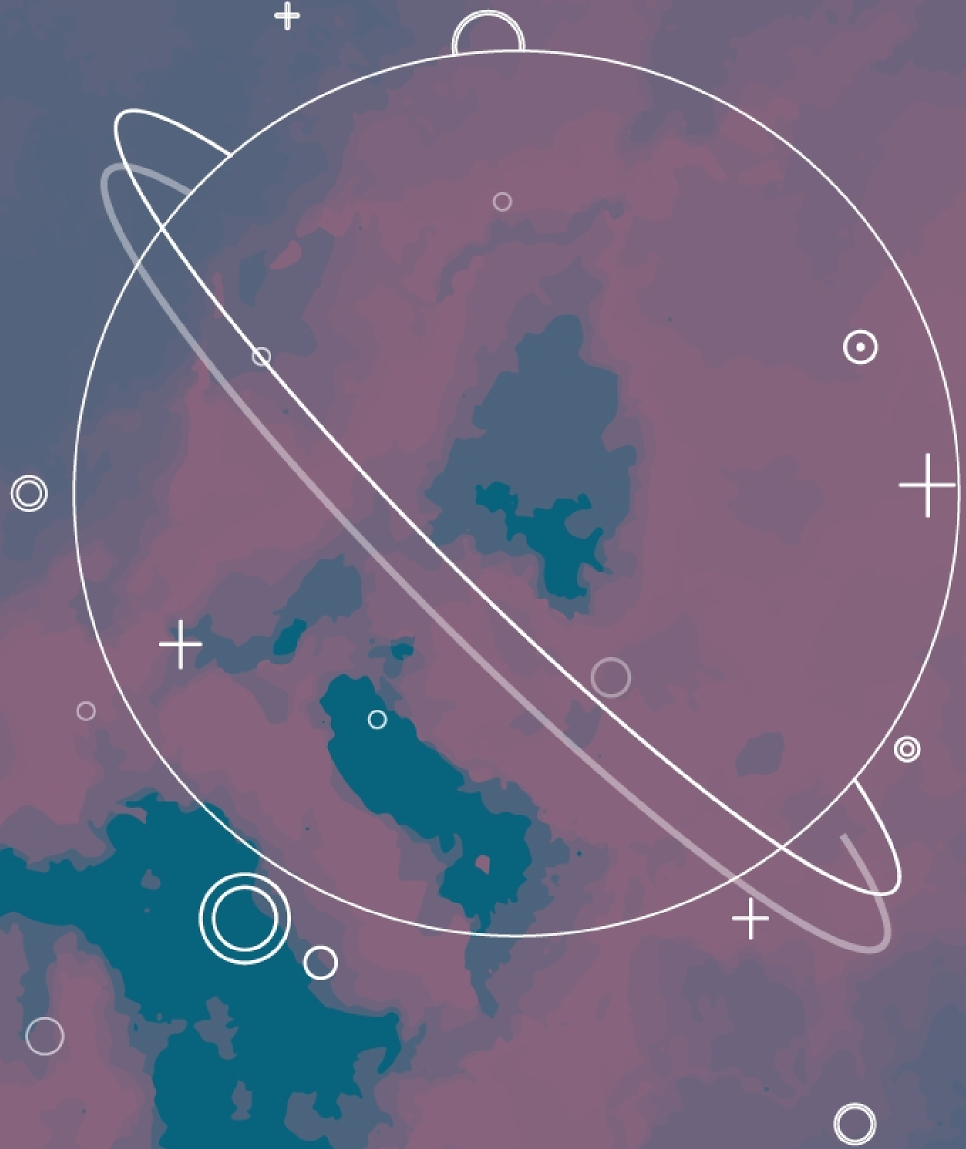

CHAPTER 9

Electrostatics

CONTENTS

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April 20, 2021

1 INTRODUCTION

In Grade 10, you learnt about the force between charges. In this chapter you will learn exactly how to determine this force and about a basic law of electrostatics.

KEY CONCEPTS

- Ratio and proportion - Physical Sciences, Grade 10, Science skills
- Equations - Mathematics, Grade 10, Equations and inequalities
- Units and unit conversions - Physical Sciences, Grade 10, Science skills
- Scientific notation - Physical Sciences, Grade 10, Science skills

2 COULOMB'S LAW

Like charges repel each other while unlike charges attract each other. If the charges are at rest then the force between them is known as the **electrostatic force**. The electrostatic force between charges increases when the magnitude of the charges increases or the distance between the charges decreases.

The electrostatic force was first studied in detail by Charles-Augustin de Coulomb around 1784. Through his observations he was able to show that the **magnitude** of the electrostatic force between two point-like charges is inversely proportional to the square of the distance between the charges. He also discovered that the **magnitude** of the force is proportional to the product of the charges. That is:

$$F \propto \frac{Q_1 Q_2}{r^2}$$

where Q_1 and Q_2 are the magnitudes of the two charges respectively and r is the distance between them. The magnitude of the electrostatic force between two point-like charges is given by *Coulomb's law*.

DEFINITION

Coulomb's law

Coulomb's law states that the magnitude of the electrostatic force between two point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.

$$F = \frac{kQ_1 Q_2}{r^2}$$

The proportionality constant k is called the *electrostatic constant* and has the value:
$9,0 \times 10^9 N \cdot m^2 \cdot C^-2$ in free space.

Similarity of Coulomb's law to Newton's universal law of gravitation.

Notice how similar in form Coulomb's law is to Newton's universal law of gravitation between two point-like particles:

$$F_G = \frac{Gm_1 m_2}{d_2},$$

where m_1 and m_2 are the masses of the two point-like particles, d is the distance between them, and G is the gravitational constant. Both are inverse-square laws.

Both laws represent the force exerted by particles (point masses or point charges) on each other that interact by means of a field.

WORKED EXAMPLE 1: COULOMB'S LAW

QUESTION

Two point-like charges carrying charges of $+3×10^{-9}\ C$ and $-5×10^{-9}\ C$ are $2\ m$ apart. Determine the magnitude of the force between them and state whether it is attractive or repulsive.

SOLUTION

Step 1: Determine what is required

We are required to determine the force between two point charges given the charges and the distance between them.

Step 2: Determine how to approach the problem

We can use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1 Q_2}{r^2}$$

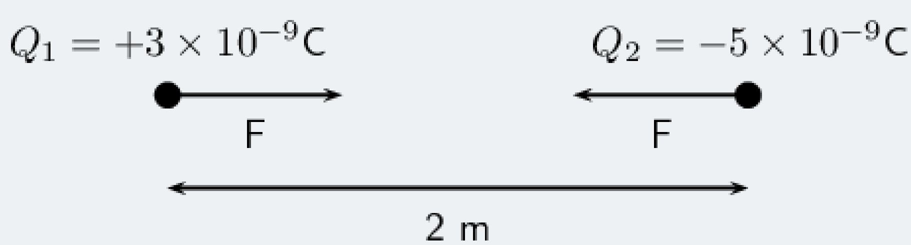
Step 3: Determine what is given

We are given:

- $Q_1 = +3×10^{-9}\ C$
- $Q_2 = -5×10^{-9}\ C$
- $r = 2\ m$

We know that $k = 9,0×10^9 N \cdot m^2 \cdot C^{-2}$.

We can draw a diagram of the situation.



Step 4: Check units

All quantities are in SI units.

WORKED EXAMPLE 1: COULOMB'S LAW (continued)

Step 5: Determine the magnitude of the force

Using Coulomb's law we have

$$
\begin{aligned}
F &= k\frac{Q_1 Q_2}{r^2} \\
&= \frac{(9,0\times10^{9})(3\times10^{-9})(5\times10^{-9})}{(2)^2} \\
&= 3,37\times10^{-8}\ N
\end{aligned}
$$

Thus the *magnitude* of the force is $3,37\times10^{-8}\ N$. However since the point charges have opposite signs, the force will be attractive.

Step 6: Free body diagram

We can draw a free body diagram to show the forces. Each charge experiences a force with the same magnitude and the forces are attractive, so we have:

Next is another example that demonstrates the difference in magnitude between the gravitational force and the electrostatic force.

WORKED EXAMPLE 2: COULOMB'S LAW

QUESTION

Determine the magnitudes of the electrostatic force and gravitational force between two electrons $10^{-10}\ m$ apart (i.e. the forces felt inside an atom) and state whether the forces are attractive or repulsive.

