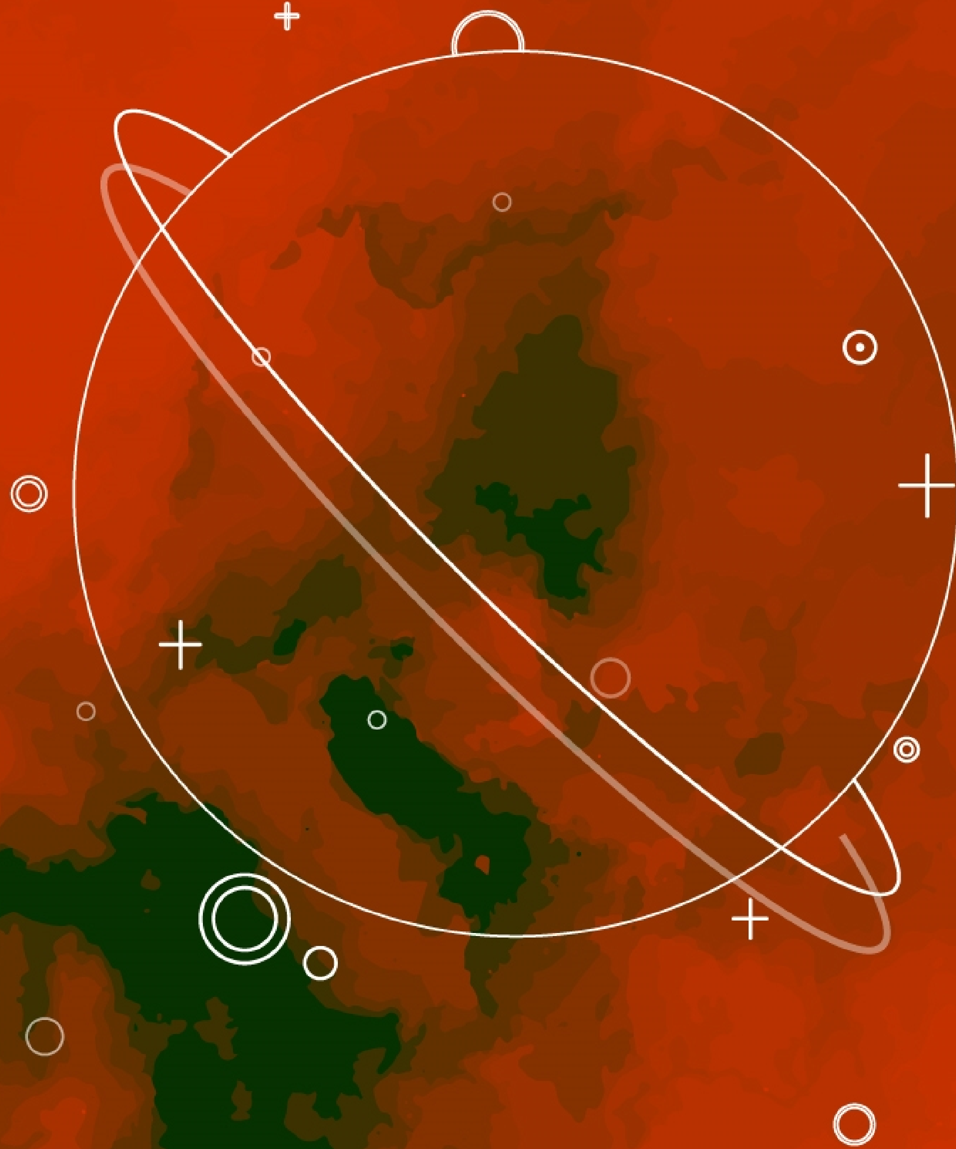

CHAPTER 9

Longitudinal Waves

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CONTENTS

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1 INTRODUCTION AND KEY CONCEPTS

We have already studied transverse pulses and waves. In this chapter we look at another type of wave called a *longitudinal* wave. In transverse waves, the motion of the particles in the medium was perpendicular to the direction of the wave. In longitudinal waves, the particles in the medium move *parallel* (in the *same* direction as) to the motion of the wave. Examples of transverse waves (discussed in the previous chapter) are water waves. An example of a longitudinal wave is a sound wave.

