.
- The unit for energy is the joule (J).
- The Law of Conservation of Energy states that energy cannot be created or destroyed, but can only be changed from one form into another.
- The Law of Conservation of Mechanical Energy states that the total mechanical energy of an isolated system (i.e. no friction or air resistance) remains constant.
- The table below summarises the most important equations:

Potential Energy	$E_P = mgh$
Kinetic Energy	$E_K = \frac{1}{2}mv^2$
Mechanical Energy	$E_M = E_K + E_P$

Physical Quantities		
Quantity	**Unit name**	**Unit symbol**
Potential energy (E_P)	joule	J
Kinetic energy (E_K)	joule	J
Mechanical energy (E_M)	joule	J

7 EXERCISES

7.1 Exercise 1

1. Describe the relationship between an object's gravitational potential energy and its:

    1.1 mass and

    1.2 height above a reference point.

2. A boy, of mass 30 kg, climbs onto the roof of a garage. The roof is $2,5$ m from the ground.

    2.1 How much potential energy did the boy gain by climbing onto the roof?

    2.2 The boy now jumps down. What is the potential energy of the boy when he is 1 m from the ground?

    2.3 What is the potential energy of the boy when he lands on the ground?

3. A hiker, of mass 70 kg, walks up a mountain, 800 m above sea level, to spend the night at the top in the first overnight hut. The second day she walks to the second overnight hut, 500 m above sea level. The third day she returns to her starting point, 200 m above sea level.

    3.1 What is the potential energy of the hiker at the first hut (relative to sea level)?

    3.2 How much potential energy has the hiker lost during the second day?

    3.3 How much potential energy did the hiker have when she started her journey (relative to sea level)?

    3.4 How much potential energy did the hiker have at the end of her journey when she reached her original starting position?

7.2 Exercise 2

1. Describe the relationship between an object's kinetic energy and its:

    1.1 mass and

    1.2 velocity

2. A stone with a mass of 100 g, cis thrown up into the air. It has an initial velocity of 3 m $\cdot$ s^{-1}. Calculate its kinetic energy:

    2.1 as it leaves the thrower's hand.

    2.2 when it reaches its turning point.

3. A car with a mass of 700 kg is travelling at a constant velocity of 100 km$\cdot$hr^{-1}. Calculate the kinetic energy of the car.

7.3 Exercise 3

1. A tennis ball, of mass 120 kg, is dropped from a height of 5 m. Ignore air friction.

   1.1 What is the potential energy of the ball when it has fallen 3 m ?

   1.2 What is the velocity of the ball when it hits the ground?

2. A bullet, mass 50 g, is shot vertically up in the air with a muzzle velocity of 200 m · s^{-1}. Use the Principle of Conservation of Mechanical Energy to determine the height that the bullet will reach. Ignore air friction.

3. skier, mass 50 kg, is at the top of a $6,4$ m ski slope.

   3.1 Determine the maximum velocity that she can reach when she skis to the bottom of the slope.

   3.2 Do you think that she will reach this velocity? Why/Why not?

4. A pendulum bob of mass $1,5$ kg, swings from a height A to the bottom of its arc at B. The velocity of the bob at B is 4 m · s^{-1}. Calculate the height A from which the bob was released. Ignore the effects of air friction.

5. Prove that the velocity of an object, in free fall, in a closed system, is independent of its mass.

8 ANSWERS TO EXERCISES

8.1 Exercise 1

1.1 The relationship is directly proportional.

1.2 The relationship is directly proportional.

2.1 735 J

2.2 294 J

2.3 0 J

3.1 705600 J

3.2 362600 J

3.3 137200 J

3.4 137200 J

8.2 Exercise 2

1.1 The relationship is directly proportional.

1.2 The relationship is directly proportional.

2.1 $0,45$ J

2.2 0 J

3. $210104,94$ J

8.3 Exercise 3

1.1 3528 J

1.2 $9,90$ m $\cdot$ s^{-1}

2. $2040,82$ m

3.1 $1,120$ m $\cdot$ s^{-1}

3.2 No, due to the presence of resistance forces like friction and air resistance.

4. $0,82$ m

5. $F = ma$ For any body force is mass multiplied by acceleration due to force of that body. So what force does is accelerate any mass. Bodies are attracted towards earth due to gravity which is gravitational force of the earth. Hence both the objects will take equal time and will have equal velocities irrespective of their masses.