SOLUTION

Step 1: Determine what is required

We are required to calculate the electrostatic and gravitational forces between two electrons, a given distance apart.

Step 2: Determine how to approach the problem

We can use:

$$F_e = k\frac{Q_1 Q_2}{r^2}$$

to calculate the electrostatic force and

$$F_g = \frac{G m_1 m_2}{d^2}$$

to calculate the gravitational force.

Step 3: Determine what is given

We are given:

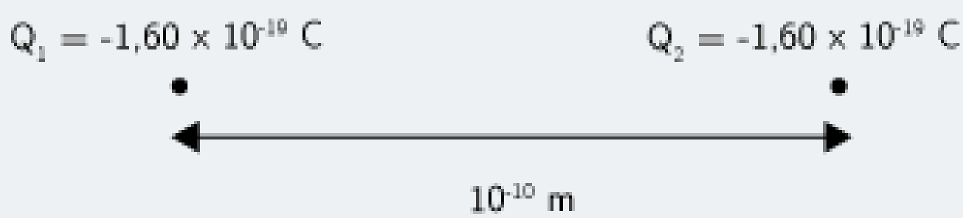
- $Q_1 = Q_2 = 1,6 \times 10^{-19}\ C$ (The charge on an electron)
- $m_1 = m_2 = 9,1 \times 10^{-31}\ kg$ (The mass of an electron)
- $r = d = 1 \times 10^{-10}\ m$

We know that:

- $k = 9,0 \times 10^{9}\ N \cdot m^2 \cdot C^{-2}$
- $G = 6,67 \times 10^{-11}\ N \cdot m^2 \cdot kg^{-2}$

All quantities are in SI units.

We can draw a diagram of the situation.



WORKED EXAMPLE 2: COULOMB'S LAW (continued)

Step 4: Calculate the electrostatic force

$$\begin{aligned} F_e &= k\frac{Q_1Q_2}{r^2} \\ &= \frac{(9,0\times10^{9})(1,60\times10^{-19})(1,60\times10^{-19})}{(10^{-10})^2} \\ &= 2,30\times10^{-8}\ N \end{aligned}$$

Hence the *magnitude* of the electrostatic force between the electrons is $2,30\times10^{-8}\ N$. Since electrons carry like charges, the force is repulsive.

Step 5: Calculate the gravitational force

$$\begin{aligned} F_g &= \frac{Gm_1m_2}{d^2} \\ &= \frac{(6,67\times10^{-11})(9,11\times10^{-31})(9,11\times10^{-31})}{(10^{-10})^2} \\ &= 5,54\times10^{-51}\ N \end{aligned}$$

The magnitude of the gravitational force between the electrons is $5,54\times10^{-51}\ N$. Remember that the gravitational force is always an attractive force.

Notice that the gravitational force between the electrons is **much smaller** than the electrostatic force.

TIP

We can apply Newton's third law to charges because two charges exert forces of equal magnitude on one another in opposite directions.

TIP

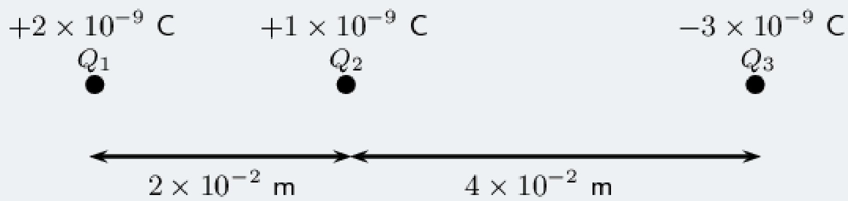
Choosing a positive direction

When substituting into the Coulomb's law equation, one may choose a positive direction thus making it unnecessary to include the signs of the charges. Instead, select a positive direction. Those forces that tend to move the charge in this direction are added, while forces acting in the opposite direction are subtracted.

WORKED EXAMPLE 3: COULOMB'S LAW

QUESTION

Three point charges are in a straight line. Their charges are $Q_1 = +2\times10^{-9}\ C$, $Q_2 = +1\times10^{-9}\ C$ and $Q_3 = -3\times10^{-9}\ C$. The distance between Q_1 and Q_2 is $2\times10^{-2}\ m$ and the distance between Q_2 and Q_3 is $4\times10^{-2}\ m$. What is the net electrostatic force on Q_2 due to the other two charges?



SOLUTION

Step 1: Determine what is required

We need to calculate the net force on Q_2. This force is the sum of the two electrostatic forces - the forces between Q_1 on Q_2 and Q_3 on Q_2.

Step 2: Determine how to approach the problem

- We need to calculate the two electrostatic forces on Q_2, using Coulomb's law.
- We then need to add up the two forces using our rules for adding vector quantities, because force is a vector quantity.

Step 3: Determine what is given

We are given all the charges and all the distances.

Step 4: Calculate the magnitude of the forces.

Force on Q_2 due to Q_1:

$$
\begin{aligned}
F_1 &= \frac{kQ_1Q_2}{r^2} \\
&= \frac{(9,0\times10^{9})(2\times10^{-9})(1\times10^{-9})}{(2\times10^{-2})^2} \\
&= \frac{(9,0\times10^{9})(2\times10^{-9})(1\times10^{-9})}{(4\times10^{-4})} \\
&= 4,5\times10^{-5}\ N
\end{aligned}
$$

WORKED EXAMPLE 3: COULOMB'S LAW (continued)

Force on Q_2 due to Q_3:

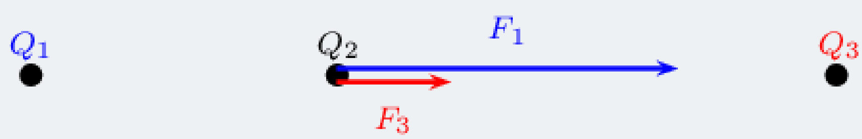
$$\begin{aligned} F_3 &= \frac{kQ_2Q_3}{r^2} \\ &= \frac{(9,0\times10^{9})(1\times10^{-9})(3\times10^{-9})}{(4\times10^{-2})^2} \\ &= \frac{(9,0\times10^{9})(1\times10^{-9})(3\times10^{-9})}{(16\times10{-}4)} \\ &= 1,69\times10^{-5}\ N \end{aligned}$$

Step 5: Vector addition of forces

We know the force magnitudes but we need to use the charges to determine whether the forces are repulsive or attractive. It is helpful to draw the force diagram to help determine the final direction of the net force on Q_2. We choose the positive direction to be to the right (the positive x-direction).

The force between Q_1 and Q_2 is repulsive (like charges). This means that it pushes Q_2 to the right, or in the positive direction.

The force between Q_2 and Q_3 is attractive (unlike charges) and pulls Q_2 to the right.



Therefore both forces are acting in the positive direction.

Therefore,

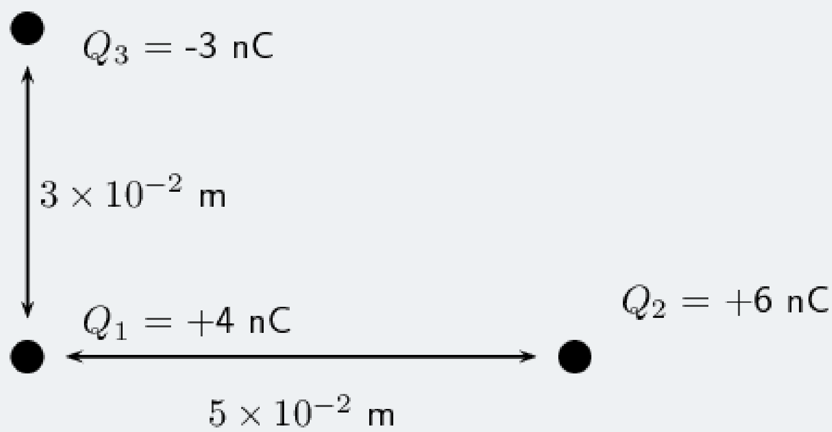
$$\begin{aligned} F_R &= 4,5\times10^{-5}N + 1,69\times10^{-5}N \\ &= 6,19\times10^{-5}\ N \end{aligned}$$

The resultant force acting on Q_2 is $6,19\times10^{-5}\ N$ to the right.