1.1 What is a *longitudinal* wave?

Definition : Longitudinal waves

A longitudinal wave is a wave where the particles in the medium move parallel to the direction of propagation of the wave.

When we studied transverse waves we looked at two different motions: the motion of the particles of the medium and the motion of the wave itself. We will do the same for longitudinal waves.

The question is how do we construct such a wave?

A longitudinal wave is seen best in a slinky spring. Do the following investigation to find out more about longitudinal waves.

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Activity

Investigating longitudinal waves

Take a slinky spring and lay it on a table. Hold one end and pull the free end of the spring and flick it back and forth once in the direction of the spring. Observe what happens.

In which direction does the disturbance move?

A slinky spring

Tie a ribbon to the middle of the spring. Watch carefully what happens to the ribbon when the end of the spring is flicked. Describe the motion of the ribbon.

Flick the spring back and forth continuously to set up a train of pulses, a longitudinal wave.

From the investigation you will have noticed that the disturbance moves parallel to the direction in which the spring was pulled. The ribbon in the investigation represents one particle in the medium. The particles in the medium move in the same direction as the wave.

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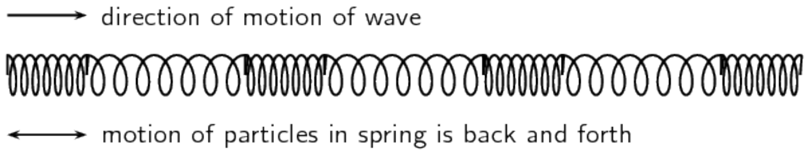

Figure 1: Longitudinal wave through a spring

As in the case of transverse waves the following properties can be defined for longitudinal waves: wavelength, amplitude, period, frequency and wave speed.

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2 COMPRESSION AND RAREFACTION

Unlike transverse waves which have crests and troughs, longitudinal waves have *compressions* and *rarefactions*.

Definition : Compression

A **compression** is a region in a longitudinal wave where the particles are closest together.

Definition : Rarefaction

A **rarefaction** is a region in a longitudinal wave where the particles are furthest apart.

As seen in Figure 2 , there are regions where the medium is compressed and other regions where the medium is spread out in a longitudinal wave. The region where the medium is compressed is known as a **compression** and the region where the medium is spread out is known as a **rarefaction**.

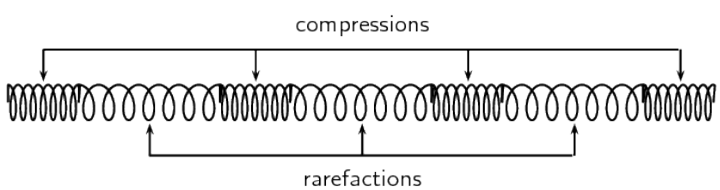

Figure 2: Compressions and rarefactions on a longitudinal wave

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3 WAVELENGTH AND AMPLITUDE

Definition : Wavelength

The **wavelength** in a longitudinal wave is the distance between two consecutive points that are in phase.

The wavelength in a longitudinal wave refers to the distance between two consecutive compressions or between two consecutive rarefactions.

Definition : Amplitude

The **amplitude** is the maximum displacement from equilibrium. For a longitudinal wave which is a pressure wave this would be the maximum increase (or decrease) in pressure from the equilibrium pressure that is caused when a compression (or rarefaction) passes a point.

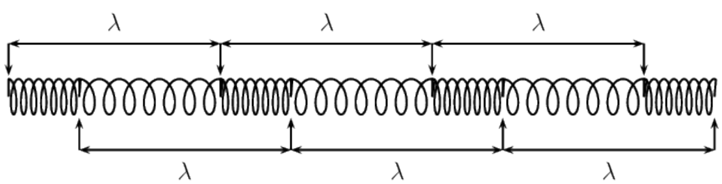

Figure 3: Wavelength of a longitudinal wave

The amplitude is the distance from the equilibrium position of the medium to a compression or a rarefaction.