WORKED EXAMPLE 4: COULOMB'S LAW

QUESTION

Three point charges form a right-angled triangle. Their charges are $Q_1 = 4×10^{-9}\ C = 4\ nC$, $Q_2 = 6×10^{-9}\ C = 6\ nC$ and $Q_3 = -3×10^{-9}\ C = -3\ nC$. The distance between Q_1 and Q_2 is $5×10^{-2}\ m$ and the distance between Q_1 and Q_3 is $3×10^{-2}\ m$. What is the net electrostatic force on Q_1 due to the other two charges if they are arranged as shown?



SOLUTION

Step 1: Determine what is required

We need to calculate the net force on Q_1. This force is the sum of the two electrostatic forces - the forces of Q_2 on Q_1 and Q_3 on Q_1.

Step 2: Determine how to approach the problem

- We need to calculate, using Coulomb's law, the electrostatic force exerted on Q_1 by Q_2, and the electrostatic force exerted on Q_1 by Q_3.
- We then need to add up the two forces using our rules for adding vector quantities, because force is a vector quantity.

Step 3: Determine what is given

We are given all the charges and two of the distances.

WORKED EXAMPLE 4: COULOMB'S LAW (continued)

Step 4: Calculate the magnitude of the forces.

The magnitude of the force exerted by Q_2 on Q_1, which we will call F_2, is:

$$\begin{aligned} F_2 &= \frac{kQ_1Q_2}{r_2} \\ &= \frac{(9,0\times10^{9})(4\times10^{-9})(6\times10^{-9})}{(5\times10^{-2})^2} \\ &= \frac{(9,0\times10^{9})(4\times10^{-9})(6\times10^{-9})}{(25\times10^{-4})} \\ &= 8,630\times10^{-5}\ N \end{aligned}$$

The magnitude of the force exerted by Q_3 on Q_1, which we will call F_3, is:

$$\begin{aligned} F_3 &= \frac{kQ_1Q_3}{r^2} \\ &= \frac{(9,0\times10^{9})(4\times10^{-9})(3\times10^{-9})}{(3\times10^{-2})^2} \\ &= \frac{(9,0\times10^{9})(4\times10^{-9})(3\times10^{-9})}{(9\times10^{-4})} \\ &= 1,199\times10^{-4}\ N \end{aligned}$$

Step 5: Vector addition of forces

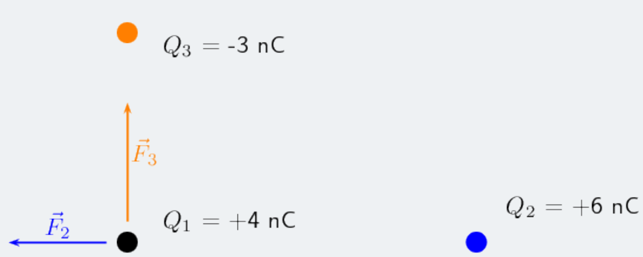
This is a two-dimensional problem involving vectors. We have already solved many two-dimensional force problems and will use precisely the **same procedure** as before. Determine the vectors on the Cartesian plane, break them into components in the x- and y-directions, and then sum components in each direction to get the components of the resultant.

We choose the positive directions to be to the right (the positive x-direction) and up (the positive y-direction). We know the magnitudes of the forces but we need to use the signs of the charges to determine whether the forces are repulsive or attractive. Then we can use a diagram to determine the directions.

The force between Q_1 and Q_2 is repulsive (like charges). This means that it pushes Q_1 to the left, or in the negative x-direction.

The force between Q_1 and Q_3 is attractive (unlike charges) and pulls Q_1 in the positive y-direction.

We can redraw the diagram as a free-body diagram illustrating the forces to make sure we can visualise the situation:



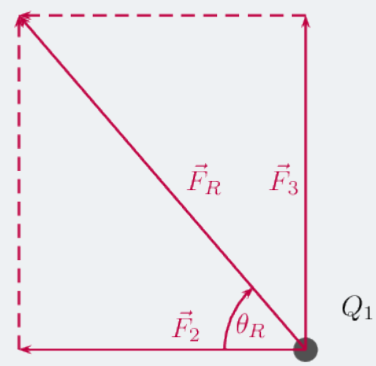
WORKED EXAMPLE 4: COULOMB'S LAW (continued)

Step 6: Resultant force

The magnitude of the resultant force acting on Q_1 can be calculated from the forces using Pythagoras' theorem because there are only two forces and they act in the x- and y-directions:

$$F_R^2 = F_2^2 + F_2^3 \text{ by Pythagoras' theorem}$$
$$F_R = \sqrt{(8,630\times10^{-5})^2 + (1,199\times10^{-4})^2}$$
$$F_R = 1,48\times10^{-4}\ N$$

and the angle, θ_R made with the x-axis can be found using trigonometry.



$$tan(\theta_R) = \frac{y - component}{x - component}$$
$$tan(\theta_R) = \frac{1,199\times10^{-4}}{8,630\times10^{-5}}$$
$$\theta_R = tan^1\left(\frac{1,199\times10^{-4}}{8,630\times10^{-5}}\right)$$
$$\theta_R = 54,25^\circ \text{ to } 2 \text{ decimal places}$$

The final resultant force acting on Q_1 is $1,48\times10^{-4}\ N$ acting at an angle of $54,25^\circ$ to the negative x-axis or $125,75^\circ$to the positive x-axis.

We mentioned in grade 10 that charge placed on a spherical conductor spreads evenly along the surface. As a result, if we are far enough from the charged sphere, electrostatically, it behaves as a point-like charge. Thus we can treat spherical conductors (e.g. metallic balls) as point-like charges, with all the charge acting at the centre.

3 ELECTRIC FIELD

We have seen in the previous section that point charges exert forces on each other even when they are far apart and not touching each other. How do the charges 'know' about the existence of other charges around them?

The answer is that you can think of every charge as being surrounded in space by an electric field. The electric field is the region of space in which an electric charge will experience a force. The direction of the electric field represents the direction of the force a positive test charge would experience if placed in the electric field. In other words, the direction of an electric field at a point in space is the same direction in which a positive test charge would move if placed at that point.

DEFINITION

Electric field

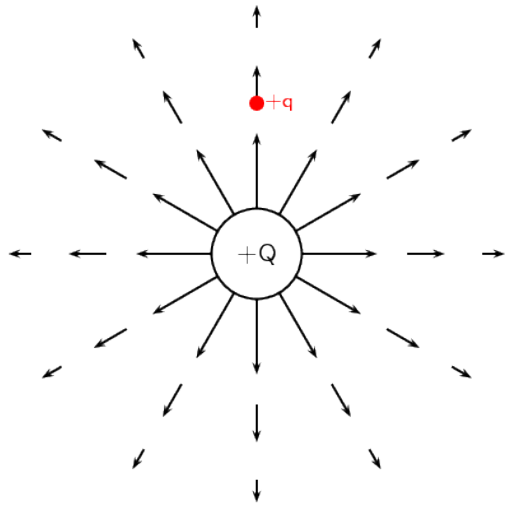
A region of space in which an electric charge will experience a force. The direction of the field at a point in space is the direction in which a positive test charge would moved if placed at that point.

3.1 Representing electric fields

We can represent the strength and direction of an electric field at a point using **electric field lines**. This is similar to representing magnetic fields around magnets using magnetic field lines as you studied in Grade 10. In the following we will study what the electric fields look like around isolated charges.

Positive charge acting on a test charge

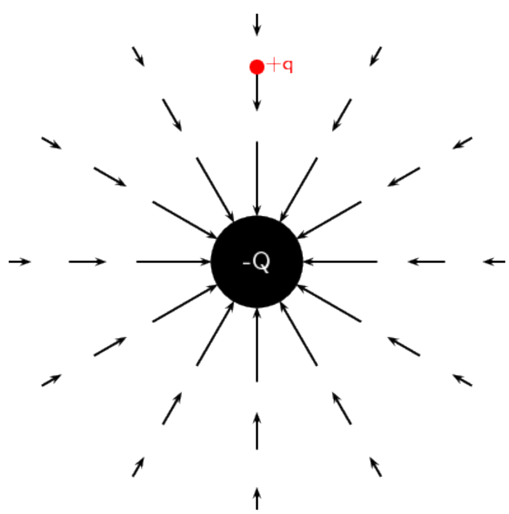
The magnitude of the force that a test charge experiences due to another charge is governed by Coulomb's law. In the diagram below, at each point around the positive charge, $+Q$, we calculate the force a positive test charge, $+q$, would experience, and represent this force (a vector) with an arrow. The force vectors for some points around $+Q$ are shown in the diagram along with the positive test charge $+q$ (in red) located at one of the points.



At every point around the charge $+Q$, the positive test charge, $+q$, will experience a force pushing it away. This is because both charges are positive and so they repel each other. We cannot draw an arrow at every point but we include enough arrows to illustrate what the field would look like. The arrows represent the force the test charge would experience at each point. Coulomb's law is an inverse-square law which means that the force gets weaker the greater the distance between the two charges. This is why the arrows get shorter further away from $+Q$.

Negative charge acting on a test charge

For a negative charge, –Q, and a positive test charge, $+q$, the force vectors would look like:

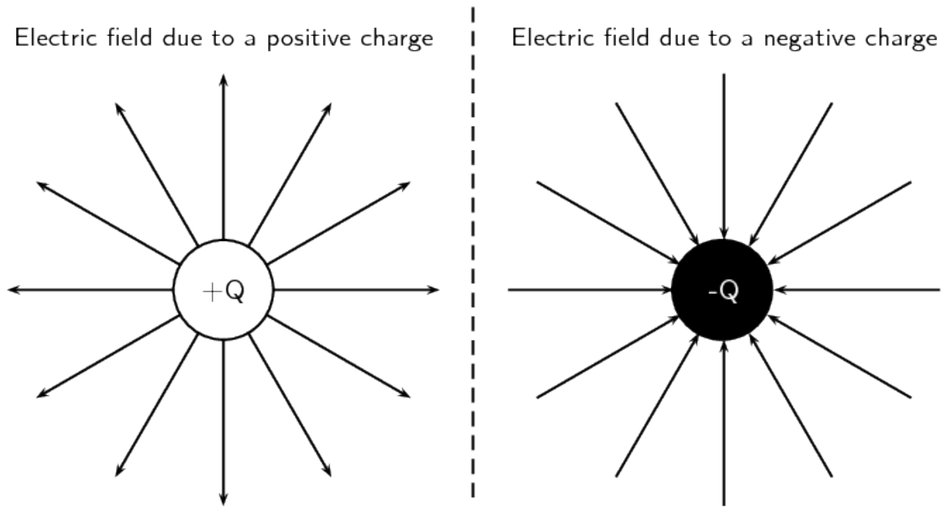


Notice that it is almost identical to the positive charge case. The arrows are the same lengths as in the previous diagram because the absolute magnitude of the charge is the same and so is the magnitude of the test charge.

Thus the magnitude of the force is the same at the same points in space. However, the arrows point in the opposite direction because the charges now have opposite signs and attract each other.

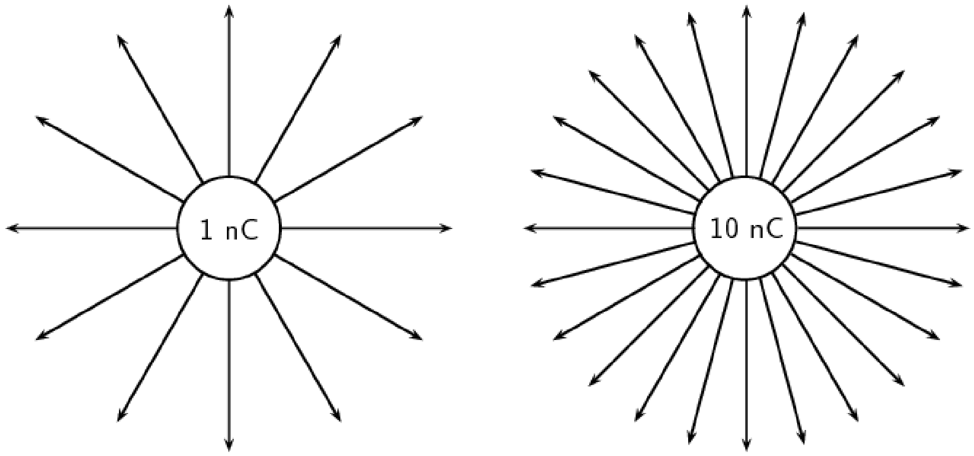
Electric fields around isolated charges - summary

Now, to make things simpler, we draw continuous lines that are tangential to the force that a test charge would experience at each point. The field lines are closer together where the field is stronger. Look at the diagram below: close to the central charges, the field lines are close together. This is where the electric field is strongest. Further away from the central charges where the electric field is weaker, the field lines are more spread out from each other.



We use the following conventions when drawing electric field lines:

- Arrows on the field lines indicate the direction of the field, i.e. the direction in which a positive test charge would move if placed in the field.
- Electric field lines point away from positive charges (like charges repel) and towards negative charges (unlike charges attract).
- Field lines are drawn closer together where the field is stronger.
- Field lines do not touch or cross each other.
- Field lines are drawn perpendicular to a charge or charged surface.
- The greater the magnitude of the charge, the stronger its electric field. We represent this by drawing more field lines around the greater charge than for charges with smaller magnitudes.



Some important points to remember about electric fields:

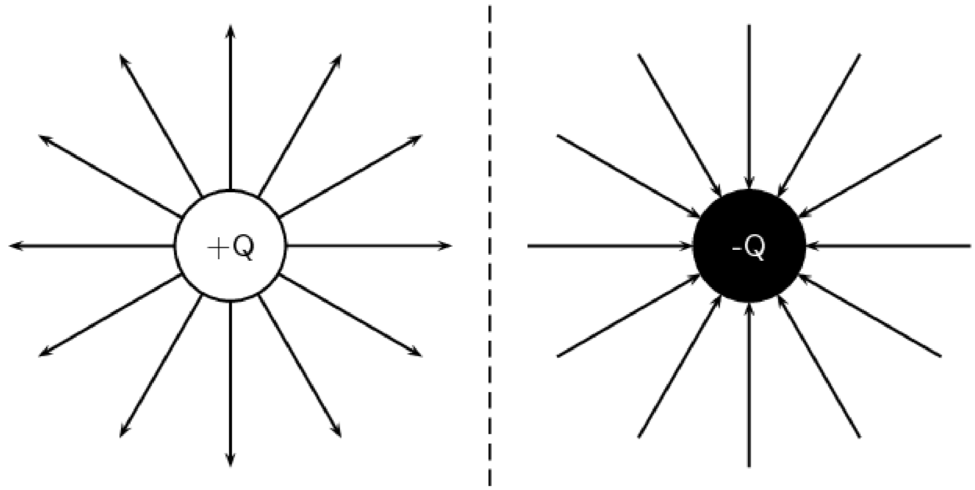
- There is an electric field at **every point** in space surrounding a charge.
- Field lines are merely a **representation** – they are not real. When we draw them, we just pick convenient places to indicate the field in space.
- Field lines exist in three dimensions, not only in two dimension as we've drawn them.
- The number of field lines passing through a surface is proportional to the charge contained inside the surface.

3.2 Electric fields around different charge configurations

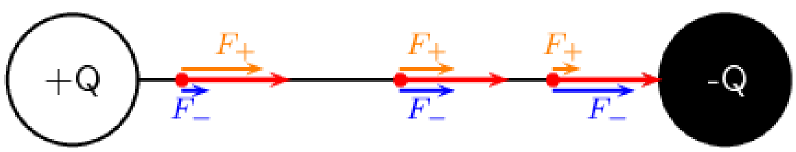
We have seen what the electric fields look like around isolated positive and negative charges. Now we will study what the electric fields look like around combinations of charges placed close together.

Electric field around two unlike charges

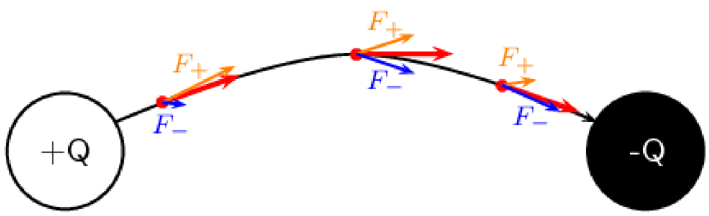
We will start by looking at the electric field around a positive and negative charge placed next to each other. Using the rules for drawing electric field lines, we will sketch the electric field one step at a time. The net resulting field is the sum of the fields from each of the charges. To start off let us sketch the electric fields for each of the charges separately.