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4 PERIOD AND FREQUENCY

Definition : Period

The **period** of a wave is the time taken by the wave to move one wavelength.

Definition : Frequency

The **frequency** of a wave is the number of wavelengths per second.

The *period* of a longitudinal wave is the time taken by the wave to move one wavelength. As for transverse waves, the symbol T is used to represent period and period is measured in seconds (s).

The frequency f of a wave is the number of wavelengths per second. Using this definition and the fact that the period is the time taken for 1 wavelength, we can define:

$$f = \frac{1}{T} \tag{1}$$

or alternately,

$$T = \frac{1}{f} \tag{2}$$

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5 SPEED OF A LONGITUDINAL WAVE

The speed of a longitudinal wave is defined in the same was as the speed of transverse waves:

Definition : Wave speed

Wave speed is the distance a wave travels per unit time.

Quantity: Wave speed (v) Unit name: speed Unit: m · s^{-1}

The distance between two successive compressions is 1 wavelength, λ. Thus in a time of 1 period, the wave will travel 1 wavelength in distance. Thus the speed of the wave, v, is:

$$v = \frac{\text{distance travelled}}{\text{time taken}} = \frac{\lambda}{T} \tag{3}$$

If Equation 1 is substituted into Equation 3 then we can also write:

$$\begin{aligned} v &= \frac{\lambda}{T} \\ &= \lambda \cdot \frac{1}{T} \\ &= \lambda \cdot f \end{aligned}$$

We call this equation the *wave equation*. To summarise, we have that $v = \lambda \cdot f$ where

- $v =$ speed in m · s^{-1}
- $\lambda =$ wavelength in m
- $f =$ frequency in Hz

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WORKED EXAMPLE 1: SPEED OF LONGITUDINAL WAVES

QUESTION

The musical note "A" is a sound wave. The note has a frequency of 440 Hz and a wavelength of $0{,}784$ m. Calculate the speed of the musical note. note.

SOLUTION

Step 1: Determine what is given and what is required

Using:

$$f = 440 \text{ Hz}$$
$$\lambda = 0{,}784 \text{ m}$$

We need to calculate the speed of the musical note "A".

Step 2: Determine how to approach based on what is given

We are given the frequency and wavelength of the note. We can therefore use:

$$v = \lambda \cdot f$$

Step 3: Calculate the wave speed

$$\begin{aligned} v &= f \cdot \lambda \\ &= (440 \text{ Hz})(0{,}784 \text{ m}) \\ &= 344,96 \text{ m} \cdot \text{s}^{-1} \end{aligned}$$

Step 4: Write the final answer

The musical note "A" travels at $344,96 \text{ m} \cdot \text{s}^{-1}$.

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WORKED EXAMPLE 2: SPEED OF LONGITUDINAL WAVES

QUESTION

A longitudinal wave travels into a medium in which its speed increases. How does this affect its... (write only increases, decreases, stays the same).

1. period?
2. wavelength?

SOLUTION

Step 1: Determine what is required

We need to determine how the period and wavelength of a longitudinal wave change when its speed increases.

Step 2: Determine how to approach based on what is given

We need to find the link between period, wavelength and wave speed.

Step 3: Discuss how the period changes

We know that the frequency of a longitudinal wave is dependent on the frequency of the vibrations that lead to the creation of the longitudinal wave. Therefore, the frequency is always unchanged, irrespective of any changes in speed. Since the period is the inverse of the frequency, the period remains the same.

Step 4: Discuss how the wavelength changes

The frequency remains unchanged. According to the wave equation

$$v = f \cdot \lambda$$

if f remains the same and v increases, then λ, the wavelength, must also increase.