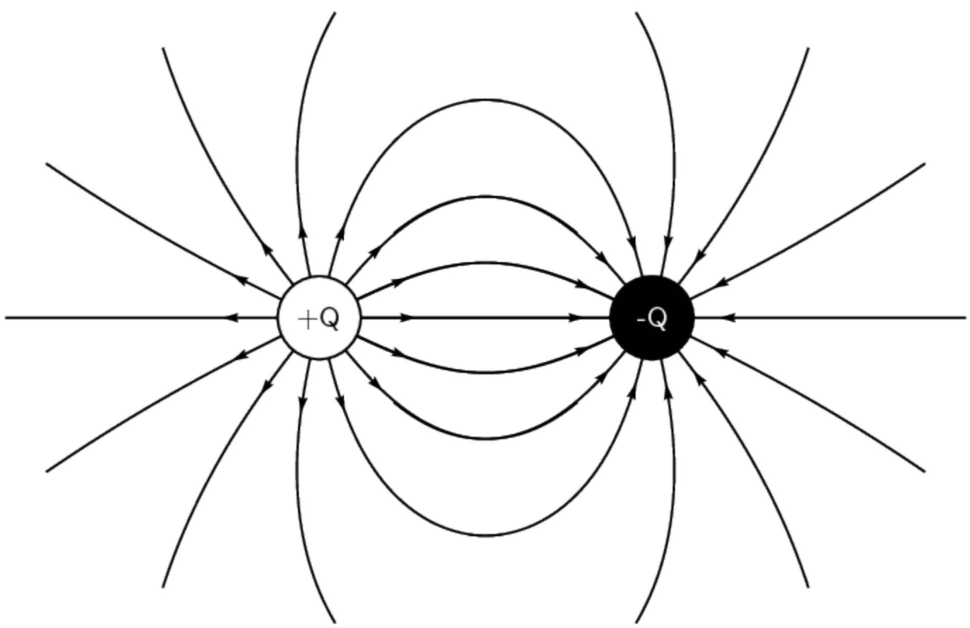
A positive test charge (red dots) placed at different positions directly between the two charges would be pushed away (orange force arrows) from the positive charge and pulled towards (blue force arrows) the negative charge in a straight line. The orange and blue force arrows have been drawn slightly offset from the dots for clarity. In reality they would lie on top of each other. Notice that the further from the positive charge, the smaller the repulsive force, $F+$ (shorter orange arrows) and the closer to the negative charge the greater the attractive force, $F-$ (longer blue arrows). The resultant forces are shown by the red arrows. The electric field line is the black line which is tangential to the resultant forces and is a straight line between the charges pointing from the positive to the negative charge.



Now let's consider a positive test charge placed slightly higher than the line joining the two charges. The test charge will experience a repulsive force ($F+$ in orange) from the positive charge and an attractive force ($F-$ in blue) due to the negative charge. As before, the magnitude of these forces will depend on the distance of the test charge from each of the charges according to Coulomb's law. Starting at a position closer to the positive charge, the test charge will experience a larger repulsive force due to the positive charge and a weaker attractive force from the negative charge. At a position half-way between the positive and negative charges, the magnitudes of the repulsive and attractive forces are the same. If the test charge is placed closer to the negative charge, then the attractive force will be greater and the repulsive force it experiences due to the more distant positive charge will be weaker. At each point we add the forces due to the positive and negative charges to find the resultant force on the test charge (shown by the red arrows). The resulting electric field line, which is tangential to the resultant force vectors, will be a curve.

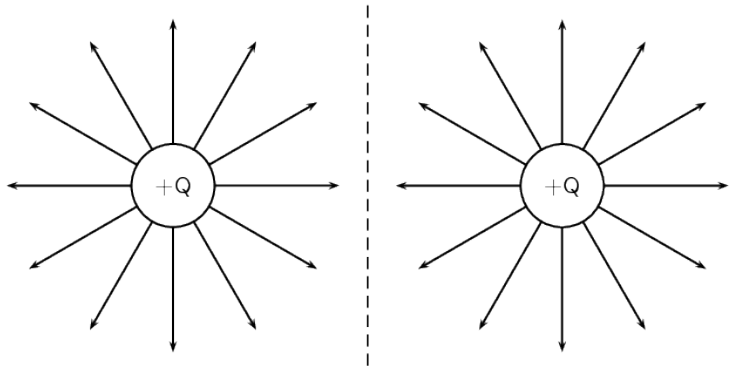


Now we can fill in the other field lines quite easily using the same ideas. The electric field lines look like:

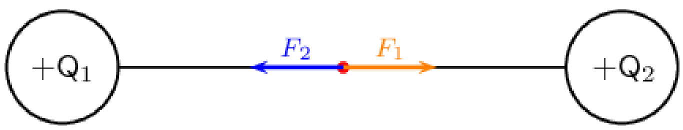


Electric field around two like charges (both positive)

For the case of two positive charges Q_1 and Q_2 of the same magnitude, things look a little different. We can't just turn the arrows around the way we did before. In this case the positive test charge is repelled by both charges. The electric fields around each of the charges in isolation looks like.

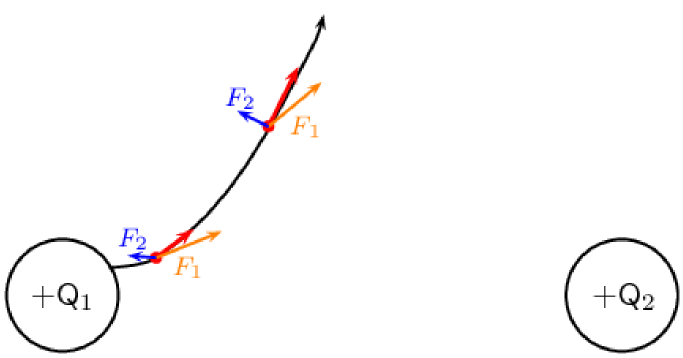


Now we can look at the resulting electric field when the charges are placed next to each other. Let us start by placing a positive test charge directly between the two charges. We can draw the forces exerted on the test charge due to Q_1 and Q_2 and determine the resultant force.

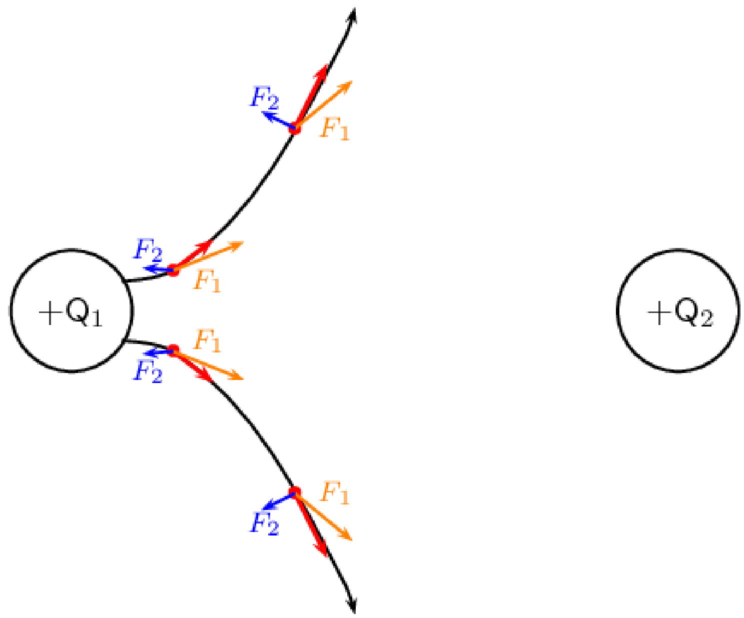


The force F_1 (in orange) on the test charge (red dot) due to the charge Q_1 is equal in magnitude but opposite in direction to F_2 (in blue) which is the force exerted on the test charge due to Q_2. Therefore they cancel each other out and there is no resultant force. This means that the electric field directly between the charges cancels out in the middle. A test charge placed at this point would not experience a force.

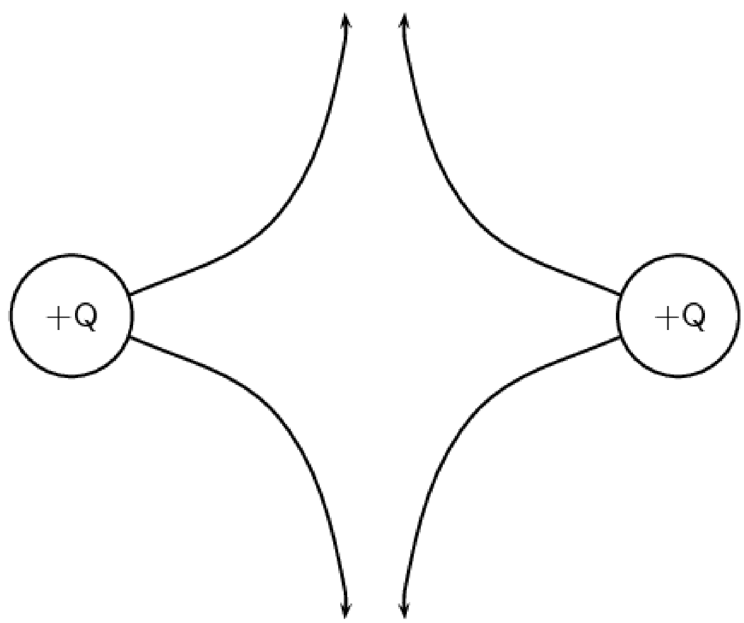
Now let's consider a positive test charge placed close to Q_1 and above the imaginary line joining the centres of the charges. Again we can draw the forces exerted on the test charge due to Q_1 and Q_2 and sum them to find the resultant force (shown in red). This tells us the direction of the electric field line at each point. The electric field line (black line) is tangential to the resultant forces.



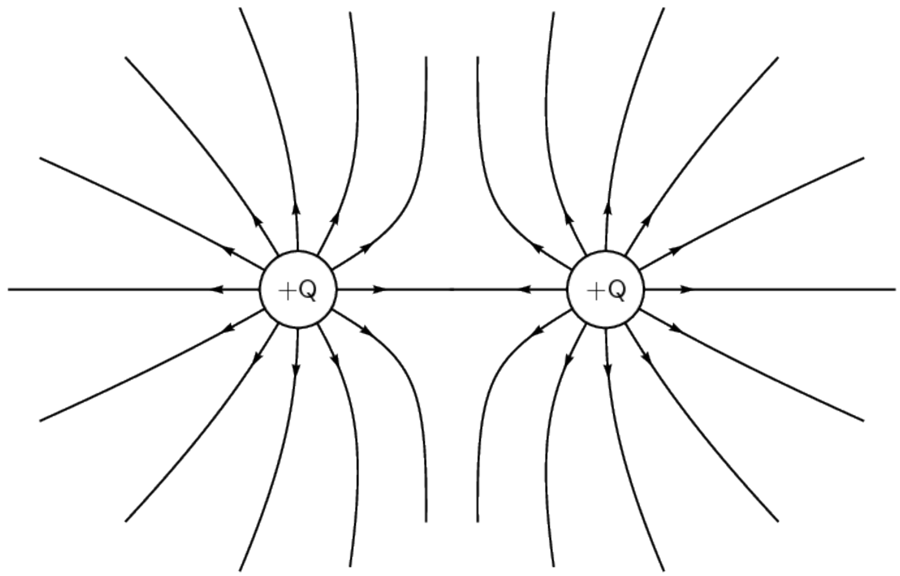
If we place a test charge in the same relative positions but *below* the imaginary line joining the centres of the charges, we can see in the diagram below that the resultant forces are reflections of the forces above. Therefore, the electric field line is just a reflection of the field line above.



Since Q_2 has the same charge as Q_1, the forces at the same relative points close to Q_2 will have the same magnitudes but opposite directions i.e. they are also reflections . We can therefore easily draw the next two field lines as follows:

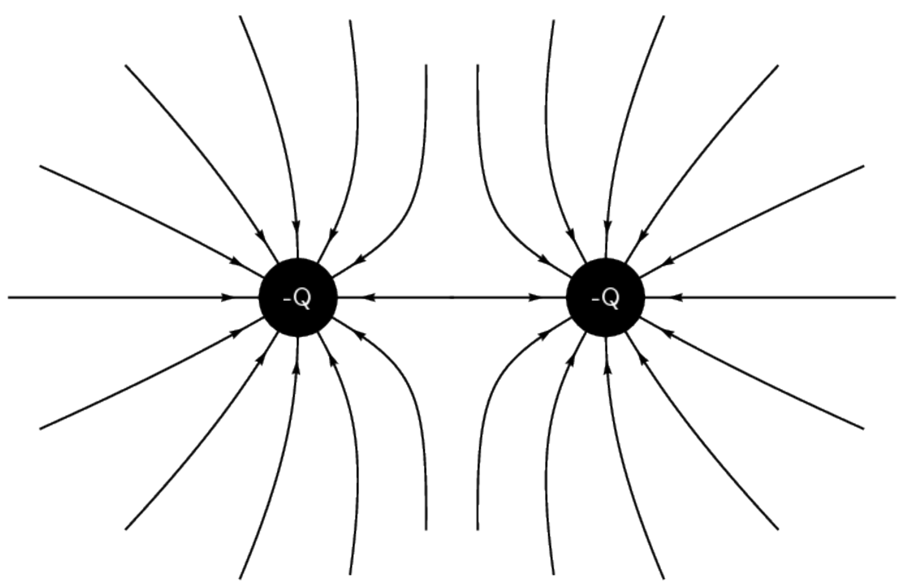


Working through a number of possible starting points for the test charge we can show the electric field can be represented by:



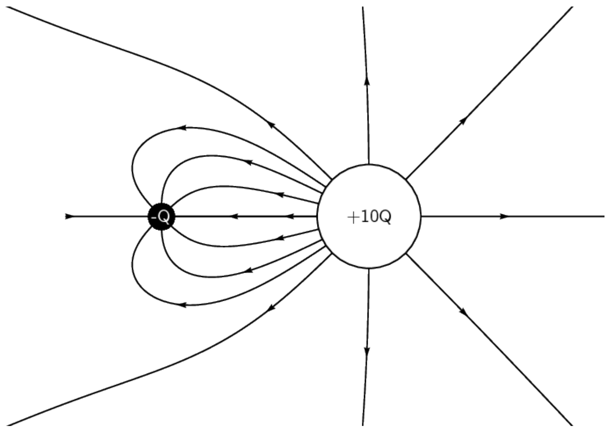
Electric field around two like charges (both negative)

We can use the fact that the direction of the force is reversed for a test charge if you change the sign of the charge that is influencing it. If we change to the case where both charges are negative we get the following result:



Charges of different magnitudes

When the magnitudes are not equal the larger charge will influence the direction of the field lines more than if they were equal. For example, here is a configuration where the positive charge is much larger than the negative charge. You can see that the field lines look more similar to that of an isolated charge at greater distances than in the earlier example. This is because the larger charge gives rise to a stronger field and therefore makes a larger relative contribution to the force on a test charge than the smaller charge.



3.3 Electric field strength

In the previous sections we have studied how we can represent the electric fields around a charge or combination of charges by means of electric field lines. In this representation we see that the electric field strength is represented by how close together the field lines are. In addition to the drawings of the electric field, we would also like to be able to quantify (put a number to) how strong an electric field is and what its direction is at any point in space.

A small test charge q placed near a charge Q will experience a force due to the electric field surrounding Q. The magnitude of the force is described by Coulomb's law and depends on the magnitude of the charge Q and the distance of the test charge from Q. The closer the test charge q is to the charge Q, the greater the force it will experience. Also, at points closer to the charge Q, the stronger is its electric field. We define the electric field at a point as the force per unit charge.

DEFINITION

Electric field The magnitude of the electric field, E, at a point can be quantified as the force per unit charge We can write this as:

$$E = \frac{F}{q}$$

where F is the Coulomb force exerted by a charge on a test charge q.

The units of the electric field are newtons per coulomb: $N \cdot C^{-1}$.

Since the force F is a vector and q is a scalar, the electric field, E, is also a vector; it has a magnitude and a direction at every point.

Given the definition of electric field above and substituting the expression for Coulomb's law for F:

$$E = \frac{F}{q}$$
$$E = \frac{kQq}{r^2 q}$$
$$E = \frac{kQ}{r^2}$$

we can see that the electric field E only depends on the charge Q and not the magnitude of the test charge.

If the electric field is known, then the electrostatic force on any charge q placed into the field is simply obtained by rearranging the definition equation:

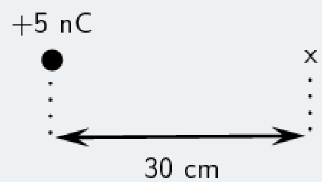
$$F = qE$$

.

WORKED EXAMPLE 5: ELECTRIC FIELD 1

QUESTION

Calculate the electric field strength $30\ cm$ from a $5\ nC$ charge.