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6 CHAPTER SUMMARY

- A longitudinal wave is a wave where the particles in the medium move parallel to the direction in which the wave is travelling.
- Most longitudinal waves consist of areas of higher pressure, where the particles in the medium are closest together (compressions) and areas of lower pressure, where the particles in the medium are furthest apart (rarefactions).
- The wavelength of a longitudinal wave is the distance between two consecutive compressions, or two consecutive rarefactions.
- The relationship between the period (T) and frequency (f) is given by

$$T = \frac{1}{f}$$

or

$$f = \frac{1}{T}$$

- The relationship between wave speed (v), frequency (f) and wavelength (λ) is given by

$$v = f \cdot \lambda$$

Table 1: Units used in **longitudinal waves**

Physical Quantities		
Quantity	**Unit name**	**Unit symbol**
Amplitude (A)	metre	m
Wavelength (λ)	metre	m
Period (T)	second	s
Frequency (f)	hertz	Hz (s^{-1})
Wave speed (v)	metre per second	$\text{m} \cdot \text{s}^{-1}$

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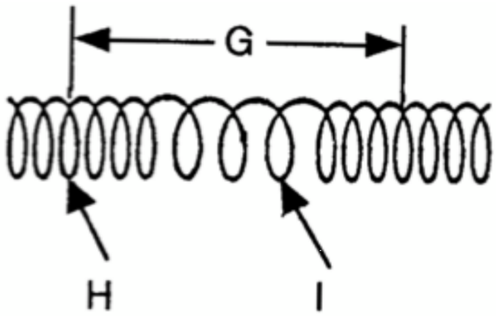
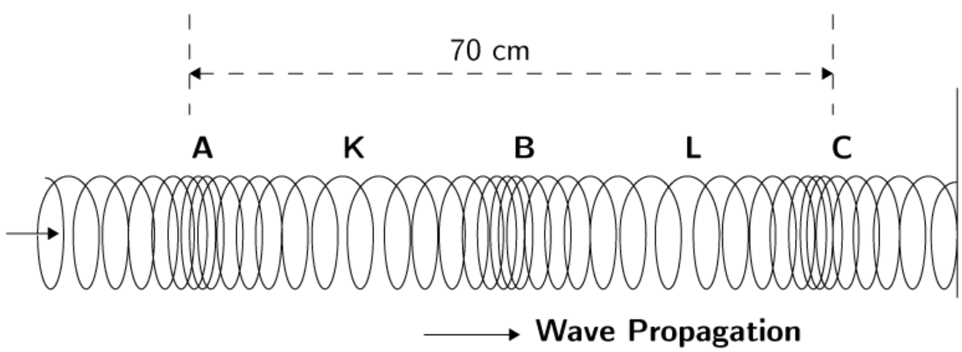
7 EXERCISES

7.1 Exercise 1



Study the diagram above and answer Questions 1-2.

1. Is the wave represented by the diagram above a transverse or a longitudinal wave?
2. What do letters G, H and I represent in the diagram above?
3. The following wave pattern is produced when two learners move a slinky as indicated in the diagram. Study the diagram and then answer Questions 3-7.



3.1 Name the type of wave produced on the slinky.

3.2 K and L are areas of low pressure where the particles are far apart. What are those areas called?

3.3 A, B and C are areas of high pressure where the particles are close together. What do we call these areas?

3.4 Use the diagram and calculate the wavelength of the wave. Give your answer in meters.

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3.5 The velocity of the wave is 5 m·s^{-1}. Calculate the period of the wave.

4. The siren of a stationary ambulance emits a longitudinal sound wave of frequency 500 Hz. The wavelength of the wave is $0{,}8$ m. Calculate the speed of the longitudinal wave.

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8 ANSWERS TO EXERCISES

8.1 Exercise 1

1. Longitudinal wave

2. **G**: Wavelength
   H: Compression
   I: Rarefaction

3.1 Longitudinal wave

3.2 Rarefactions

3.3 Compressions

3.4 $\lambda = 0.35$ m

3.5 T $= 0.07$ s

4. $v = 400$ m $\cdot$ s^{-1}

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