SOLUTION

Step1: Determine what is required

We need to calculate the electric field a distance from a given charge.

Step 2: Determine what is given

We are given the magnitude of the charge and the distance from the charge.

Step 3: Determine how to approach the problem

We will use the equation: $E = \frac{kQ}{r^2}$.

Step 4: Solve the problem

$$\begin{aligned} E &= \frac{kQ}{r^2} \\ &= \frac{(9,0\times10^9)(5\times10^{-9})}{0,3}^2 \\ &= 4,99\times10^2\ N \cdot C{-}1 \end{aligned}$$

WORKED EXAMPLE 6: ELECTRIC FIELD 2

QUESTION

Two charges of $Q_1 = +3nC$ and $Q_2 = -4nC$ are separated by a distance of $50\ cm$. What is the electric field strength at a point that is $20\ cm$ from Q_1 and $50\ cm$ from Q_2? The point lies between Q_1 and Q_2.

SOLUTION

Step1: Determine what is required

We need to calculate the electric field a distance from two given charge.

Step 2: Determine what is given

We are given the magnitude of the charges and the distances from the charges.

Step 3: Determine how to approach the problem

We will use the equation: $E = \frac{kQ}{r^2}$.

We need to calculate the electric field for each charge separately and then add them to determine the resultant field.

Step 4: Solve the problem

We first solve for Q_1:

$$\begin{aligned} E &= \frac{kQ}{r^2} \\ &= \frac{(9,0\times10^9)(3\times10^{-9})}{0,2^2} \\ &= 6,74\times10^2\ N \cdot C^{-1} \end{aligned}$$

Then for Q2:

$$\begin{aligned} E &= \frac{kQ}{r^2} \\ &= \frac{(9,0\times10^9)(4\times10^{-9})}{0,3^2} \\ &= 3,99\times10^2\ N \cdot C^{-1} \end{aligned}$$

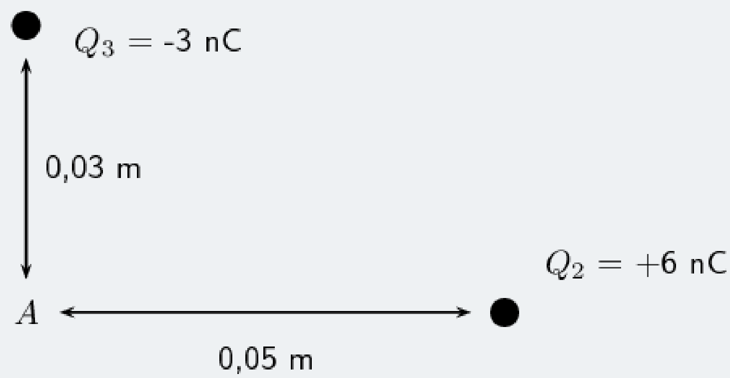
We need to add the two electric fields because both are in the same direction. The field is away from Q_1 and towards Q_2. Therefore,

$$E_{total} = 6,74\times10^2 + 3,99\times10^2 = 1,08\times10^2\ N \cdot C{-1}$$

WORKED EXAMPLE 7: TWO-DIMENSIONAL ELECTRIC FIELDS

QUESTION

Two point charges form a right-angled triangle with the point A at the origin. Their charges are $Q_2 = 6\times10^{-9}\ C = 6\ nC$ and $Q_3 = -3\times10^{-9}\ C = -3\ nC$. The distance between A and Q_2 is $5\times10^{-2}\ m$ and the distance between A and Q_3 is $3\times10^{-2}\ m$. What is the net electric field measured at A from the two charges if they are arranged as shown?



SOLUTION

Step1: Determine what is required

We are required to calculate the net electric field at A. This field is the sum of the two electric fields - the field from Q_2 at A and from Q_3 at A.

Step 2: Determine how to approach the problem

- We need to calculate the two fields at A, using $E = \frac{kQ}{r^2}$ for the magnitude and determining the direction from the charge signs.
- We then need to add up the two fields using our rules for adding vector quantities, because the electric field is a vector quantity.

Step3: Determine what is given

We are given all the charges and the distances.

WORKED EXAMPLE 7: TWO-DIMENSIONAL ELECTRIC FIELDS (continued)

Step 4: Calculate the magnitude of the fields.

The magnitude of the field from Q_2 at A, which we will call E_2, is:

$$\begin{aligned} E_2 &= \frac{kQ_2}{r^2} \\ &= \frac{(9,0\times10^9)(6\times10^{-9})}{(5\times10^{-2})^2} \\ &= \frac{(9,0\times10^9)(6\times10^{-9})}{(25\times10^{-4})} \\ &= 2,158\times10^4\ N\cdot C^{-1} \end{aligned}$$

The magnitude of the electric field from Q_3 at A, which we will call E_3, is:

$$\begin{aligned} E_3 &= \frac{kQ_3}{r^2} \\ &= \frac{(9,0\times10^9)(3\times10^{-9})}{(3\times10^{-2})^2} \\ &= \frac{(9,0\times10^9)(3\times10^{-9})}{(9\times10^{-4})} \\ &= 2,997\times10^4\ N\cdot C^{-1} \end{aligned}$$

Step 5: Vector addition of electric fields

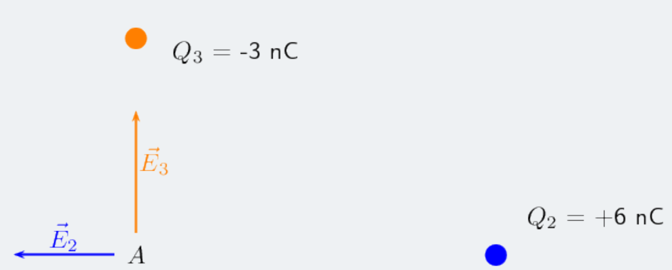
We will use precisely the **same procedure** as before. Determine the vectors on the Cartesian plane, break them into components in the x- and y-directions, sum components in each direction to get the components of the resultant.

We choose the positive directions to be to the right (the positive x-direction) and up (the positive y-direction). We know the electric field magnitudes but we need to use the charges to determine the direction. Then we can use the diagram to determine the directions.

The force between a positive test charge and Q_2 is repulsive (like charges). This means that the electric field is to the left, or in the negative x-direction.

The force between a positive test charge and Q_3 is attractive (unlike charges) and the electric field will be in the positive y-direction.

We can redraw the diagram illustrating the fields to make sure we can visualise the situation:



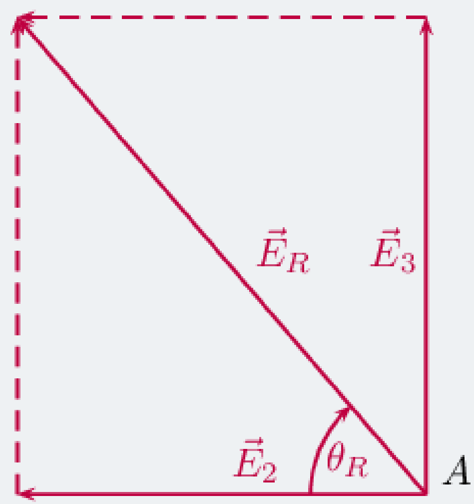
WORKED EXAMPLE 7: TWO-DIMENSIONAL ELECTRIC FIELDS (continued)

Step 6: Resultant force

The magnitude of the resultant force acting on Q_1 can be calculated from the forces using Pythagoras' theorem because there are only two forces and they act in the x- and y-directions:

$$E_R^2 = E_2^2 + E_3^2 \text{ Pythagoras' theorem}$$
$$E_R = \sqrt{(2,158\times10^4)^2 + (2,997\times10^4)^2}$$
$$E_R = 3,693\times10^4 \; N \cdot C^{-1}$$

and the angle, θ_R made with the x-axis can be found using trigonometry.



$$tan(\theta_R) = \frac{y - component}{x - component}$$
$$tan(\theta_R) = \frac{2,997\times10^4}{2,158\times10^4}$$
$$\theta_R = tan^{-1}\left(\frac{2,997\times10^4}{2,158\times10^4}\right)$$
$$\theta_R = 54,24°$$

The final resultant electric field acting at A is $3,693\times10^4 \; N \cdot C^{-1}$ acting at $54,24°$ to the negative x-axis or $125,76°$ to the positive x-axis.

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4 CHAPTER SUMMARY

- Objects can be **positively, negatively** charged or **neutral**.
- Coulomb's law describes the electrostatic force between two point charges and can be stated as: the magnitude of the electrostatic force between two point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.

$$F = k\frac{Q_1 Q_2}{r_2}$$

- An electric field is a region of space in which an electric charge will experience a force. The direction of the field at a point in space is the direction in which a positive test charge would moved if placed at that point.
- We can represent the electric field using field lines. By convention electric field lines point away from positive charges (like charges repel) and towards negative charges (unlike charges attract).
- The magnitude of the electric field, E, at a point can be quantified as the force per unit charge We can write this as:

$$E = \frac{F}{q}$$

  where F is the Coulomb force exerted by a charge on a test charge q. The units of the electric field are newtons per coulomb: $N \cdot C^{-1}$.
- The electric field due to a point charge Q is defined as the force per unit charge:

$$E = \frac{F}{q} = \frac{kQ}{r^2}$$

- The electrostatic force is attractive for unlike charges and repulsive for like charges.

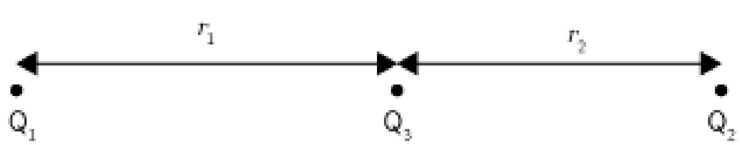
Physical Properties		
Quantity	**Unit name**	**Uni symbol**
Charge (q)	Coulomb	C
Distance (r)	meters	m
Electric field (E)	Newtons per Coulomb	$N \cdot C^{-1}$
Force (F)	Newtons	N

Table 1: Units used in **electrostatics**

5 EXERCISES

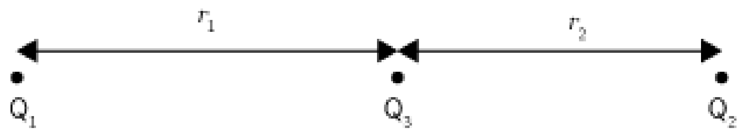
5.1 Exercise 1

1. Calculate the electrostatic force between two charges of $+6$nC and $+1$nC if they are separated by a distance of 2 mm
2. What is the magnitude of the repulsive force between two pith balls (a pith ball is a small, light ball that can easily be charged) that are 8 cm apart and have equal charges of -30nC?
3. How strong is the attractive force between a glass rod with a $0,7\mu$C charge and a silk cloth with a $-0,6\mu$C charge, which are 12 cm apart, using the approximation that they act like point charges?
4. Two point charges exert a 5 N force on each other. What will the magnitude of the resulting force be if the distance between them is increased by a factor of three?
5. Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased?
6. If two equal charges each of 1 C each are separated in air by a distance of 1 km, what is the magnitude of the force acting between them?
7. Calculate the distance between two charges of $+4$ nC and -3 nC if the electrostatic force between them is $0,005$ N.
8. For the charge configuration shown, calculate the resultant force on Q_2 if:
   - $Q_1 = 2,3 \times 10^{-7}$ C
   - $Q_2 = 4 \times 10^{-6}$ C
   - $Q_3 = 3,3 \times 10^{-7}$ C
   - $r_1 = 2,5 \times 10^{-1}$ m
   - $r_2 = 3,7 \times 10^{-2}$ m

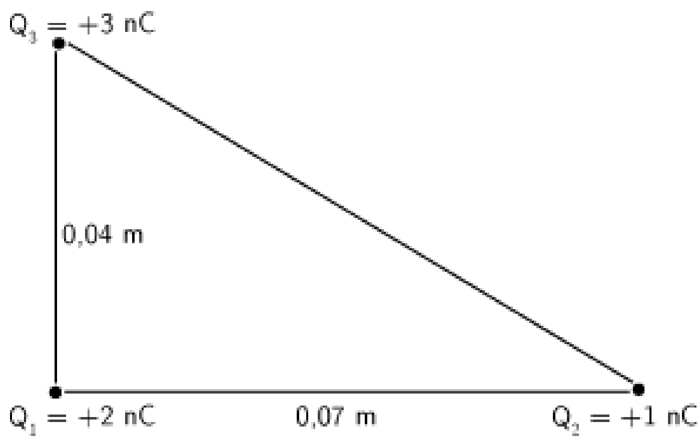


9. For the charge configuration shown, calculate the charge on Q_3 if the resultant force on Q_2 is $6,3 \times 10^{-1}$ N to the right and:

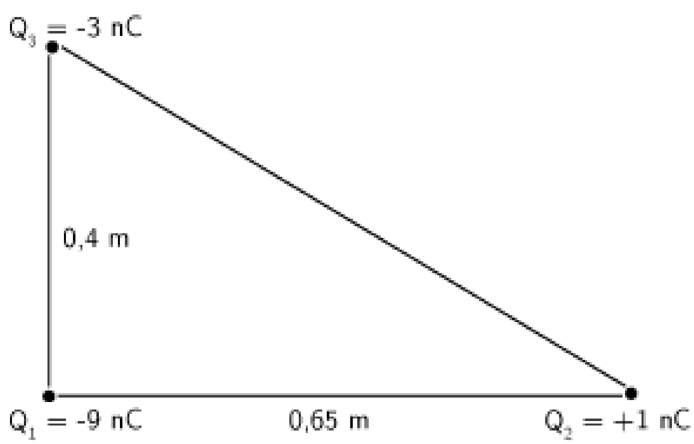
- $Q_1 = 4,36 \times 10^{-6}$ C
- $Q_2 = -7 \times 10^{-7}$ C
- $r_1 = 1,85 \times 10^{-1}$ m
- $r_2 = 4,7 \times 10^{-2}$ m



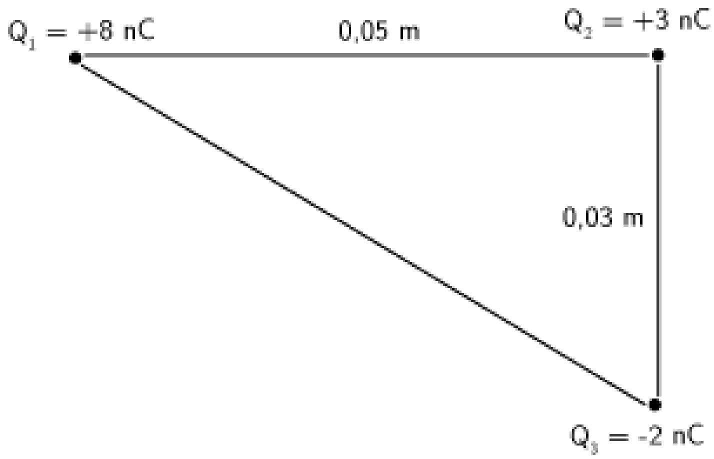
10. Calculate the magnitude of the resultant force on Q_1 given this charge configuration:



11. Calculate the magnitude of the resultant force on Q_1 given this charge configuration:

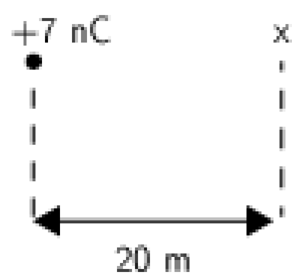


12. Calculate the resultant force on Q_2 given this charge configuration:



5.2 Exercise 2

1. Calculate the electric field strength 20 m from a 7 nC charge.



2. Two charges of $Q_1 = -6$ pC and $Q_2 = -8$ pC are separated by a distance of 3 km. What is the electric field strength at a point that is 2 km from Q_1 and 1 km from Q_2? The point lies between Q_1 and Q_2.

6 ANSWERS TO EXERCISES

6.1 Exercise 1

1. $1{,}35 \times 10^{-2}$ N This force is repulsive since it is between two like charges.
2. $1{,}27 \times 10^{-3}$ N
3. 0,26 N This force is attractive since it is between two unlike charges.
4. $F_{e2} =$ 0,56 N
5. 0,2
6. 9000 N
7. $4{,}6 \times 10^{-3}$ m
8. 8,77 N to the right.
9. -4×10^{-7} C
10. $3,42 \times 10^{-5}$ N
11. $1,52 \times 10^{-6}$ N
12. $1{,}05 \times 10^{-4}$ N, 34,78°

6.2 Exercise 2

1. $0,15$ N · C^{-1}
2. $-5{,}8 \times 10^{-8}$ N · C^{-1} in the direction of the -8 pC charge.